

## Lecture 2: Basic Properties of Dielectric Materials

In electromagnetics we classify materials generally into four broad categories:

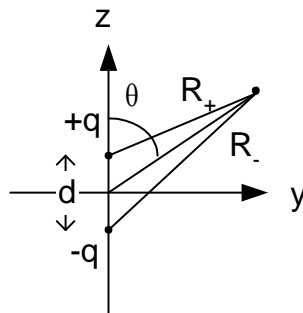
1. Conductor – free charge moves easily through the material
2. Semi-conductor – free charge moves somewhat
3. Dielectric (insulators) – no free charge, but material produces change to electric field
4. Magnetic – material produces change to magnetic field

The electric dipole is used to model the effects that a dielectric material produces on an external electric field. Before discussing dielectric materials, we will first quickly review the **electric dipole moment** model.

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### Electric Dipole

An **electric dipole** is formed from two charges of opposite sign and equal magnitude located close together (wrt to observation distance).



The absolute electric potential is the sum

$$\Phi_e(\bar{r}) = \frac{q}{4\pi\epsilon_0 R_+} - \frac{q}{4\pi\epsilon_0 R_-} \quad (1)$$

where

$$R_{\pm} = \sqrt{y^2 + \left(z \mp \frac{d}{2}\right)^2}$$

Now, if the observation distance is much greater than  $d$ , then (1) can be approximated as

$$\Phi_e(\bar{r}) \approx \frac{qd \cos\theta}{4\pi\epsilon_0 r^2} \quad (2)$$

Defining the **electric dipole moment** as

$$\bar{p} = q\bar{d} \quad (3)$$

then (2) can be expressed as

$$\Phi_e(\bar{r}) \approx \frac{\bar{p} \cdot \bar{a}_R}{4\pi\epsilon_0 R^2} \quad (4)$$

This is the absolute potential at point  $\bar{r}$  of an electric dipole of moment  $\bar{p}$  located at coordinates  $\bar{r}'$ , where  $R = |\bar{r} - \bar{r}'|$ .

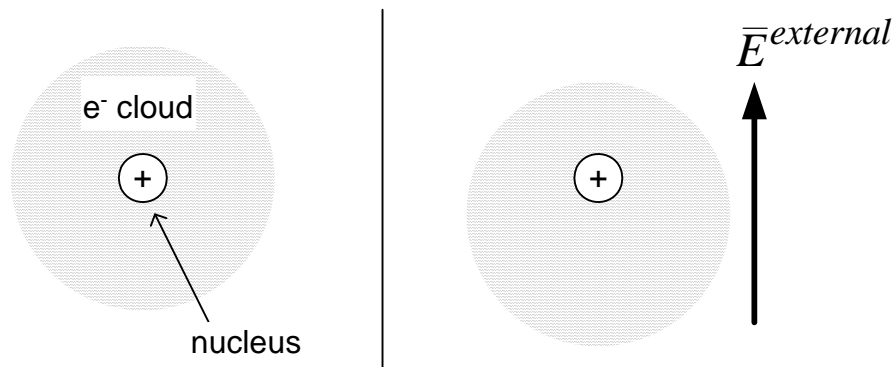
Equation (4) is an accurate representation of the potential and electric field produced by the dipole provided the observation distance is approximately 2.5 times  $d$ . (See *VisualEM*, Section 3.6.2 worksheet.)

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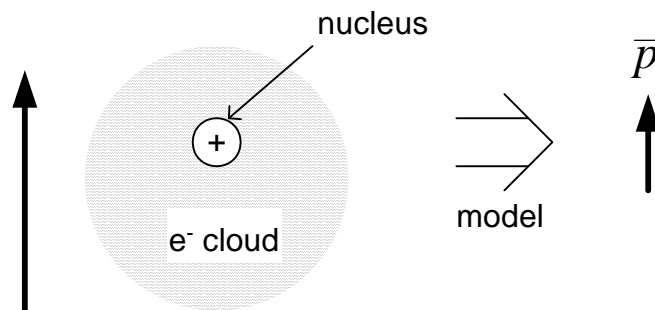
## Bound Charge

When a dielectric material is placed in an external electric field, the dielectric alters this electric field due to **bound** or **polarization charges** that are formed in the dielectric. A capacitor is an example of this.

A simplistic model of the atomic conditions that produce this bound charge is the displacement of the electron cloud around a nucleus. In an electric field, the negatively charged electron cloud becomes **displaced very slightly** from the positively charge nucleus:

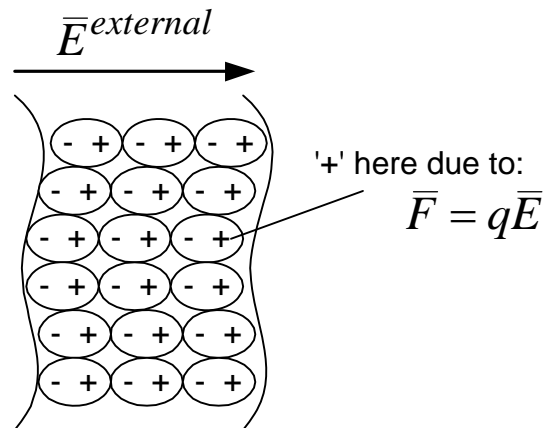


The electrostatic effects of this displacement (potential and electric field) are model by an **electric dipole moment**  $\vec{p}$ :



This is only an approximation, but for charge neutral molecules and relatively “small” electric fields, it is a very good model.

Using this model, a polarized material can be visualized as



These **bound charges** cannot move. Unlike free charge, bound charge is **induced** by an external  $\vec{E}$  field and vanishes when the external  $\vec{E}$  field is removed.

## Multipole Expansion

Imagine that a localized distribution of electric charge density  $\rho_e(\vec{r}')$  is centered at the origin of a coordinate system.

The electric potential outside of an imaginary sphere of radius  $R$  that fully contains the charge can be written as an expansion in so-called **spherical harmonics** as (Jackson, 3<sup>rd</sup> ed., p. 145)

$$\Phi_e(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{4\pi}{2l+1} q_{lm} \frac{Y_{lm}(\theta, \phi)}{r^{l+1}}$$

where  $q_{lm}$  is the multipole moment coefficients and  $Y_{lm}$  is the spherical harmonic function.

After a bit of manipulation, this equation reduces to

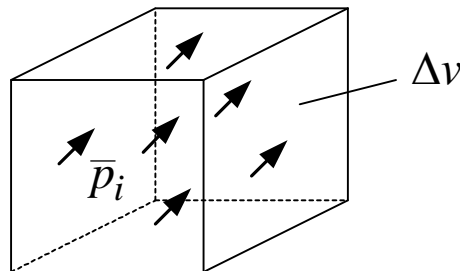
$$\Phi_e(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left( \frac{Q_e}{r} + \frac{\vec{p} \cdot \vec{r}}{r^3} + \text{H.O.T.} \right)$$

The higher order terms in this expression decay as  $1/r^4$  or quicker. Hence, the potential produced by these terms is much smaller than the  $1/r^2$  term.

Further, if the entire distribution of charge is neutral so that  $Q_e=0$ , then the **potential is dominated** by the  $1/r^2$  term, which is the **electric dipole term**. That is why we model the bound charge in a material by the dipole term only.

## Polarization Vector

Consider a polarized volume with a density of  $\vec{p}$ 's:



A **polarization vector**  $\vec{P}$  is defined as

$$\bar{P} = \lim_{\Delta v \rightarrow 0} \frac{\sum_{i=1}^N \bar{p}_i}{\Delta v} \quad [\text{C/m}^2] \quad (5)$$

where  $N$  is the number of molecules in  $\Delta v$ .

The macroscopic effects of a polarized dielectric material are modeled by  $\bar{P}$ , which really is an average **dipole moment per unit volume** of the material.

In summary, when a dielectric material is placed in an external electric field, as we saw earlier, the dielectric alters this electric field due to **bound** or **polarization charges** that are formed in the dielectric.

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# Types of Dielectric Polarization

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Ref. Solymar and Walsh & Von Hippel.

## Electronic Polarization -

All matter consists, at a fundamental level, of positively charged atomic nuclei surrounded by negatively charged electron clouds. An applied  $\vec{E}$  displaces cloud wrt nucleus, creating an induced dipole. This is called electronic polarization.

Molecular (or atomic) polarization - Bonds between atoms are stretched by an applied  $\vec{E}$  creating induced dipole moments.

Orientalional (or dipolar) polarization - Certain molecules formed by different types of atoms will share electrons in an asymmetrical manner, causing the electron clouds to be displaced towards atoms with the stronger binding force. These molecules will have a permanent electric dipole, which could then be aligned to an applied  $\vec{E}$ , giving rise to orientational polarization.

Such materials are called polar. Materials w/o molecules with permanent dipoles are called non-polar.

Interfacial (or space-charge) polarization - free charge that has migrated or otherwise been deposited on outer surfaces of a dielectric. Creates polarization in dielectric indistinguishable from a real increase in permittivity.

These four polarization mechanisms add to form the total polarization:

$$\bar{P} = \bar{P}_e + \bar{P}_a + \bar{P}_d + \bar{P}_s$$



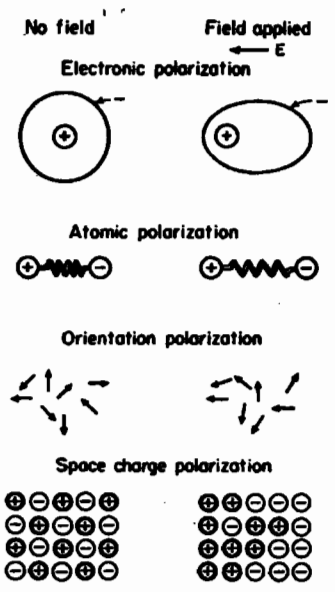


Fig. 1.1. Mechanisms of polarization.

Ref.: A. von Hippel, ed. *Dielectric Materials and Applications*. Boston: Artech House, second ed., 1995.

## Electric Susceptibility and Permittivity

It is customary in electromagnetics to “bury” the effects of bound polarization effects in materials through the electric flux density vector,  $\bar{D}$ . The polarization effects of a dielectric can be accounted for by defining  $\bar{D}$  as

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} \quad [\text{C/m}^2] \quad (6)$$

What we desire now is to know  $\bar{P}$  in terms of  $\bar{E}$ . Basically, without knowing  $\bar{P}$  this theory is not very useful.

It has been found through experimentation that for many materials with “small”  $\bar{E}$  that

$$\bar{P} = \varepsilon_0 \chi_e \bar{E} \quad (7)$$

where  $\chi_e$  is the **electric susceptibility** of a material (dimensionless).

Substituting (7) into (6) gives

$$\bar{D} = \varepsilon_0 \bar{E} + \varepsilon_0 \chi_e \bar{E} = \varepsilon_0 (1 + \chi_e) \bar{E}$$

We can rewrite this as

$$\bar{D} = \varepsilon \bar{E} \quad [\text{C/m}^2] \quad (8)$$

This is called a **constitutive equation**. The constant  $\varepsilon$  is called the **permittivity** of the material where

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi_e) \varepsilon_0 \quad [\text{F/m}] \quad (9)$$

and

$$\varepsilon_r = 1 + \chi_e \quad (10)$$

is called the **relative permittivity** of the material (dimensionless).

The relative permittivity  $\epsilon_r$  is usually measured for different materials and then tabulated. (A good reference book is A. von Hippel, *Dielectric Materials and Applications*, Artech House, 1995.)

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## Definitions

- **Linear** material means  $\chi_e \neq f(\bar{E})$
  - **Homogeneous** material means  $\chi_e \neq f(\bar{r})$
  - **Isotropic** material means  $\chi_e \neq f(\bar{a}_E)$  where  $\bar{a}_E$  is the direction of  $\bar{E}$
  - **Simple** material means that it is linear, homogeneous and isotropic.
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## Gauss' Law

In the presence of dielectric materials, there is a slight change that must be made to Gauss' law:

$$\oint_s \bar{D} \cdot d\bar{s} = Q_{free} \quad (11)$$

where  $Q_{free}$  is the **net free charge** enclosed by  $s$ .

Applying the divergence theorem to (11) and simplifying gives the point form of Gauss' law:

$$\nabla \cdot \bar{D} = \rho_v \quad (12)$$

where it is understood that  $\rho_v$  is the free charge density only.

These two equations (11) and (12) are **ALWAYS** true.

From these two equations, we can deduce that sources of  $\bar{D}$  are free charge only. Conversely, the sources of  $\bar{E}$  are both free and bound charge.