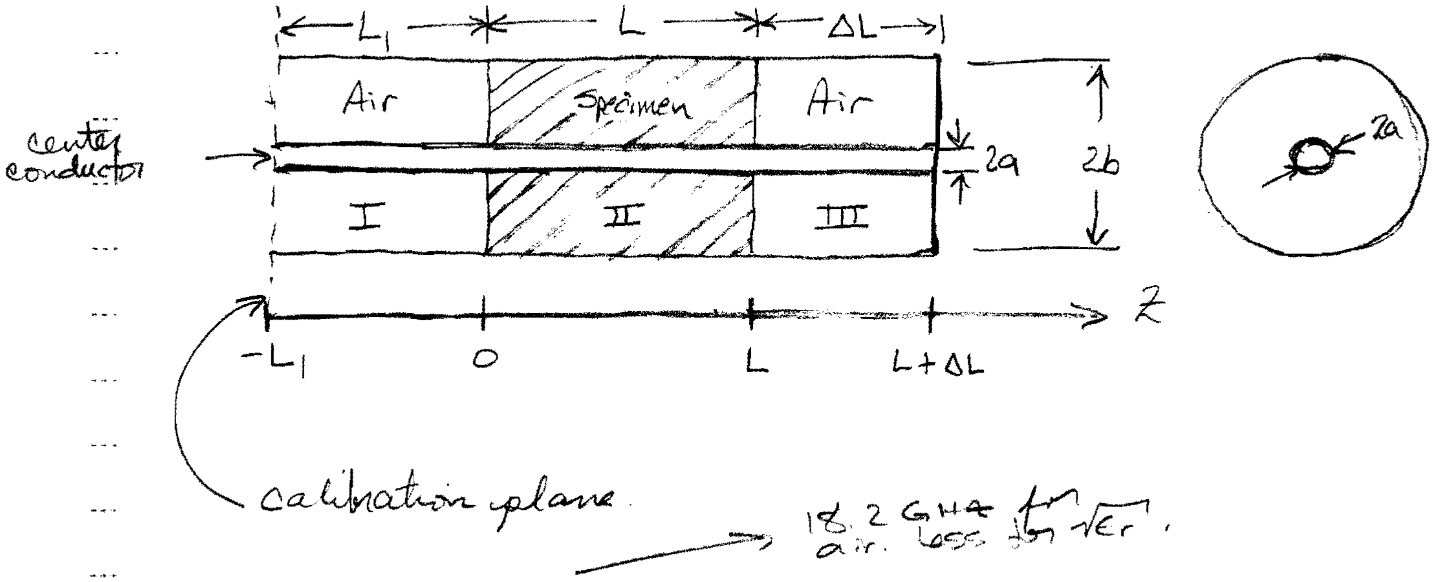
Short Circuit Line Measurements

The first test fixture we'll study is a short circuited TL with a specimen located some distance away from the short as shown below.

We will closely follow the exposition of Baker-Jarvis, "Transmission/Reflection and Short-Circuit Line Permittivity Measurements," NIST Tech. Report 1341, July. 1990. This approach also used in your text, pp. 143-144.



For a 7-mm test fixture, $2a \approx 3mm$ & $2b \approx 7mm$.

A signal is assumed incident from the left and is incident on this three-region TL structure. The voltages in each of these three regions will be written as

Reason for ΔL is E will be smaller near the short. May wish to place near an E max for better sensitivity device.

$$V_I(z) = e^{-\gamma_0 z} + C_1 e^{+\gamma_0 z} \qquad (1)$$

$$V_{II}(z) = C_2 e^{-\gamma z} + C_3 e^{\gamma z} \qquad (2)$$

and

$$V_{III}(z) = C_4 e^{-\gamma_0(z-L)} + C_5 e^{\gamma_0(z-L)} \qquad (3)$$

where γ_0 and γ are prop. constants in air and specimen, respectively. We've assumed incident wave has amplitude = 1.

These coefficients $C_1 - C_5$ can be solved for by applying appropriate b.c's at $z = 0$, L, and $L + \Delta L$. (See hmwk)

Of particular interest is the voltage reflection coeff. at the front face of the specimen at $z = 0$.

$$\Gamma \equiv \left.\frac{V_I^-}{V_I^+}\right|_{z=0} = C_1 \qquad (4)$$

From Baker-Jarvis & text eqn (3.6)

$$\Gamma = C_1 = \frac{-2\tilde{\beta}\delta + [(\delta+1) + (\delta-1)\tilde{\beta}^2]\tanh(\gamma L)}{2\tilde{\beta} + [(\delta+1) - (\delta-1)\tilde{\beta}^2]\tanh(\gamma L)} \qquad (5)$$

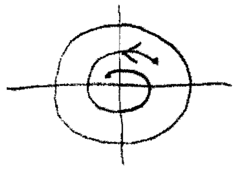
where $\tilde{\beta} = \dfrac{\gamma \mu_0}{\gamma_0 \mu}$ and $\delta = e^{-2\gamma_0 \Delta L}$ (6)

An alternate form is given in text eqn. (3.9).

Eqn. (5) was derived for reflection from the front face of the sample ($z = 0$).

Alternatively, perhaps the calibration has been performed at the end of the cable, and the fixture extends at $z = -L_1$. In this case, we can use the simple relation

$$\Gamma' = \Gamma e^{-2\gamma_0 L_1} \qquad (7)$$



where Γ' is the reflection coeff referenced to a calibration plane at the opening to the fixture

The complex permittivity of a nonmagnetic specimen ($\mu = \mu_0$) can be determined from (5) and measured S_{11} data using numerical root finding methods. No explicit expressions for ϵ. Will discuss this later.

Correcting for Systemic Errors

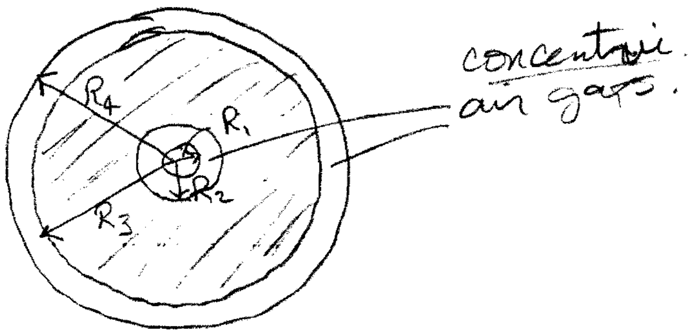
Once the ϵ' & ϵ'' have been extracted from the measurement data, one should also apply corrections due to systematic errors related to the specimen & fixture.

We'll briefly discuss corrections due to gaps between the specimen and the conducting walls of the fixture.

Gap Correction

This is an important source of error to understand and minimize. Samples that are assumed to completely fill a cross-section must be manufactured to exacting tolerances. In this coaxial air line system, we can expect air gaps between the sample and the inner and outer conductors. Because of the larger E near the center conductor, this fit is more critical. (Perhaps cool center conductor before sliding in?)

Here, will consider a simple model for gap correction.



For a coaxial line supporting TEM mode, with voltage V between inner & outer conductors,

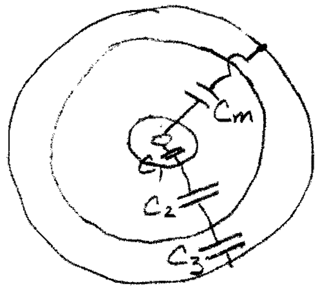
$$E_r = \frac{V}{\ln(\frac{b}{a})\, r} \quad \text{\&} \quad \Phi_e = -\int_a^r E_r(r)\, dr = \frac{V}{\ln(\frac{b}{a})}\left(-\ln(r)\right)\Big|_a^r$$

$$= \frac{V}{\ln(\frac{b}{a})} \cdot \ln\left(\frac{r}{a}\right) \qquad (B)$$

Φ_e is not a fct. of ϕ. Hence, can treat any r = constant surface as an <u>equipotential</u> surface. Between any two equipotential surfaces, can associate an <u>effective</u> cap.

even though there's no storage of free charge. Trick!

So, will treat the measured "capacitance" as that produced by 3 caps, in series.



$$\therefore \quad \frac{1}{C_m} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{9}$$

For a coaxial line of length L, $C = \dfrac{2\pi\epsilon' L}{\ln(b/a)}$ $\quad b > a$ (10)

Sub (10) → (9)

$$\frac{\ln\left(\frac{R_4}{R_1}\right)}{\epsilon_m'} = \frac{\ln\left(\frac{R_2}{R_1}\right)}{\epsilon_1'} + \frac{\ln\left(\frac{R_3}{R_2}\right)}{\epsilon_c'} + \frac{\ln\left(\frac{R_4}{R_3}\right)}{\epsilon_1'} \tag{11}$$

measured $\uparrow$ (ϵ_m'), corrected $\uparrow$ (ϵ_c'), air (ϵ_1')

Solve for the corrected value ϵ_c':

$$\frac{\ln\left(\frac{R_3}{R_2}\right)}{\epsilon_c'} = \frac{\ln\left(\frac{R_4}{R_1}\right)}{\epsilon_m'} - \frac{\ln\left(\frac{R_2}{R_1}\right)}{\epsilon_1'} - \frac{\ln\left(\frac{R_4}{R_3}\right)}{\epsilon_1'} \tag{12}$$

define
$$L_1 = \ln\left(\frac{R_2}{R_1}\right) + \ln\left(\frac{R_4}{R_3}\right)$$
$$L_2 = \ln\left(\frac{R_3}{R_2}\right)$$
$$L_3 = \ln\left(\frac{R_4}{R_1}\right)$$

Sub in (12):

$$\frac{L_2}{\epsilon_c'} = \frac{L_3}{\epsilon_m'} - \frac{L_1}{\epsilon_1'} \Rightarrow \epsilon_c' = \frac{L_2}{\frac{L_3}{\epsilon_m'} - \frac{L_1}{\epsilon_1'}}$$

s.t. $\epsilon_c' = \epsilon_m' \epsilon_1' \dfrac{L_2}{\epsilon_1' L_3 - \epsilon_m' L_1}$

In the case $\epsilon_1' = \epsilon_0$ then $\underline{\epsilon_c' = \epsilon_0 \epsilon_m' \dfrac{L_2}{\epsilon_0 L_3 - \epsilon_m' L_1}}$ (13)

Given measured ϵ_m', obtain gap corrected ϵ_c' from (13).
This is a low frequency gap correction method.

We've assumed, though, that the gap thicknesses are uniform along the entire length of the specimen, that the gaps are concentric with specimen & the fixture, and that we have values for these gaps.
All big assumptions!