The second type of measurement method—and test fixture—uses both transmission and reflection data to extract the material parameters. Using the same geometry as Baker-Jawors (except for the spatial variables), a toroidally-shaped specimen in a coaxial airline fixture is defined as: 

![Diagram of coaxial airline fixture with ports and measurement calibration planes.]

Similar to our analysis for the shielded airline fixture, the Tl equivalent voltages in the three regions are:

\[ V_1(z) = e^{-\gamma_0 z} + c_1 e^{+\gamma_0 z} \]  

(1)

\[ V_{II}(z) = c_2 e^{-\gamma_2 z} + c_3 e^{+\gamma_2 z} \]  

(2)

and

\[ V_{III}(z) = c_4 e^{-\gamma_0 z} \]  

(3)

We've assumed here that a wave is incident from port 1 with amplitude \( 1 \) \( V \), and that both ports are attached to matched lines.
The coefficients $C_1 - C_4$ can be determined by applying the BC's in the equivalent TL model.
Continuity of $V$ & $I$ at $z = L_1$, i.e., $z = L_1 + L$.

For a reciprocal material, the 5 parameters for this specimen referenced to the faces of the specimen are:

$$S_{01} = \frac{\Gamma_p (1 - \rho^2)}{1 - \rho^2 \rho^2} = S_{22}$$

(4)

$$S_{21} = \frac{\rho (1 - \rho^2)}{1 - \rho^2 \rho^2} = S_{12}$$

(5)

where $\rho = e^{-\gamma L}$ is the propagation factor "through the specimen" (6).

and $\Gamma_p$ = "partial reflection coefficient" at sample face when incident from air to specimen filled TL:

$$\Rightarrow \Gamma_p = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{Z_0/Z_2 - 1}{Z_0/Z_2 + 1}$$

(7)

For any structure guiding a TEM wave (i.e., a TL), the p.u.l. inductance $\omega L_0$ (from definition $L = \frac{\omega L_0}{\omega}$) and the p.u.l. capacitance $\omega C_0$ (from def. $C = \frac{\omega C_0}{\omega}$).

Consequently, $\frac{Z_2}{Z_0} = \sqrt{\frac{1}{\varepsilon_r}} \equiv \eta_r \quad \forall \text{TLs}$

(8)
Sub (8) \rightarrow (7) yields

\[ \Gamma_p = \frac{\rho_r - 1}{\rho_r + 1} \]  

(9)

The material parameters of the specimen under test can be determined from eqns. (4) \& (5) using the measured S parameters, \( S_{ji}^m \). The measured S parameters are assumed referenced to the calibration planes shown in the figure.

![Diagram showing measurement and calibration planes]

Using the same prime notation we employed for shifted measurements, the measured S parameters at the calibration planes are related to those referenced to the specimen faces, \( S_{ji}^m \), as

\[ S_{ji}^m = e^{-\gamma_0 (l_i + L_i)} S_{ji}^m \]  

(10)

Following Baker-Tarvis's notation, we will define

\[ R_{li} = e^{-\gamma_0 L_i} \]  

(11)
So that from (10): \[ S_{ji}^m = R_j R_i S_{ji}^{m'} \quad i,j=1,2 \] (12)

To determine material parameters from measured S parameters, we equate \( S_{ji}^{m'} \) to \( S_{ji} \) in (4) \& (5) giving

\[ S_{ii}^m = R_1^2 \frac{\Gamma_1 (1-P_1^2)}{1-P_1^2 P_2^2} \] (13)
\[ S_{22}^m = R_2^2 \frac{\Gamma_2 (1-P_2^2)}{1-P_1^2 P_2^2} \] (14)
\[ S_{21}^m = R_1 R_2 \frac{\Gamma_1 (1-P_2^2)}{1-P_1^2 P_2^2} = S_{12}^m \] (15)

An additional measurement that can be made is the "Thru" with the specimen removed:

\[ S_{21}^{m_0} = R_1 R_2 E_0 L \] (16)

These equations (13)-(16) contain lengths \( L, L_1, \) and \( L_2 \). We'll define the total length of the airline fixture as \( L_{air} \) which is

\[ L_{air} = L + L_1 + L_2 \] (17)

To start, we'll presume that \( L_{air} \) is known, whereas \( L, L_1, \) and \( L_2 \) are not known, i.e., they're unknowns we may need to solve for.