Electromagnetic Scattering by an Impedance Sheet with a 1-D Inhomogeneity in a Rectangular Waveguide

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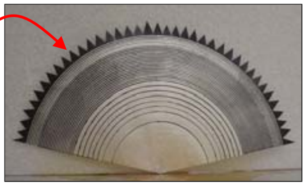
Adjustable and Tapered R card

- One aspect of this work is to produce lossy films with adjustable sheet impedance:
  - Perhaps isotropic and uniform
  - Perhaps spatially varying
  - Perhaps anisotropic; etc.
- As will be discussed shortly, this will be accomplished by physically altering commercially available resistive films.
- Applications include:
  - Reduce backscattering (Tapered R card: Senior and Liepa, 1984; Haupt and Liepa, 1987)
  - Adaptive reflector antenna (Haupt, 2006)
  - Wu-King taper for ultrawideband antenna:
    - Radially inhomogeneous sheet impedance
    - Printed conductor onto 370 OPS Kapton XC



(Glover, Kirschenmann, and Whites, “Engineering R-Card Surface Resistivity with Printed Metallic Patterns, *Metamaterials'2007*, Oct. 2007.)

Outline

This work requires a method for characterizing the scattering by films that are spatially varying, which is the topic of this talk. Eventually would like to measure the effective sheet impedance .

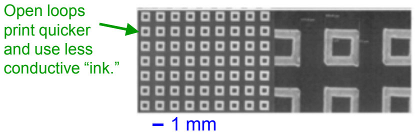
- Creating effective sheet impedance films
- Mode matching solution
  - scattering by spatially varying sheet impedance films in a rectangular waveguide
- Using perforations to create spatial variation
  - accuracy of the Maxwell/Maxwell Garnett mixing rule
- Properties of DuPont Kapton 370 XC
- Measurements for two region perforated Kapton 370 XC
- Conclusions

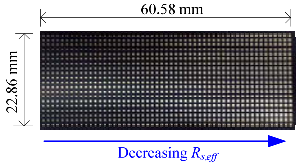
Creating Effective Films

Methods to create tapered sheet impedance films include:

- Printing electrically small, metallic patterns on commercially available films to **reduce** the effective surface resistivity. [Films include DuPont's Kapton XC® or Ohmega Technologies OHMEGA-PLY® or 200 Ω/sq stainless steel coated PET (polyethylene terephthalate), for example.]

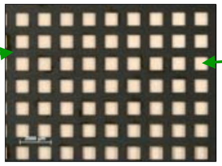
Open loops print quicker and use less conductive "ink."





- Perforating commercially available films to **increase** the effective surface resistivity.

Kapton 370 XC



Laser cut square holes

- Directly manufacture. Expensive, difficult to realize spatially varying.

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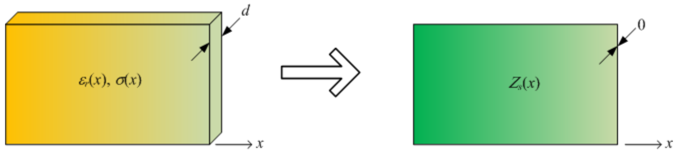
Waveguide Characterization

- Quest is to develop a method for characterizing spatially varying impedance films. In this work, will assume variation in only one Cartesian dimension.
- Will develop a waveguide technique hoping for higher accuracy in a confined measurement system.
- In actuality, the film is a thin slab of material. Maybe multilayered in the case of printed film (though not the case with perforated).



- In inhomogeneous materials, difficult to obtain mode expansion of electromagnetic fields.
- However, here we have very thin, high contrast films. Model these with sheet impedance boundary condition.
- Effect sheet impedance $Z_s(x)$ will vary across the surface.

Mode match solution - 1

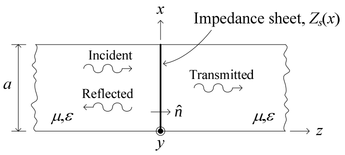
- Solve with mode match approach.
- Incident TE_{10} mode:

$$E_y^i = \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{z,10} z}$$

$$H_x^i = -\frac{\beta_{z,10}}{\omega\mu} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{z,10} z}$$

$$\beta_{z,m0}^2 = \omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2$$

Top View



- With no *y* variation in the incident fields or the specimen, then only TE_{m0} modes (and no TM modes) will be scattered by the specimen.
- Subsequently, non-zero components of the reflected fields include

$$E_y^r = \sum_{m=1}^{\infty} A_m \sin\left(\frac{m\pi x}{a}\right) e^{j\beta_{z,m0} z} \qquad H_x^r = \sum_{m=1}^{\infty} \frac{\beta_{z,m0}}{\omega\mu} A_m \sin\left(\frac{m\pi x}{a}\right) e^{j\beta_{z,m0} z}$$

- Non-zero components of the transmitted fields include

$$E_y^t = \sum_{m=1}^{\infty} B_m \sin\left(\frac{m\pi x}{a}\right) e^{-j\beta_{z,m0} z} \qquad H_x^t = -\sum_{m=1}^{\infty} \frac{\beta_{z,m0}}{\omega\mu} B_m \sin\left(\frac{m\pi x}{a}\right) e^{-j\beta_{z,m0} z}$$

Mode match solution - 2

- Employing the impedance sheet boundary condition at *z* = 0 in which the tangential electric field is continuous across the sheet while the tangential magnetic field is discontinuous:

$$\hat{n} \times \bar{E} = Z_s(x)\, \hat{n} \times \left[\hat{n} \times \left(\bar{H}^+ - \bar{H}^-\right)\right]$$

- Leads to a matrix equation for the vector of transmitted field amplitudes B_i

$$\bar{\bar{Q}} \cdot \bar{B} = \bar{P}$$

where

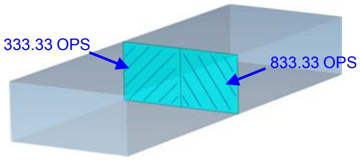
$$Q_{ij} = \frac{a}{2}\delta_{ij} + \frac{2\beta_{z,j0}}{\omega\mu} I_{R,ij} \qquad P_i = \frac{2\beta_{z,10}}{\omega\mu} I_{R,i1}$$

$$I_{R,ij} \equiv \int_0^a Z_s(x) \sin\left(\frac{i\pi x}{a}\right) \sin\left(\frac{j\pi x}{a}\right) dx$$

and δ_{ij} is the Kronecker delta function.

Compare with CST *MWS* Simulation

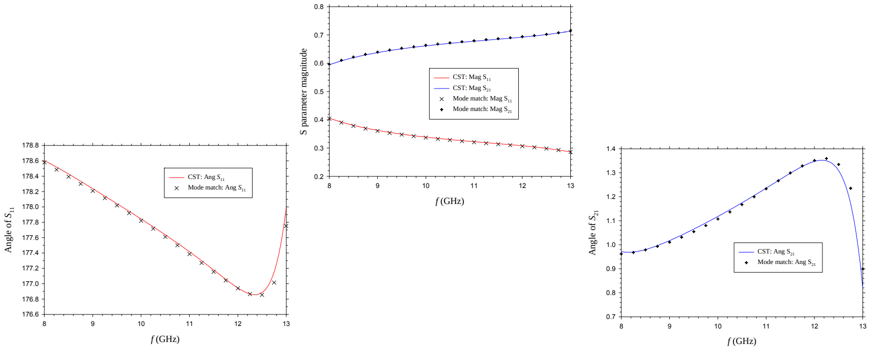
- To help validate the accuracy of this mode matching solution, comparisons were made with the frequency domain solver of CST *Microwave Studio*.
- Considering a two region resistive film:



- Within the frequency domain solver, CST *MWS* provides for an **ideal** impedance sheet boundary condition, with infinitesimal thickness, in an integral equation solution.
- No such boundary condition is available with the time domain solver.

Two Region Specimen Results

- Used 20 modes in the mode matching solution (both transmitted and reflected), and ~70k mesh cells in the CST *MWS* solution.



- Effects of higher order modes directly observed in phase angle data: would be 180º or 0º for uniform resistive sheets.
- Excellent agreement: good confidence in mode match solution.

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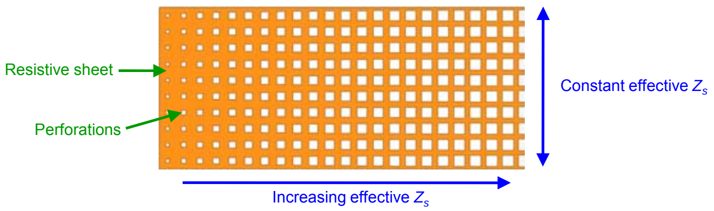
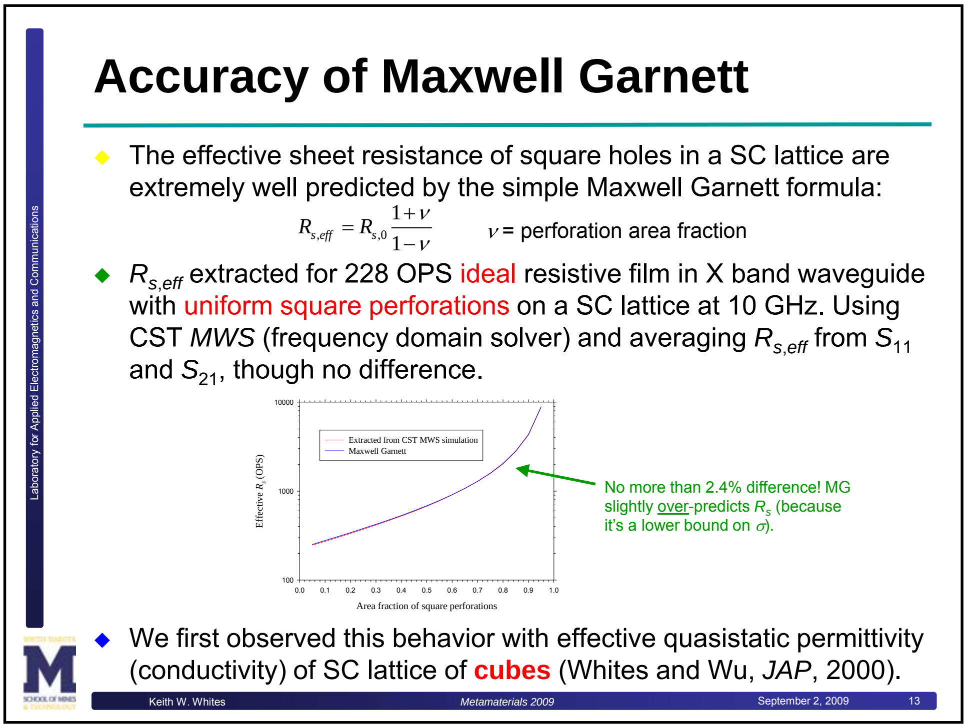
Spatially Varying Sheet Impedance

- Ideal resistive sheets with constant sheet resistance were used in the previous simulations.
- How can one create physically realizable films with varying sheet impedance?
- One approach is to use the concept of "effective film," as mentioned at the beginning of this presentation, and print metallic particles onto a resistive film, or else perforate the film:



- Perforations in a host resistive film more closely matches an effective film than printing particles on top of the film.



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Accuracy of Maxwell Garnett

- The effective sheet resistance of square holes in a SC lattice are extremely well predicted by the simple Maxwell Garnett formula:

$$R_{s,eff} = R_{s,0}\frac{1+\nu}{1-\nu} \qquad \nu = \text{perforation area fraction}$$

- $R_{s,eff}$ extracted for 228 OPS ideal resistive film in X band waveguide with uniform square perforations on a SC lattice at 10 GHz. Using CST *MWS* (frequency domain solver) and averaging $R_{s,eff}$ from S_{11} and S_{21}, though no difference.

No more than 2.4% difference! MG slightly over-predicts R_s (because it's a lower bound on σ).

- We first observed this behavior with effective quasistatic permittivity (conductivity) of SC lattice of **cubes** (Whites and Wu, *JAP*, 2000).

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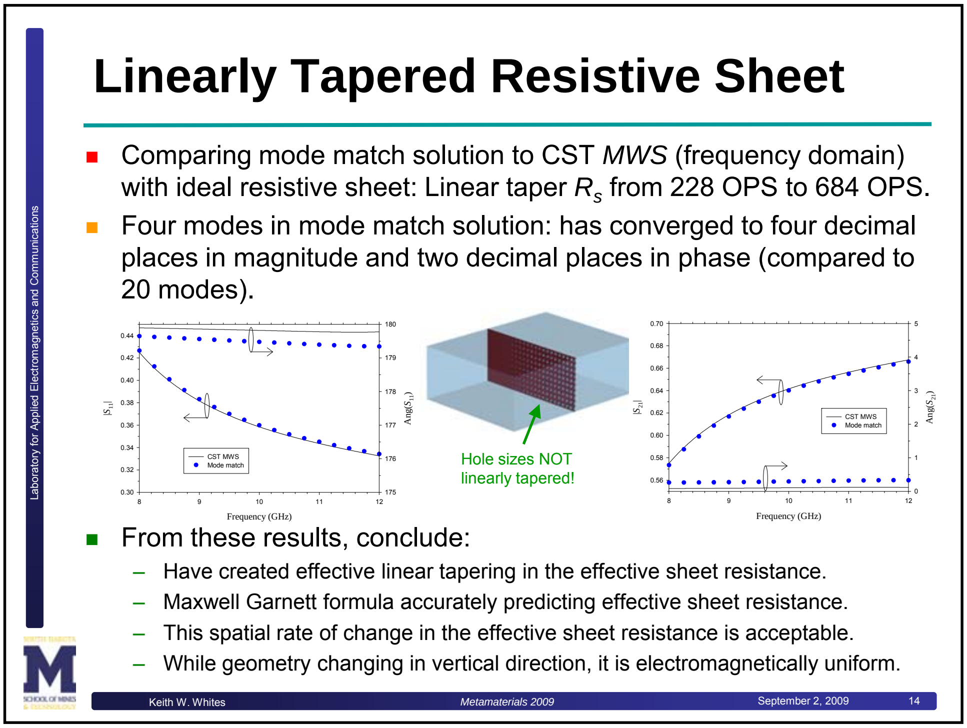

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Linearly Tapered Resistive Sheet

- Comparing mode match solution to CST *MWS* (frequency domain) with ideal resistive sheet: Linear taper R_s from 228 OPS to 684 OPS.
- Four modes in mode match solution: has converged to four decimal places in magnitude and two decimal places in phase (compared to 20 modes).

Hole sizes NOT linearly tapered!

- From these results, conclude:
  - Have created effective linear tapering in the effective sheet resistance.
  - Maxwell Garnett formula accurately predicting effective sheet resistance.
  - This spatial rate of change in the effective sheet resistance is acceptable.
  - While geometry changing in vertical direction, it is electromagnetically uniform.

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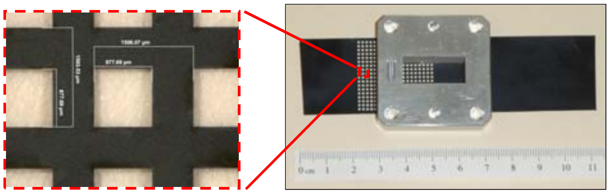
Two Region Perforated Specimen

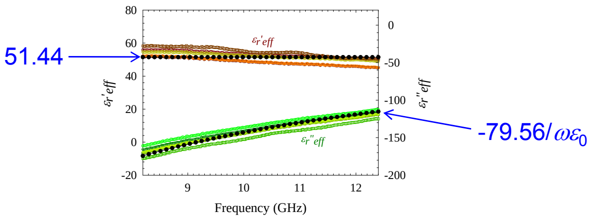
- The last set of data we will show involves the measurement of a two region specimen in a WR-90 rectangular waveguide.
- Using Kapton 370 XC as the resistive sheet (~40 μm thick).
- Specimen was laser cut to produce a SC lattice of square holes:



- Perforation area fraction = 0.340. Using Maxwell Garnett, expect Z_s to increase by 2.03 [=(1+0.340)/(1-0.340)] over neat R card.
- Some discoloration around edges of perforations. Localized heating during laser cutting? Heating will change properties of Kapton XC.

Properties of Kapton XC

- Kapton XC is a fairly complex material. Carbon black dispersed in polyimide in a multilayered structure.
- Complex ε_r from 5 specimens measured in a flanged waveguide:



- Appreciable variation between specimens: a well-known property of Kapton XC. (Also known to be somewhat anisotropic.)
- A parallel RC model with $\varepsilon_{r,eff} = 51.44\text{-}j79.56/\omega\varepsilon_0$ approximately describes the measured complex permittivity of Kapton XC within the X-band, though a more sophisticated model is likely necessary especially for broadband modeling.

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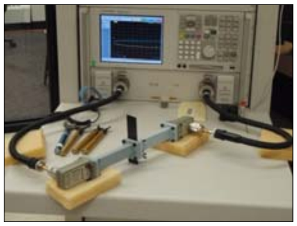
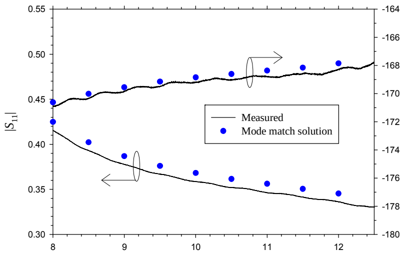
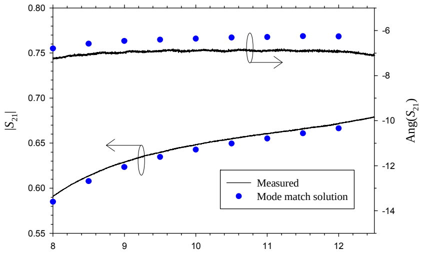
Two Region Specimen Measurement

- For mode match calculations, extracted Z_s from waveguide measurement of uniform specimens (neat R card, then perforated specimen).
- Used these measured Z_s values at each frequency to calculate scattering for the two region specimen.





Measurements by T. Amert



Conclusions

- ✓ Overall quest of this work is to develop a method for characterizing the scattering by spatially varying impedance films.
- ✓ Presented a rectangular waveguide mode matched-based solution for characterizing films with a 1-D variation in sheet impedance.
- ✓ Showed a method for spatially tailoring such films by using perforations of varying size on a uniform grid.
- ✓ Presented evidence that the change in sheet impedance is extremely well predicted by the Maxwell Garnett formula.
- ✓ Measured results with a two region specimen of neat and perforated Kapton XC were shown.

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Thank You

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