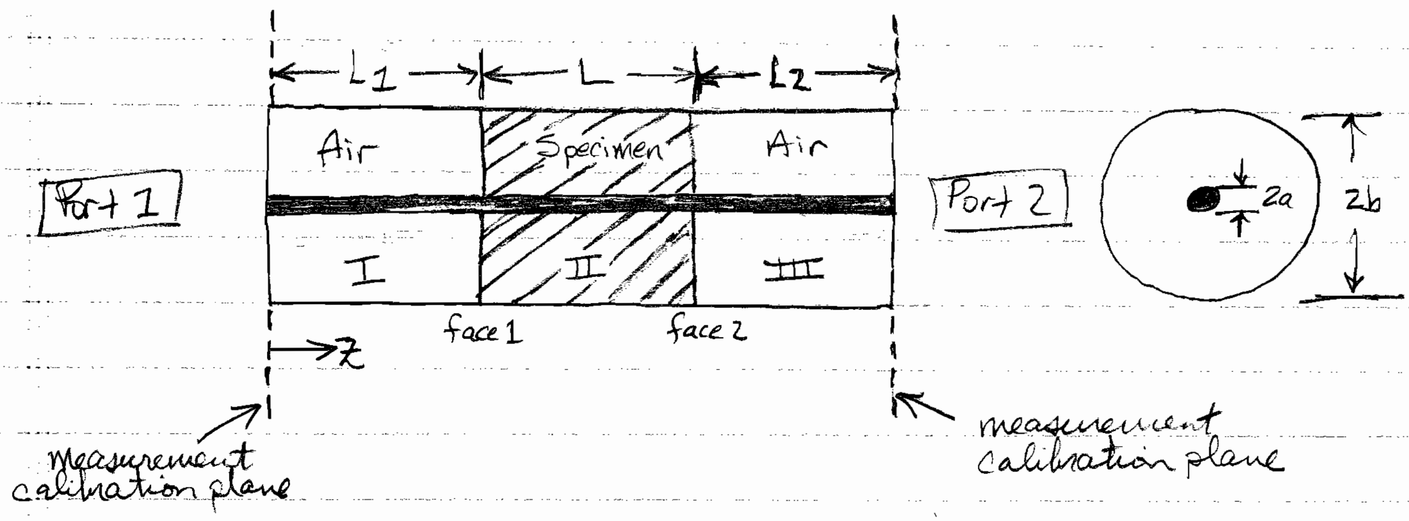
# Transmission-Reflection Measurements for $\epsilon$ & $\mu$

The second type of measurement method - and test fixture - uses both transmission and reflection data to extract the material parameters. Using the same geometry as Baker-Jarvis (except for the spatial variable), a toroidally-shaped specimen in a coaxial air line fixture is defined as:




Similar to our analysis for the shorted air line fixture, the TL equivalent voltages in the three regions are:

$$V_{I}(z) = e^{-\gamma_0 z} + C_1 e^{+\gamma_0 z} \tag{1}$$

$$V_{II}(z) = C_2 e^{-\gamma z} + C_3 e^{+\gamma z} \tag{2}$$

and

$$V_{III}(z) = C_4 e^{-\gamma_0 z} \tag{3}$$

We've assumed here that a wave is incident from port 1 w/ amplitude 1 V, and that both ports are attached to matched lines.

The coefficients $C_1 - C_4$ can be determined by applying the b.c.'s in the equivalent TL model: Continuity of V & I at $z = L_1$ & $z = L_1 + L$.

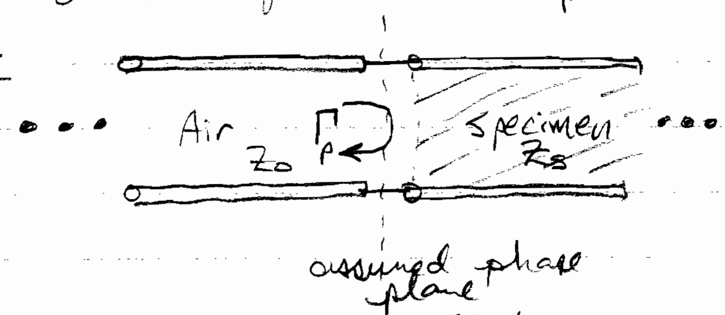
For a reciprocal material, the S parameters for this specimen referenced to the faces of the specimen are:

$$S_{11} = \frac{\Gamma_p (1 - P^2)}{1 - \Gamma_p^2 P^2} = S_{22} \qquad (4)$$

$$S_{21} = \frac{P(1 - \Gamma_p^2)}{1 - \Gamma_p^2 P^2} = S_{12} \qquad (5)$$

where $P \equiv e^{-\gamma L}$ is the "propagation factor" through the specimen (6)

and $\Gamma_p$ = "partial reflection coefficient" at sample face when incident from air to specimen filled TL:




$$\Rightarrow \Gamma_p = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{Z_S/Z_0 - 1}{Z_S/Z_0 + 1} \qquad (7)$$

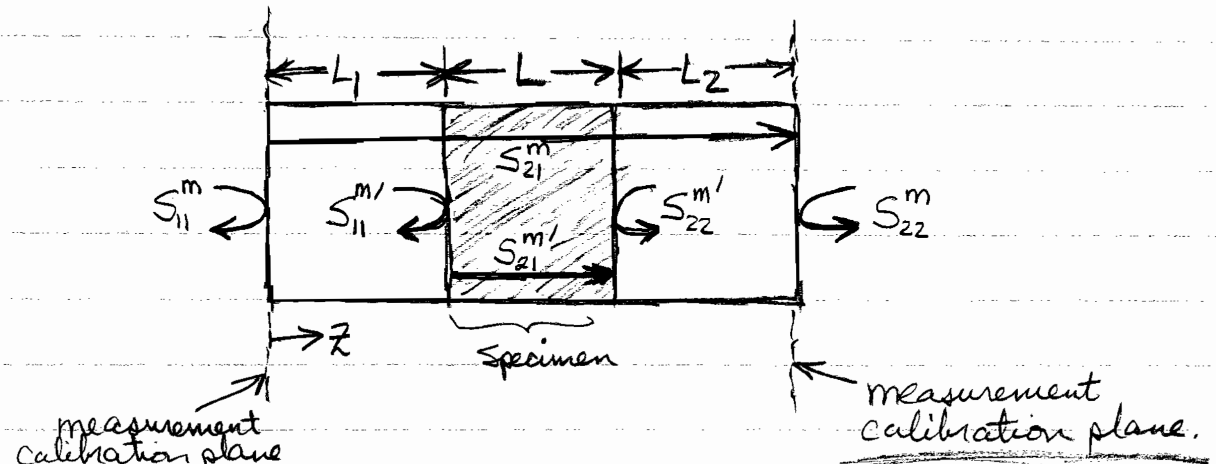
ref. Paul MTL's.

For any structure guiding a TEM wave (i.e., a TL), the p.u.l. inductance $\propto \mu$ (from definition $L = \frac{\Psi}{I}$) and the p.u.l. capacitance $\propto \epsilon$ (from def. $C = \frac{Q_{enc}}{V}$).

Consequently, $\frac{Z_S}{Z_0} = \sqrt{\frac{\mu_r}{\epsilon_r}} \equiv \eta_r$ $\forall$ TLs (8)

Sub. (8) → (7) yields $\Gamma_p = \frac{\eta_r - 1}{\eta_r + 1}$ (9).

The material parameters of the specimen under test can be determined from eqns. (4) & (5) using the measured S parameters, $S_{ji}^m$. The measured S parameters are assumed referenced to the calibration planes shown in the figure.




Using the same prime notation we employed for shorted measurements, the measured S parameters at the calibration planes, $S_{ji}^m$, are related to those referenced to the specimens faces, $S_{ji}^{m'}$, as

$$S_{ji}^m = e^{-\gamma_0(L_j+L_i)} S_{ji}^{m'} \qquad i,j=1,2 \qquad (10)$$

Following Baker-Jarvis's notation, we'll define

$$R_i \equiv e^{-\gamma_0 L_i} \qquad i=1,2 \qquad (11)$$

So that from (10): $$S_{ji}^{m} = R_j R_i S_{ji}^{m\prime} \qquad i,j=1,2 \qquad (12)$$

To determine material parameters from measured S parmeters, we equate $S_{ji}^{m\prime}$ to $S_{ji}$ in (4) & (5) giving

Ref.: Baker-Jarvis, et al, MTT, Aug. 1990.

$$S_{11}^{m} = R_1^2 \frac{\Gamma_p (1-P^2)}{1-\Gamma_p^2 P^2} \qquad (13)$$

$$S_{22}^{m} = R_2^2 \frac{\Gamma_p (1-P^2)}{1-\Gamma_p^2 P^2} \qquad (14)$$

$$S_{21}^{m} = R_1 R_2 \frac{P(1-\Gamma_p^2)}{1-\Gamma_p^2 P^2} = S_{12}^{m} \qquad (15)$$

An additional measurement that can be made is the "Thru" with the specimen removed:

$$S_{21}^{m,0} = R_1 R_2 e^{-\gamma_0 L} \qquad (16)$$

These equations (13)-(16) contain lengths $L$, $L_1$ and $L_2$. We'll define the total length of the air line fixture as $L_{air}$ which is

$$L_{air} = L + L_1 + L_2 \qquad (17).$$

To start, we'll presume that $L_{air}$ is known, whereas $L$, $L_1$, and $L_2$ are not known, i.e., they're unknowns we may need to solve for.