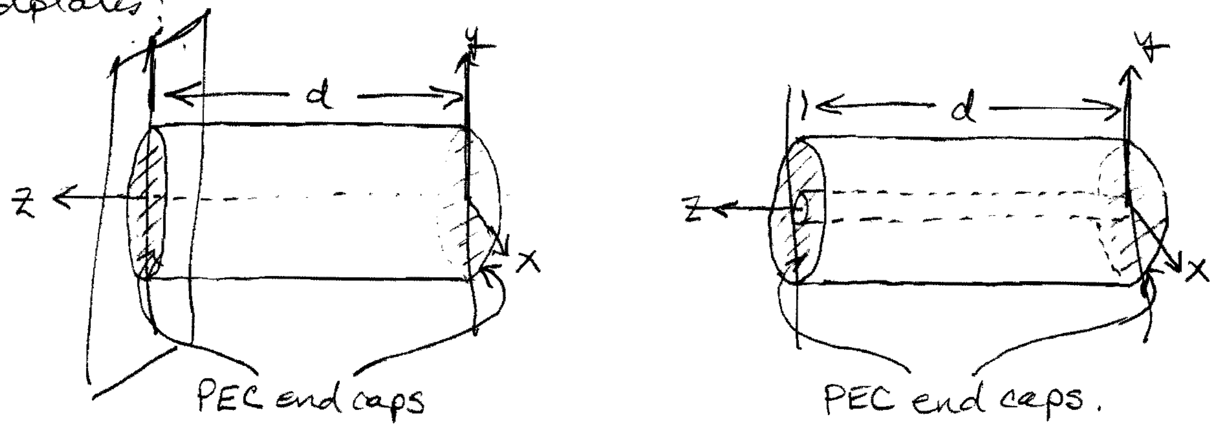
# Circular Waveguide Resonant Cavities



[Ref. Pozar, 3rd Sec. 6.4]

Waldron pp. 273-277.

We can use circular waveguides & coaxial waveguides as resonant cavities by adding PEC endplates:




The analysis of these structures proceeds the same as for the rectangular waveguide cavities. In general, we will incorporate all of our previous analyses for infinitely long guides by requiring the end caps to be perpendicular to the axes of the waveguide. This keeps the interior of the cavity a separable geometry.

Next, we'll use the simple normal modes (SNM) of the infinitely long guide – as these are also the normal modes of the resonant cavity – and apply b.c.'s at the end caps. This process will lead us to constrain the axial wavenumber $\beta_z$ to certain discrete values, which will only occur at discrete frequencies.

# Circular Waveguide Resonant Cavity

We will apply b.c.'s to $E_{tan}$ at $z=0$ & $d$. We expect wave propagation in $+z$ & $-z$ directions:

$$\bar{E}_{tan}(\rho,\phi,z) = \bar{e}(\rho,\phi)\left[A^+ e^{-j\beta_{znm}z} + A^- e^{+j\beta_{znm}z}\right] \qquad (1)$$

We aren't concerned about the specific form of $\bar{e}(\rho,\phi)$ at this time. These satisfy appropriate b.c. internally to the waveguide.

For $\bar{E}_{tan}=0$ at $z=0$ & $d$ requires from (1) that

$$A^+ = -A^-$$

and that

$$A^+ \sin(\beta_{znm} d) = 0 \qquad (2)$$

To satisfy (2)

$$\beta_{znml} = \frac{l\pi}{d} \qquad l=1,2,3,\ldots \qquad (3)$$

This is the same constraint equation we derived for rectangular waveguide resonators.

From the dispersion equation for the waveguide

$$\beta_c^2 + \beta_z^2 = \beta^2 = \omega^2\mu\epsilon \qquad (4)$$

or

$$f_{cnml} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\beta_{cnm}^2 + \beta_{znml}^2}$$

or

$$f_{c_{nml}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\beta_{c_{nm}}^2 + \left(\frac{l\pi}{d}\right)^2} \qquad (5)$$

• $TE^z_{nml}$ modes. For $TE^z_{nm}$ modes in a circular waveguide we found that

$$\beta_{c_{nm}} = \frac{p'_{nm}}{a} \qquad \begin{matrix} n=0,1,2,\cdots \\ m=1,2,3,\cdots \end{matrix} \qquad (6)$$

Sub. (6) → (5):

$$\underline{\underline{f_{c_{nml}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}}} \qquad (7)$$

$$n=0,1,2,\cdots$$
$$m,l=1,2,3,\cdots$$

• $TM^z$ modes. For the $TM^z_{nm}$ modes in a circular waveguide, we found that

$$\beta_{c_{nm}} = \frac{p_{nm}}{a} \qquad \begin{matrix} n=0,1,2,\cdots \\ m=1,2,3,\cdots \end{matrix} \qquad (8)$$

Sub. (8) → (5):

$$f_{c_{nml}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \qquad \begin{matrix} n,l=0,1,2,\cdots \\ m=1,2,3,\cdots \end{matrix} \qquad (9)$$

which are the resonant frequencies of the circular waveguide resonant cavity.

Note that, as with rectangular waveguide resonators, $l(=p) \neq 0$ for $TE^z$ modes while $l(=p)$ can equal 0 for $TM^z$ modes.

A consequence of this is that while the $TE_{11}$ mode is the dominant mode of the circular waveguide, the $TE_{110}$ mode is not the dominant resonant cavity mode since $l=0$ is not allowed for $TE^z$ modes.

Rather, the $TM_{010}$ mode is the dominant resonant cavity mode in a circular waveguide resonant cavity for $\frac{d}{a} < 2$. For $\frac{d}{a} \geq 2$, $TE_{111}$ is the dominant mode.

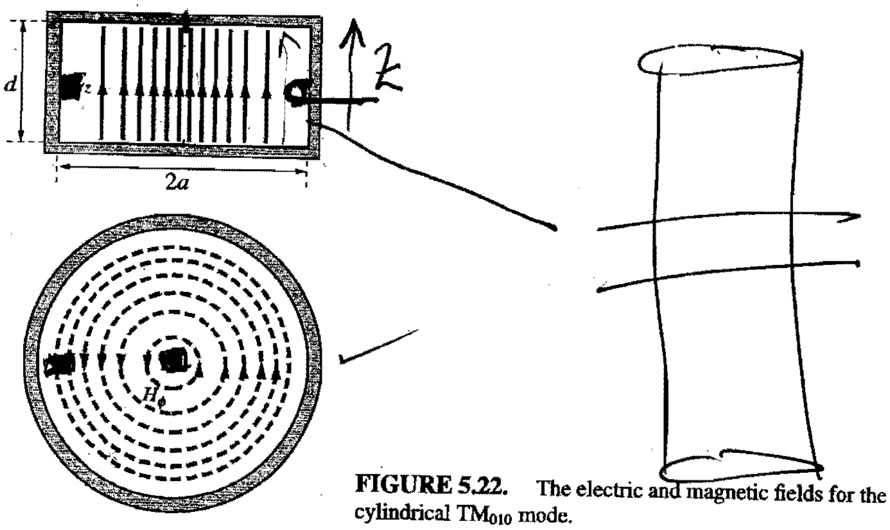



**FIGURE 5.22.** The electric and magnetic fields for the cylindrical $TM_{010}$ mode.

Reference: U. S. Inan and A. S. Inan, *Electromagnetic Waves*. Upper Saddle River, NJ: Prentice-Hall, 2000.

(See Pozar's Fig. 6.9. Confused? Is $TM_{010}$ always dominant mode?)

→ see Harrington, "THEF," p. 214, Table 5-4. If $\frac{d}{a} < 2$, $TM_{010}$ is the dominant mode. If $\frac{d}{a} \geq 2$, then $TE_{111}$ is the dominant mode. (length < diameter)