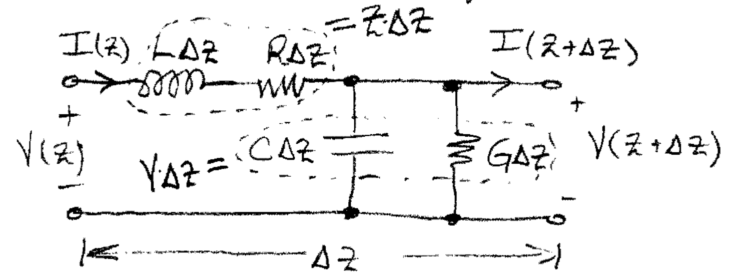
Lossy TLs

The first fixture we'll use to extract material parameters from samples placed within them is a shorted coaxial line. We model this structure as a transmission line (TL); in particular, a lossy TL.

The lumped ckt model for an infinitesimal section of a TL is



- R is due to conductor losses in the metal parts of the coax.
- G is due to losses in the material between the conductors. (Note that $G \neq \frac{1}{R}$)

The phasor-domain telegrapher's eqns for this lossy TL are:

$$\frac{dV(z)}{dz} = -Z\,I(z) \qquad (1)$$

$$\frac{dI(z)}{dz} = -Y\,V(z) \qquad (2)$$

where $Z = R + j\omega L$ & $Y = G + j\omega C$ (3), (4)

(1) and (2) can be combined in ways to produce second-order O.D.E.'s for $V(z)$ and $I(z)$ as:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \qquad (5)$$

and

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \qquad (6)$$

where $\gamma^2 = Z \cdot Y = (R + j\omega L) \cdot (G + j\omega C)$ (7)

($s.t.$ $\gamma = \pm\sqrt{\gamma^2}$ need to be aware of this!)

Solutions to (5) & (6) are

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \qquad (8)$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z} \qquad (9)$$

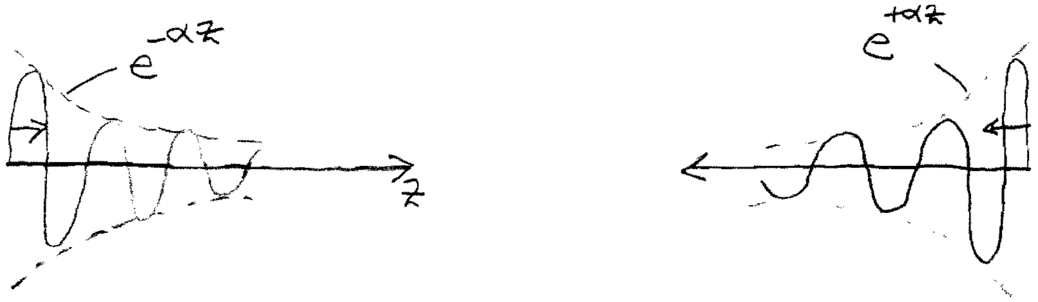
where $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$.

The propagation constant γ is a complex quantity, as we can deduce from (7). We define the real & imag. parts of γ as

$$\gamma \equiv \alpha + j\beta \qquad (10)$$

Hence, the voltage & current waves in (8) & (9) are attenuated as they propagate

ref. 322 notes for figs.



Another non-ideal characteristic of lossy TL's is the <u>signal dispersion</u>, which occurs when the signal velocity is a fct. of frequency.

Phase velocity on a lossy line is given by the same expression as a lossless line,

$$v_p = \frac{\omega}{\beta} \qquad (11)$$

where $\beta = \mathrm{Im}(\gamma)$, and will not generally be proportional to ω for a lossy line. (Exception is Heaviside dispersionless TL where $R/L = G/C$. In this case, α and v_p are not fcts. of ω!)