Noninvasive Electrical Characterization of Materials at Microwave Frequencies Using an Open-Ended Coaxial Line: Test of an Improved Calibration Technique

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Abstract—We consider dielectric measurements using a probe consisting of coaxial transmission line with an open-circuit end placed against the sample. For the 2.99 or 3.6 mm (OD) probes considered in this study, a simple lumped parameter model shows errors above 1 GHz that increase greatly with frequency. We evaluate an approximate model due originally to Marcuvitz, on the basis of measured probe impedances from 1 to 18 GHz with samples consisting of water, methanol, and dioxane-water mixtures. This model is more accurate than the lumped-parameter model, and is better suited for calibration of the automated network analyzer. Finally, we consider the errors introduced in dielectric measurements by the use of approximate models for the probe. The technique succeeds because of partial cancellation of errors in modeling the probe in ANA-based measurements.

I. INTRODUCTION

SeVERAL AUTHORS have described the use of open-ended coaxial transmission lines as sensors for measurement of the complex permittivity $\varepsilon$ of materials at microwave frequencies [1]–[6]. The probe (typically a length of the common 3.6 mm OD semirigid line) is placed against the sample and its complex reflection coefficient $S$ is measured with an automated network analyzer (ANA). The ANA is calibrated by measuring the reflection coefficient of the line with known loads, typically open and short circuits and a reference liquid such as water. A good electrical model for the probe is needed, both to calibrate the ANA and to calculate the dielectric properties of the sample from the probe admittance.

In the limit of low frequencies, the admittance $Y$ of the probe is given by a lumped-parameter model:

$$Y = C_0 + j\omega C_f (\varepsilon' - j\varepsilon'')$$  \hspace{1cm} (1)

where $C_0$ and $C_f$ are constants that depend on the dimensions of the probe, $\omega$ is the frequency (rad/s), and $\varepsilon = \varepsilon' - j\varepsilon''$ is the complex permittivity of the sample.

As the frequency is increased, this model fails because of the presence of higher order modes at the aperture and (at still higher frequencies) because of radiation effects. While no rigorous closed-form solution exists, several approximate solutions are available. Marcuvitz expressed the admittance of a probe as an integral over its aperture [7]; this approximation was rederived in equivalent forms by Levine and Papas [8] and Misra [9]. Marcuvitz's approximation can be expanded in a series that is convenient for numerical solution, and in this form has been used in experimental studies employing the probe technique [10]. Alternatively, the probe admittance can be numerically calculated using the mode-matching technique [11]. This method explicitly takes higher order modes into account and is more accurate but too time-consuming for microcomputer-based measurements.

As shown below, Marcuvitz's approximation is only a modest improvement over the lumped-parameter model for the probes considered here. However, for ANA-based measurements it significantly enhances measurement accuracy. The reason, apparently, is a cancellation of errors: errors in calibrating the ANA are partially compensated by subsequent miscalculation of the permittivity from the (mismeasured) admittance. (The ANA is still miscalibrated, however, which might lead to imperfect cancellation of resonance-type artifacts [12].)

II. THEORY

The two equivalent formulations for the admittance of the line are

$$Y = j\frac{k^2 Y_0}{\sigma k_0} \ln \left( \frac{b}{a} \right) \int_0^b \int_0^b \cos \phi' \exp \left( -jkr \right) \frac{dr}{r} dp dp$$  \hspace{1cm} (2)

where $r = (\rho^2 + \rho'^2 - 2\rho\rho' \cos \phi')^{1/2}$ and $Y_0 = 0.02 \Omega$, the
TABLE I
PARAMETERS FOR THE COAXIAL TRANSMISSION LINES USED FOR THIS STUDY

<table>
<thead>
<tr>
<th>Substance</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$r+10^3$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>5.2</td>
<td>78.5</td>
<td>8.3</td>
<td>0</td>
</tr>
<tr>
<td>80% water - 20% dioxane</td>
<td>3.75</td>
<td>61.9</td>
<td>1.1</td>
<td>0</td>
</tr>
<tr>
<td>60% water - 40% dioxane</td>
<td>3.35</td>
<td>44.5</td>
<td>1.5</td>
<td>0.1</td>
</tr>
<tr>
<td>40% water - 60% dioxane</td>
<td>3.1</td>
<td>27.2</td>
<td>1.7</td>
<td>0.1</td>
</tr>
<tr>
<td>20% water - 80% dioxane</td>
<td>3.75</td>
<td>11.9</td>
<td>2.3</td>
<td>0.1</td>
</tr>
<tr>
<td>dioxane</td>
<td>2.2</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>methanol</td>
<td>5.6</td>
<td>32.6</td>
<td>4.8</td>
<td>0</td>
</tr>
</tbody>
</table>

The outer diameter (o.d.) is nominal only and used for reference in the discussion in the text. Exponents are indicated in parentheses.

The characteristic admittance of the line (from Misra). In addition,

$$Y = G + jB$$

$$G = \frac{Y_0\sqrt{\epsilon}}{\ln(b/a)\sqrt{\epsilon_c}} \int_{0}^{\pi/2} \frac{1}{\sin \theta} \left[ J_0(k_0\sqrt{\epsilon} b \sin \theta) - J_0(k_0\sqrt{\epsilon} a \sin \theta) \right]^2 d\theta$$

$$B = \frac{Y_0\sqrt{\epsilon}}{\pi \ln(b/a)\sqrt{\epsilon_c}} \int_{0}^{\pi/2} \left[ 2 \text{Si} \left( k_0\sqrt{\epsilon} a \sin (\theta/2) \right) - \text{Si} \left( 2k_0\sqrt{\epsilon} b \sin (\theta/2) \right) \right] d\theta$$

(from Marcuvitz). In these expressions, $a$ and $b$ are the inner and outer radii of the line (Table I); $k_0$ is the propagation constant in free space, $2\pi f/c$, where $c$ is the velocity of light in vacuum and $f$ the frequency; $\epsilon$ and $\epsilon_c$ are the relative permittivities of the material under test and the dielectric in the transmission line, respectively; $J_0$ is the Bessel function of order zero; and $\text{Si}$ is the sine integral.

These results are approximate in several respects: they were derived assuming the presence of an infinite conductive flange over the aperture (not present with the coaxial lines), and they do not explicitly account for higher order modes at the aperture. In addition, the properties of any real probe will vary because of manufacturing tolerances.

In this study we evaluated (2) and (3) numerically, using either a series expansion or numerical integration. The Appendix contains a series expansion that is valid for the frequency range and probe dimensions used in this study. (Fewer terms in this expansion are reported in [10].) In the lumped-parameter model, the parameter $C_j$ corresponds to the term $B_1$ in the Appendix, divided by the frequency; $C_0$ is small and is neglected for present purposes.

III. EXPERIMENTAL

For the experiments, probes were constructed from lengths of semirigid 50 $\Omega$ coaxial line (3.6 mm OD line with a type SMA connector, or 2.99 mm OD precision line with a precision type K connector) ranging in length from 9.5 to 14 cm. The end distal to the connector was machined flat and polished with fine crocus cloth. The 2.99 mm line and K connector have considerably higher precision than 3.6 mm line and SMA connector, and were used for most of the measurements discussed below.

Measurements were conducted from 0.1 to 18 GHz on several reference liquids: water, methanol, and various water–dioxane mixtures. The dielectric properties of these
liquids are summarized by the empirical equation

\[ \epsilon = \epsilon_\infty + \frac{\epsilon_f - \epsilon_\infty}{1 + (\omega \tau)^{1-\alpha}} \]  

with parameter values summarized in Table II. Measurements were conducted at 20–25 °C.

Figs. 1 and 2 compare the measured admittance of the 2.99 mm probe when immersed in methanol and water with Marcuvitz’s approximation (eq. (3)) and the lumped-parameter model (eq. (1)). Marcuvitz’s approximation works quite well for methanol over the entire frequency range of the measurements; it is less satisfactory for water. The errors in the reflection coefficient (given as the magnitude of the difference between the measured and calculated reflection coefficients) for water at 10 GHz are 0.06 (Marcuvitz’s approximation) and 0.14 (lumped-parameter model). For methanol at 10 GHz these errors are 0.02 and 0.04 respectively. Marcuvitz’s approximation is clearly an improvement over the lumped-parameter model, although the errors involved considerably exceed those of factory-standard calibration loads for ANA calibration. (Another source of error, not considered here, is the uncertainty in the dielectric properties of the reference liquids, which can be several percent at microwave frequencies.)
mixtures. The inputs consisted of measured reflection coefficients from 2.99 mm probes in water, methanol, and dioxane \((\Gamma_1, \Gamma_2, \Gamma_3, \text{used for calibration})\) and water–dioxane mixtures. For each water–dioxane mixture, the corrected probe admittance was obtained from (6). The permittivity of the sample was then found by solving (3). Then, the elements of the presumed "scattering matrix" for system errors (which actually represent errors due to the lumped-parameter model) were achieved high accuracy in their measurements.

Finally, the correction method was applied to data obtained from a probe made from 3.6 mm semirigid coaxial line, whose connector (type SMA) had a high reflection coefficient of about 0.05 at 10 GHz. The calibration loads consisted of an open and a short circuit and water; measurements were performed on 0.3 N NaCl and methanol. The results for methanol are shown in Fig. 3. The suggested calibration procedure eliminates noticeable oscillatory behavior due to connector artifacts.

Fig. 3. Real and imaginary parts of the permittivity of methanol, measured with a 3.6 mm coaxial probe, calculated using eq. (2), compared with literature values. The calibration loads were open and short circuits and water.

V. USE OF LUMPED-PARAMETER MODEL IN PREVIOUS STUDIES

Several authors (notably, Stuchly and coworkers [2–4]) have reported extensive measurements on tissues using 3.6 mm probes at frequencies up to 10 GHz. In most cases, the ANA was calibrated from measurements on the probe when shorted, open-circuited, and immersed in water. Despite the use of the lumped-parameter model, they achieved high accuracy in their measurements.

To explore this further, we simulated the effects of miscalibration of the ANA due to the use of lumped-parameter model (eq. (1)) for the probe. The calculations assume a 2.99 mm probe, with calibration standards consisting of open and short circuits and water.

TABLE III

<table>
<thead>
<tr>
<th>Solution</th>
<th>Frequency</th>
<th>(\varepsilon)</th>
<th>(\varepsilon)</th>
<th>(\varepsilon)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GHz</td>
<td>(Eq. 1)</td>
<td>(Eq. 2)</td>
<td>(Lit.)</td>
</tr>
<tr>
<td>80% water</td>
<td>1</td>
<td>64-36</td>
<td>63-36</td>
<td>62-36</td>
</tr>
<tr>
<td>40% water</td>
<td>1</td>
<td>63-312</td>
<td>59-312</td>
<td>60-312</td>
</tr>
<tr>
<td>dioxane</td>
<td>10</td>
<td>65-324</td>
<td>40-326</td>
<td>63-327</td>
</tr>
<tr>
<td>dioxane</td>
<td>18</td>
<td>23-20</td>
<td>20-228</td>
<td>27-228</td>
</tr>
<tr>
<td>40% water</td>
<td>1</td>
<td>44-35</td>
<td>44-36</td>
<td>43-35</td>
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<tr>
<td>dioxane</td>
<td>10</td>
<td>27-312</td>
<td>22-318</td>
<td>25-318</td>
</tr>
<tr>
<td>dioxane</td>
<td>18</td>
<td>17-39</td>
<td>16-316</td>
<td>16-316</td>
</tr>
<tr>
<td>60% water</td>
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<td>26-33</td>
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<td>10-33</td>
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<td>80% water</td>
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</tr>
<tr>
<td>dioxane</td>
<td>10</td>
<td>7-31</td>
<td>6-33</td>
<td>7-33</td>
</tr>
<tr>
<td>dioxane</td>
<td>18</td>
<td>9-30</td>
<td>5-33</td>
<td>5-33</td>
</tr>
</tbody>
</table>

The reference standards consisted of data obtained with the 2.99 mm probe immersed in water, methanol, and dioxane. The literature values were calculated using (4), with the parameters given in Table II.

![Figure 4](image-url)
The results of this simulation are shown in Fig. 4. The permittivity values ($\varepsilon'$) are remarkably accurate. In contrast, the loss ($\varepsilon''$) is underestimated above a few GHz, by about 20 percent for the water–dioxane mixture at 10 GHz, and by much larger fractions for the methanol. The previous studies [2]–[4] used a reference liquid (water) that is electrically similar to the test materials (high-water-content tissues). This is likely to ensure high accuracy in the measurements, despite errors in the model of the probe. At frequencies below ca. 1 GHz, the model-dependent errors become very small and the lumped-parameter model is satisfactory.

VI. CONCLUSION

Marcuvitz's approximation improves the accuracy of dielectric measurements using coaxial probes and is useful, for the probes considered here, up to ca. 10 GHz. The probe technique succeeds because the errors in modeling the probe are partially canceled by calibration errors in the ANA. While the final measurement errors are difficult to quantify, they appear to be small if the electrical properties of the reference liquid are close to those of the sample under test or at low frequencies.

APPENDIX

SERIES APPROXIMATIONS TO EQUATION (3)

In the expressions below, all dimensions are in cm and all frequencies in Hz. The probe admittance $Y$ (in siemens) is given by

$$Y = G(f) + jB(f)$$

where

$$B(f) := B1(f) + B2(f) + B3(f) + B4(f) + B5(f) + B6(f) + B7(f)$$

where

$$B1(f) := k(f) \cdot K(f) \cdot I0$$
$$B2(f) := k(f)^2 \cdot K(f) \cdot I1$$
$$B3(f) := k(f)^3 \cdot K(f) \cdot I2$$
$$B4(f) := k(f)^4 \cdot K(f) \cdot I3$$
$$B5(f) := k(f)^5 \cdot K(f) \cdot I4$$
$$B6(f) := k(f)^6 \cdot K(f) \cdot I5$$
$$B7(f) := k(f)^7 \cdot K(f) \cdot I6$$

$$I0 := 2 \cdot \int_0^\infty \sqrt{\frac{1}{a^2 + b^2} \cdot a^2 + b^2} \cdot \cos(t) + b^2 \cdot dt - 4 \cdot (a + b)$$

$$I1 := - \int_0^\infty \left\{ \frac{1}{9} \cdot \left[ -2 \cdot a \cdot b \cdot \cos(t) + a^2 + b^2 \right]^{3/2} \cdot dt + 16 \cdot \frac{a^3 + b^3}{27} \right\}$$

$$I2 := \frac{1}{300} \cdot \int_0^\infty \left\{ -2 \cdot a \cdot b \cdot \cos(t) + a^2 + b^2 \right\}^{3/2} \cdot dt - \frac{64}{1125} \cdot [a^5 + b^5]$$

$$I3 := \frac{1}{17640} \cdot \int_0^\infty \left\{ -2 \cdot a \cdot b \cdot \cos(t) + a^2 + b^2 \right\}^{7/2} \cdot dt + \frac{256}{77175} \cdot [a^7 + b^7]$$

$$I4 := 6.124 \cdot 10^{-7} \cdot \int_0^\infty \left\{ -2 \cdot a \cdot b \cdot \cos(t) + a^2 + b^2 \right\}^{9/2} \cdot dt - 1.274 \cdot 10^{-4} \cdot [a^9 + b^9]$$

$$I5 := -4.555 \cdot 10^{-9} \cdot \int_0^\infty \left\{ -2 \cdot a \cdot b \cdot \cos(t) + a^2 + b^2 \right\}^{11/2} \cdot dt + 3.446 \cdot 10^{-6} \cdot [a^{11} + b^{11}]$$

$$I6 := 2.471 \cdot 10^{-11} \cdot \int_0^\infty \left\{ -2 \cdot a \cdot b \cdot \cos(t) + a^2 + b^2 \right\}^{13/2} \cdot dt - 6.901 \cdot 10^{-8} \cdot [a^{13} + b^{13}]$$

$$G(f) := \frac{\sqrt{\frac{\varepsilon(f)}{\varepsilon_0}}}{\ln \left[ \frac{a}{b} \right] \cdot Z0} \cdot (G1(f) + G2(f) + G3(f) + G0(f) + G4(f) + G5(f))$$
where

\[ G_0(f) := \frac{1}{24} \left[ a^2 - b^2 \right]^2 k(f)^4 \]

\[ G_1(f) := -\frac{1}{240} \left[ a^2 - b^2 \right]^2 \left[ a^4 - b^4 \right] k(f)^6 \]

\[ G_2(f) := \left[ \frac{1}{8960} \left[ a^4 - b^4 \right]^2 + \frac{1}{10080} \left[ a^2 - b^2 \right] \left[ a^6 - b^6 \right] \right] k(f)^8 \]

\[ G_3(f) := \left[ -\frac{128}{315} \left\{ \frac{1}{72} \left[ a^4 - b^4 \right] \left[ a^6 - b^6 \right] + \frac{1}{288} \left[ a^2 - b^2 \right] \left[ a^6 - b^6 \right] \right\} \right] k(f)^{10} \]

\[ G_4(f) := \left[ k(f)^{12} \right] \frac{36(a^2 - b^2)(a^{10} - b^{10}) + 225(a^4 - b^4)(a^6 - b^6) + 200(a^6 - b^6)^2}{287409600} \]

\[ G_5(f) := \left[ -k(f)^{14} \right] \frac{(a^2 - b^2)(a^{12} - b^{12}) + 9(a^4 - b^4)(a^{10} - b^{10}) + 25(a^6 - b^6)(a^8 - b^8)}{12454041600} \]

(This expansion, to fewer terms, is given in [10].) The characteristic impedance of the line is \( Z_0 \). Note that \( k(f) \) and \( K(f) \) are complex functions of the complex permittivity \( \varepsilon \), which is defined in the Introduction.

We have incorporated this series in the software to perform the probe measurements, using a Hewlett-Packard model HP8410 ANA controlled by a H-P 9836 computer. The permittivity \( \varepsilon \) can be obtained from the measured admittance within about a second. Surprisingly, the solution (using Laguerre's method [13]) showed no tendency to converge to a spurious root, perhaps because of the availability of good initial guesses for the permittivity. For measurements using the 3.6 mm probe on water at 10 GHz, all terms in this series are needed to ensure convergence of the series to (3) to within 1 percent. The rate of convergence of the series is extremely sensitive to the dimensions of the probe and to the frequency.

REFERENCES


Devendra Misra (M'74) was born in Unnao (U.P.), India, in 1949. He received the Ph.D. degree in electrical engineering from Michigan State University, East Lansing, in 1984. Since August 1983, he has been with the Faculty of Electrical Engineering, University of Wisconsin-Madison, Milwaukee. Prior to this he had been a Post-Doctoral Research Associate in Electrical Engineering, Division of Engineering Research, Michigan State University. During the years 1973-1980, he held research and teaching faculty positions in India. He worked as a Teaching and Research Assistant during his studies at Michigan State University. His current interests include industrial and biomedical applications of electromagnetic energy.
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In 1983 he joined the Microwave Research Laboratory at the David Sarnoff Research Center, Princeton, NJ, as a member of the technical staff. His work has concentrated on the development of computer-aided design, test, and manufacturing techniques for microwave circuits and systems. Other interests include microwave circuit design and research on the biophysical mechanisms of microwave absorption by tissues.

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