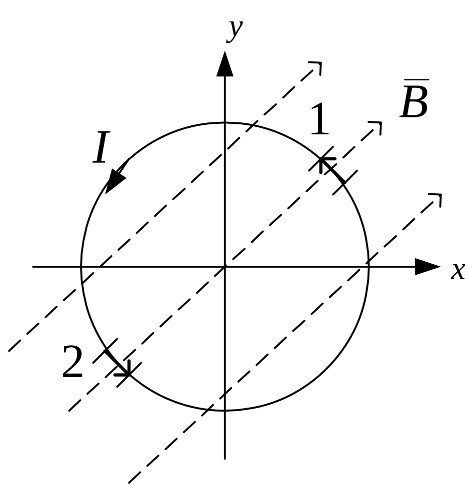
Lecture 3: Basic Properties of Magnetic Materials

Magnetic materials become "magnetized" when they are placed in a magnetic field. Examples of magnetic materials include steel and iron. (These are not necessarily magnets.)

This magnetization is analogous to the "polarization" of dielectric materials when they are placed in an electric field.

Torque on a Current Loop in a Magnetic Field

To develop an appreciation for the effects of magnetized materials, we will first consider a small loop of current in a uniform $\bar{B}$:



The net force $\bar{F}_{net}$ on the loop can be computed as

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$$\bar{F}_{net} = \oint_c \left(I d\bar{l} \times \bar{B} \right) = I \underbrace{\left(\oint_c d\bar{l} \right)}_{=0} \times \bar{B} = 0$$

We see that the net force on the loop is zero. It turns out that this is true for any shape of current loop provided the loop is immersed in a uniform $\bar{B}$.

While $\bar{F}_{net} = 0$, this loop does experience a torque, $\bar{T}$. (Recall that for a point object $\bar{T} = \bar{r} \times \bar{F}$.)

To see this effect, consider the two current segments 1 and 2 shown in the figure. Using the Lorentz force equation

$$\bar{F}_m = q\bar{v} \times \bar{B}$$

then

- at position 1, $\bar{F}_m$ is in the $-\bar{a}_z$ direction
- at position 2, $\bar{F}_m$ is in the $+\bar{a}_z$ direction

Consequently, this loop will rotate if it's free to do so.

We can compute the torque on this loop beginning with the elemental torque $d\bar{T}$ on current element $d\bar{l}$ as

$$d\bar{T} = \bar{r} \times d\bar{F}$$

where $d\bar{F} = I d\bar{l} \times \bar{B}$.

The total net torque on the entire loop is then

$$\bar{T} = \oint_{\text{loop}} d\bar{T} = I \oint_{\text{loop}} \bar{r} \times \left(d\bar{l} \times \bar{B} \right)$$

which for this loop example evaluates to

$$\bar{T} = I\pi a^2\left(-\bar{a}_x B_y + \bar{a}_y B_x\right) \tag{1}$$

Magnetic Dipole Moment

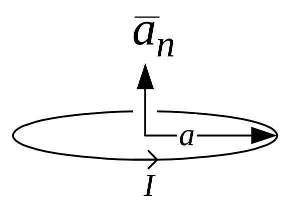
This result in (1) can be expressed in the more succinct form

$$\bar{T} = \bar{m}\times\bar{B} \quad [\mathrm{N\cdot m}] \tag{2}$$

where $\bar{m}$ is called the magnetic dipole moment of the current loop given by

$$\bar{m} = \bar{a}_n I \underbrace{\pi a^2}_{\text{area}} \quad [\mathrm{A\cdot m^2}]$$

for this circular loop. The direction of $\bar{a}_n$ is determined by the current direction and the RHR as



In general, for any small, arbitrarily-shaped and planar current loop

$$\bar{m} = \bar{a}_n m = \bar{a}_n IA \tag{3}$$

where A is the planar area of the loop.

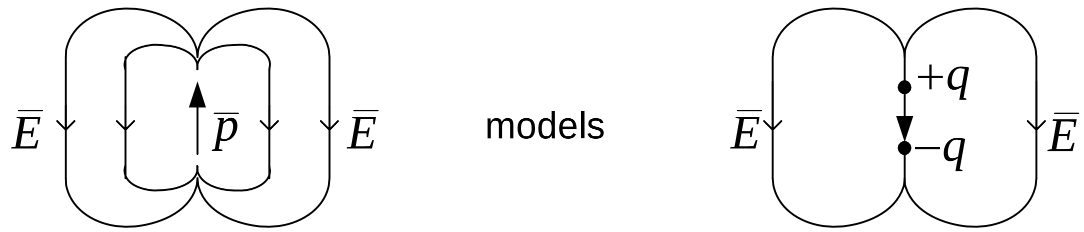
Finally, observe two points concerning this magnetic dipole moment:

1. This current loop will rotate if it is free to do so. With the thumb in the direction of $\bar{T}$, the fingers give the sense of

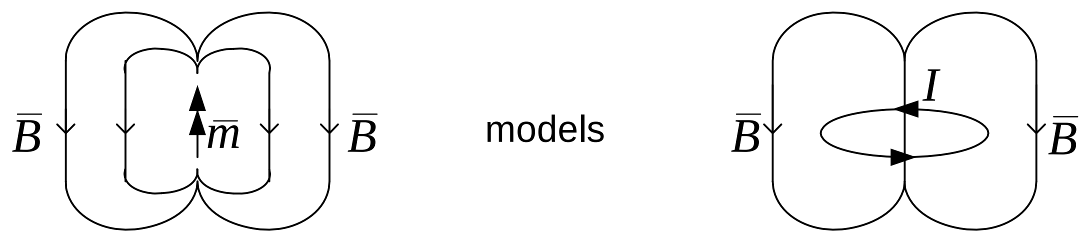
rotation. This loop rotation will continue until $\bar{m}$ and $\bar{B}$ are parallel. Then $\bar{T} = \bar{m} \times \bar{B} = 0$. (This is a general result.)

2. The magnetic dipole moment $\bar{m}$ is analogous to $\bar{p}$ in electrostatics:

Electrostatics:

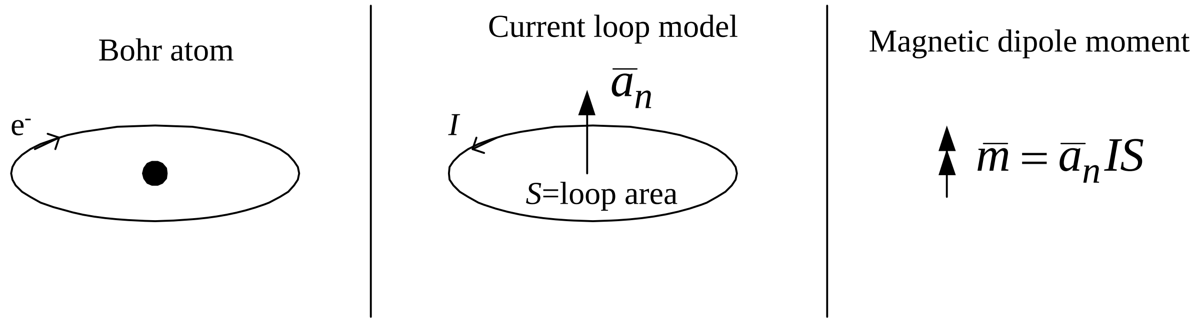


Magnetostatics:



Magnetic Material Model

The magnetic dipole moment is used to model the microscopic effects of magnetized materials. For example:

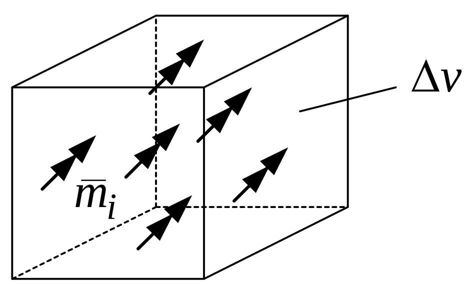


Note that "magnetized" does not mean permanent magnetization. Rather, a "magnetized" material in magnetostatics is analogous to a "polarized" dielectric in electrostatics.

There are many sources for the magnetization effect other than the "current loops" shown above. Some of these sources require a quantum mechanical description.

Magnetization Vector

Consider a magnetized volume of material containing many magnetic dipole moments $\bar{m}_i$:



A magnetization vector field $\bar{M}$ is defined as

$$\bar{M} = \lim_{\Delta v \to 0} \frac{\sum_{i=1}^{N} \bar{m}_i}{\Delta v} \quad \text{[A/m]} \tag{4}$$

where N is the number of magnetized molecules in Δv.

The macroscopic effects of a magnetized material are modeled with $\bar{M}$ in a process similar to $\bar{P}$ and a polarized dielectric.

In particular, the magnetic field intensity $\bar{H}$ is defined as

$$\bar{H} \equiv \frac{\bar{B}}{\mu_0} - \bar{M} \quad \text{[A/m]} \tag{5}$$

or equivalently,

$$\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M}$$

From experimentation, it has been found for many materials that

$$\bar{M} = \chi_m \bar{H} \tag{6}$$

χ_m is called the magnetic susceptibility, a dimensionless quantity.

Substituting (6) into (5) gives

$$\bar{B} = \mu_0 \bar{H} + \mu_0 \chi_m \bar{H} = \mu_0 \left(1 + \chi_m\right) \bar{H}$$

which can be written as

$$\bar{B} = \mu \bar{H} \tag{7}$$

This is the second constitutive equation we will use. In (7)

$$\mu = \mu_r \mu_0 = \left(1 + \chi_m\right) \mu_0 \tag{8}$$

and μ_r is called the relative permeability (dimensionless).

Types of Magnetic Materials

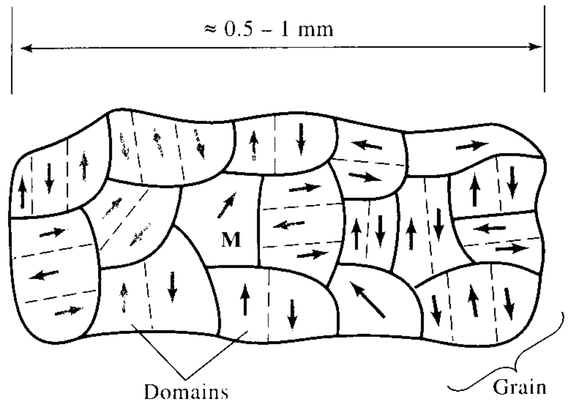
There are five main types of magnetic materials:

1. **Diamagnetic** – $\mu_r \lessapprox 1$. Examples are water and copper.
2. **Paramagnetic** – $\mu_r \gtrapprox 1$. Examples are air and aluminum.
3. **Ferromagnetic** – $\mu_r \gg 1$. Examples are cobalt, steel and nickel.
4. **Ferrimagnetic** – $\mu_r \gg 1$ (but less than ferromagnetic materials). Examples are MnZn and NiZn.
5. **Antiferromagnetic** – $\mu_r \approx 1$. Examples are chromium and manganese.

Ferromagnetic materials are a very interesting class of materials. They have extremely large μ_r reaching as high as 1,000,000!

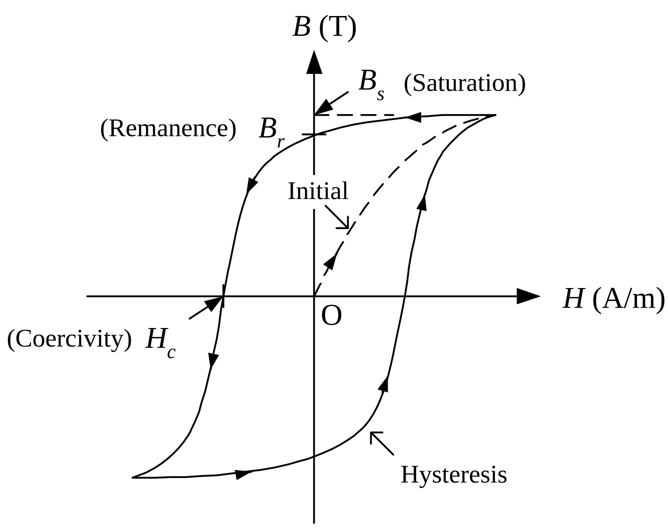
However, these materials can be highly nonlinear. They are used in electric motors and generators among other applications. Ferromagnetic materials are also used to make so called permanent magnets.

The reason ferromagnetic materials have such large μ_r is the existence of magnetized domains. These are regions of high $\bar{M}$ with dimensions on the order of 0.1 to 1 mm^3 inside the material:



Because of these magnetized domains and their interactions (which can be highly quantum mechanical), there are three distinct regimes in which the ferromagnetic material may operate:

1. Without an external $\bar{B}$, the domains are randomly orientated and the net $\bar{M} = 0$. This is point "O" in the magnetization curve:

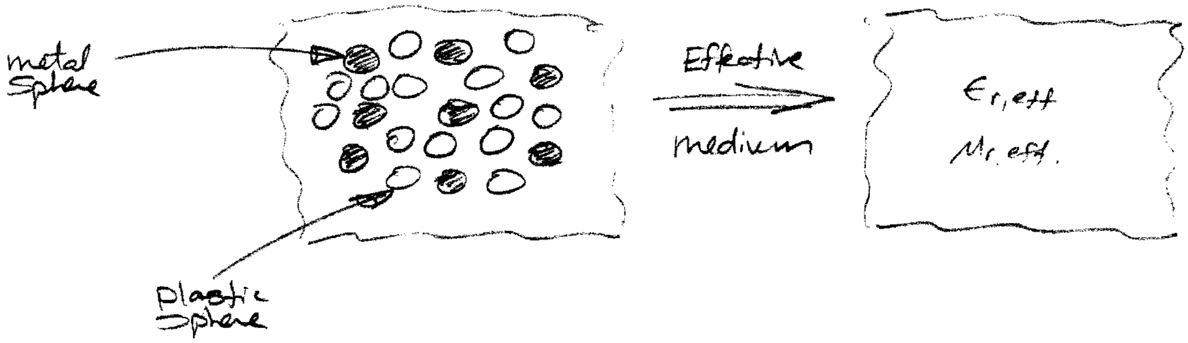


2. With a small external $\bar{B}$, the domains align and produce a large $\bar{M}$, and consequently a large μ_r. This linear region is called the "initial" magnetization curve.

3. If the external $\bar{B}$ becomes large enough, the material can "saturate" and enter the hysteresis region. This is very nonlinear. For example, when $\bar{H}$ is reduced to zero, $B = B_r$ in the material. It is not zero! This material is now a permanent magnet.

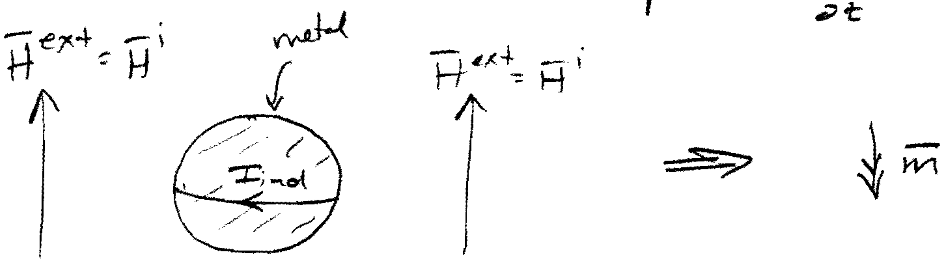
Effective Media

One can also create "artificial materials" by mixing together electrically small particles in a host material. Consider an example of metal and plastic spheres randomly dispersed in a host material.



There is obviously an effective permittivity, but there is also an effective permeability. Why $\mu_{r,eff}$? Because of electromagnetic induction: $\oint E \cdot dl = -\frac{\partial \Psi_m}{\partial t}$

$$emf = -\frac{\partial \Psi_m}{\partial t}.$$

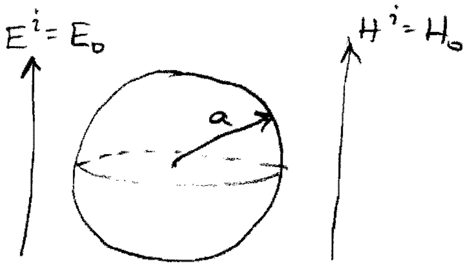


Lenz's Law.

The current on the sphere is induced, not impressed like in the current loop discussed earlier.

Referring to S. A. Schelkunoff & H. T. Friis, "Antennas, Theory and Practice," John Wiley, pp. 579-580, 1952.

For an isolated metalic sphere, they show that



$$p_e = \underbrace{4\pi\epsilon_0 a^3}_{= \chi_e^0} E_0$$

↑ electric polarizability of an isolated metal sphere of radius a.

$$\& \quad p_m = \underbrace{-2\pi\mu_0 a^3}_{= \chi_m^0} H_0$$

↑ magnetic polarizability of an isolated metal sphere.

So, the dispersion of small metallic spheres gives rise to both electric & magnetic dipole moments leading to both electric & magnetic polarizations. These can be used in so-called effective media models to estimate effective permittivity & permeability for the medium. (we'll discuss this more towards the end of this class.)

Notice in the above formulas that p_m is <u>negative</u>.

This means that the induced magnetic dipole moment is in the opposite direction as the applied magnetic field.

Consequently, the effective relative permeability of this medium will be less than one! $\mu_{r,eff} < 1$ (but > 0).

Concerning the effective index of refraction of the material, half of the increase due to the larger effective permittivity is cancelled because of this decrease in the effective permeability. Bummer.

These are general results for non-resonant metallic particles, not just spheres.