Static Moment Method Solution for a Microstrip in an Infinite Dielectric

Keith W. Whites
Last Modified on January 29, 2004

- The microstrip substrate and the infinite space above it are assumed to be the same material having relative permittivity \( \varepsilon_{\text{rbackgnd}} \). Using pulse expansion and point matching for the MM solution with \( \text{numcells} \) segments of width \( \Delta \) uniformly distributed across the infinitely-thin strip, which has a width-over-separation ratio \( \text{Woverd} \). The voltage of the strip conductor is \( \text{Vapplied} \) volts with respect to the ground plane.

- Revisions:
  - 10/1/03: First completed.

<< Graphics`Graphics`

Enter the geometrical parameters and the applied voltage.

```
ClearAll[Woverd, \( \varepsilon_{\text{rbackgnd}} \), Vapplied, d, width, \( \Delta \),
  numcells, c0, \( \varepsilon_0 \), \( \Delta x \), m, n, Zmn, \( \varepsilon_r \), Zmatrix, Vvector, \( \alpha \), \( \alpha_0 \),
  CPUL, COPUL, \( Z_0 \), \( \varepsilon_{\text{reff}} \), ee, \( \text{Wod} \), \( Z_0\text{approx} \), plot1, plot2]

Woverd := 5. ;
\( \varepsilon_{\text{rbackgnd}} \) = 1. ;
Vapplied = 1. ;
```
Compute the moment method solution. "\(\alpha\)" and "\(\alpha_0\)" are the line charge density coefficients when the background relative permittivity is \(\varepsilon_{\text{backgnd}}\) and 1, respectively. Without loss of generality, assume a spacing of \(d=1\) m between the microstrip and the ground plane and compute width for the chosen \(\text{Woverd}\).

\[
d = 1. \\
\text{width} := \text{Woverd} \times d \\
\Delta := \text{width} / \text{numcells}
\]

\[
c_0 = 2.998 \times 10^8 \\
\varepsilon_0 = 8.854 \times 10^{-12}
\]

\[
\Delta x[m_-, n_] := \Delta \ast (m-n)
\]

\[
\text{Zmn}[m_-, n_, \varepsilon_r_] := -1 / (4 \pi \varepsilon_0) \times ((\Delta x[m, n] + \Delta / 2) \ast \log((\Delta x[m, n] + \Delta / 2)^2 / ((\Delta x[m, n] + \Delta / 2)^2 + 4 \times d^2) \ast (\Delta x[m, n] - \Delta / 2) \ast \log((\Delta x[m, n] - \Delta / 2)^2 / ((\Delta x[m, n] - \Delta / 2)^2 + 4 \times d^2) \ast 4 \times d \ast (\arctan(2 \times d, \Delta x[m, n] - \Delta / 2))) / m \neq n
\]

\[
\text{Zmn}[m_-, n_, \varepsilon_r_] := \Delta / (2 \pi \varepsilon_0) \ast (1 - \log(\Delta / 2) + 1 / (4 \pi \varepsilon_0) \ast \log(\Delta^2 / 4 + 4 \times d^2) - 2 \ast \Delta + 8 \times d \ast \arctan(\Delta / (4 \times d)))
\]

\[
\text{Zmatrix}[\varepsilon_r_] := \text{Table}[\text{Zmn}[m, n, \varepsilon_r], \{m, \text{numcells}\}, \{n, \text{numcells}\}]
\]

\[
\text{Vvector} := \text{Table}[\text{Vapplied}, \{\text{numcells}\}]
\]

\[
\alpha := \text{LinearSolve}[\text{Zmatrix}[\varepsilon_{\text{backgnd}}], \text{Vvector}]
\]

\[
\alpha_0 := \text{LinearSolve}[\text{Zmatrix}[1], \text{Vvector}]
\]
Choose the number of pulse basis functions "numcells" then compute the capacitance per unit length "CPUL" assuming a background relative permittivity "erbackgnd." The characteristic impedance of the microstrip "Z0" and the effective relative permittivity "erreff" of a TEM wave propagating on this microstrip are computed from both CPUL as well as "C0PUL," which is the capacitance per unit length of the microstrip with a background relative permittivity equal to 1.

\begin{align*}
\text{numcells} & := 30 \\
\text{CPUL} & := \text{Total}[\alpha] \ast \Delta \\
\text{C0PUL} & := \text{Total}[\alpha 0] \ast \Delta \\
Z0 & := (c0 \ast \text{Sqrt}[\text{CPUL} \ast \text{C0PUL}])^{-1} \\
\text{erreff} & := \text{CPUL} / \text{C0PUL} \\
N[\text{Woverd}] & \\
N[Z0] & \\
N[\text{erreff}] & \\
5. & \\
49.7729 & \\
1. & 
\end{align*}
Plot the amplitudes of the line charge density coefficients $\alpha$. It can be shown theoretically that the line charge density should approach infinity at the edges of the strip. This is called the "edge effect." In the MM solution, we have not directly incorporated that physical characteristic. Nevertheless, this pulse expansion-point match MM solution has predicted that the line charge density is becoming very large near the edges.

```math
BarChart[\alpha_1, AxesLabel -> {"Basis ", ", \rho_1 \text{ (C/m)}"},
BarSpacing -> -0.15, PlotLabel -> "Microstrip in Infinite Dielectric"]
```

- Graphics -
The variation of $C$, $Z_0$ and $\varepsilon_{\text{reff}}$ are next observed as the number of basis functions is increased. This is called a "convergence study." It is not known a priori how many basis functions are needed in a MM solution to provide an accurate solution. A convergence study should show that the physical quantities are smoothly approaching an asymptote as the number of basis functions increases.

```
ListPlot[Table[{numcells, CPUL}, {numcells, 50}], PlotJoined -> True,
AxesLabel -> {"# basis", "C [F/m]"}, PlotLabel -> "Infinite Dielectric",
PlotRange -> All, PlotStyle -> RGBColor[1, 0, 0]]
```
\textbf{ListPlot[\{\text{numcells, } Z0\}, \{\text{numcells, 50}\}], PlotJoined \to \text{True,} \\
\text{AxesLabel} \to \{"\# basis", "Z0 [\Omega]\}, PlotLabel \to \text{"Infinite Dielectric",} \\
\text{PlotRange} \to \text{All, PlotStyle} \to \text{RGBColor[1, 0, 0]}

- Graphics -

\textbf{ListPlot[\{\text{numcells, } \varepsilon_{reff}\}, \{\text{numcells, 50}\}], PlotJoined \to \text{True,} \\
\text{AxesLabel} \to \{"\# basis", "\varepsilon_{reff}\}, PlotLabel \to \text{"Infinite Dielectric",} \\
\text{PlotRange} \to \{0, \varepsilon_{\text{backgnd}} + 1\}, PlotStyle \to \text{RGBColor[1, 0, 0]}

- Graphics -

\textbf{\textcolor{red}{\textbf{\textbullet}}} \textbf{ Lastly, plot and list } Z0 \textbf{ from the MM solution as a function of } W/d \textbf{ and compare with the approximate solution given in (3.196) of Pozar, "Microwave Engineering." This approximate expression was presumably obtained by researchers curve fitting}
numerically accurate results, such as those from a MM solution like this one. These two solutions should be in close agreement only for $\varepsilon_{\text{backgnd}}=1$.

- Approximate formula for the characteristic impedance of a quasi-static microstrip from (3.196) in Pozar.

\[
Z_0\approx_{\text{Approx}} (\varepsilon + 1) / 2 + (\varepsilon - 1) / 2 * 1 / \sqrt{1 + 12 / W_d}
\]

\[
Z_0\approx_{\text{Approx}} (\varepsilon + 1.393 + 0.667 * \log(W_d + 1.444)) / W_d
\]

\[
Z_0\approx_{\text{Approx}} (8 / W_d + \varepsilon_{\text{backgnd}}) * (W_d + 1.393 + 0.667 * \log(W_d + 1.444)) / W_d
\]

- Plot the results of both methods.

```mathematica
ee[Wod_, \varepsilon_] := (\varepsilon + 1) / 2 + (\varepsilon - 1) / 2 * 1 / Sqrt[1 + 12 / Wod]
Z0approx[Wod_, \varepsilon_] :=
  60 / Sqrt[ee[Wod, \varepsilon]] * Log[8 / Wod + Wod / 4] /; Wod \leq 1
Z0approx[Wod_, \varepsilon_] :=
  120 * Pi / (Sqrt[ee[Wod, \varepsilon]] * (Wod + 1.393 + 0.667 * Log[Wod + 1.444])) /;
  Wod > 1

numcells := 50
plot1 := ListPlot[Table[{Woverd, Z0}, {Woverd, 0.1, 10, 0.2}],
  AxesLabel -> {"W/d", "Z0 [\Omega]"}, PlotJoined -> True,
  PlotLabel -> "Moment Method/Infinite Dielectric",
  PlotStyle -> RGBColor[1, 0, 0]] ;
plot2 := ListPlot[Table[{Woverd, Z0approx[Woverd, \varepsilon_{\text{backgnd}}]}],
  {Woverd, 0.1, 10, 0.2}], AxesLabel -> {"W/d", "Z0approx [\Omega]"},
  PlotJoined -> True, PlotLabel -> "Approximate Formula",
  PlotStyle -> RGBColor[0, 0, 1]] ;
Show[{plot1, plot2}, AxesLabel -> {"W/d", "Z0 [\Omega]"}, PlotLabel -> "Both"]
```
numcells := 50

TableForm@Table[
  {Woverd, Z0, Z0approx[{Woverd, εrbackgnd}]], {Woverd, 0.2, 10, 0.4}],
  TableHeadings -> {None, {"W/d", "Z0 [Ω]", "Z0approx [Ω]"}}

<table>
<thead>
<tr>
<th>W/d</th>
<th>Z0 [Ω]</th>
<th>Z0approx [Ω]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>221.672</td>
<td>221.408</td>
</tr>
<tr>
<td>0.6</td>
<td>156.375</td>
<td>156.087</td>
</tr>
<tr>
<td>1</td>
<td>126.82</td>
<td>126.613</td>
</tr>
<tr>
<td>1.4</td>
<td>108.104</td>
<td>108.016</td>
</tr>
<tr>
<td>1.8</td>
<td>94.7633</td>
<td>94.7707</td>
</tr>
<tr>
<td>2.2</td>
<td>84.6318</td>
<td>84.6128</td>
</tr>
<tr>
<td>2.6</td>
<td>76.6168</td>
<td>76.5471</td>
</tr>
<tr>
<td>3</td>
<td>70.0886</td>
<td>69.9704</td>
</tr>
<tr>
<td>3.4</td>
<td>64.6522</td>
<td>64.4942</td>
</tr>
<tr>
<td>3.8</td>
<td>60.045</td>
<td>59.8562</td>
</tr>
<tr>
<td>4.2</td>
<td>56.0844</td>
<td>55.8728</td>
</tr>
<tr>
<td>4.6</td>
<td>52.6389</td>
<td>52.411</td>
</tr>
<tr>
<td>5</td>
<td>49.6112</td>
<td>49.372</td>
</tr>
<tr>
<td>5.4</td>
<td>46.9275</td>
<td>46.6811</td>
</tr>
<tr>
<td>5.8</td>
<td>44.5309</td>
<td>44.2801</td>
</tr>
<tr>
<td>6.2</td>
<td>42.3765</td>
<td>42.1237</td>
</tr>
<tr>
<td>6.6</td>
<td>40.4284</td>
<td>40.1753</td>
</tr>
<tr>
<td>7</td>
<td>38.6576</td>
<td>38.4057</td>
</tr>
<tr>
<td>7.4</td>
<td>37.0406</td>
<td>36.7908</td>
</tr>
<tr>
<td>7.8</td>
<td>35.5577</td>
<td>35.3107</td>
</tr>
<tr>
<td>8.2</td>
<td>34.1925</td>
<td>33.949</td>
</tr>
<tr>
<td>8.6</td>
<td>32.9312</td>
<td>32.6916</td>
</tr>
<tr>
<td>9</td>
<td>31.7624</td>
<td>31.5268</td>
</tr>
<tr>
<td>9.4</td>
<td>30.6758</td>
<td>30.4446</td>
</tr>
<tr>
<td>9.8</td>
<td>29.6631</td>
<td>29.4363</td>
</tr>
</tbody>
</table>