TL Resonators

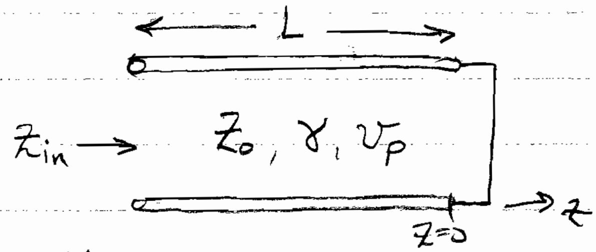
Certain EM waveguiding structures can be used as resonators, and in some cases with extremely high Q's. These are used in filters, oscillators, and material measurement systems. In the latter case, cavity-type resonators provide the highest accuracy measurements for low-loss materials.

Ref. Pozar, 3rd ed., Sections 6.1 & 6.2

TL Resonators

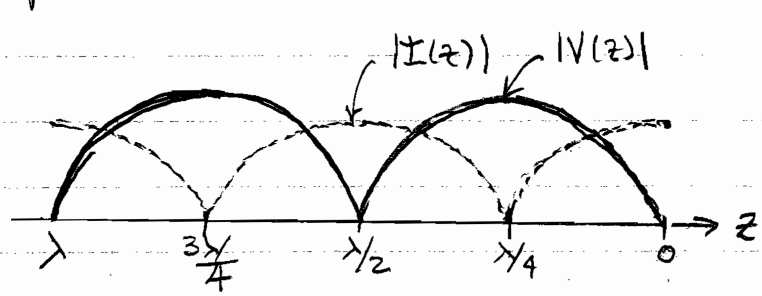
Transmission lines can be constructed to operate as resonators when the "load" is an open or short circuit.

- Short-circuited TL –



Since we're interested in the Q's of this resonator as well as its resonate frequencies, we will include effects of small loss.

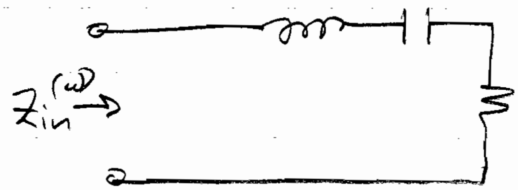
For a nearly lossless s.c. TL:



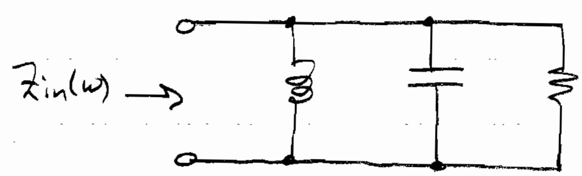
→ At $z = \frac{n\lambda}{2}$, $Z_{in} \approx 0$, $n = 1, 2, 3, \cdots$

→ At $z = \frac{n\lambda}{4}$, $Z_{in} \approx \infty$, $n = 1, 3, 5, \cdots$

Pozar shows that near the frequencies where $L \approx \frac{n\lambda}{2}$, $n=1,2,\cdots$, $Z_{in}(\omega)$ has the form of a series resonant circuit:



while at the frequencies where $L \approx \frac{n\lambda}{4}$, $n=1,3,5,\cdots$, $Z_{in}(\omega)$ has the form of a parallel resonant circuit:



Rutledge, ch. 4.

To estimate the Q of these resonators, we'll follow a different method than is traditionally used. We'll begin with the general definition of Q for a circuit in steady state:

$$Q = 2\pi \frac{\text{Max. energy stored at some time } t}{\text{Total energy lost per period}} \qquad (1)$$

$$= 2\pi \frac{E_{max}}{P_a \cdot T}$$

or $$Q = \omega \frac{E_{max}}{P_a} \qquad (2)$$

where P_a is average power dissipated in the circuit.

For a TL, where is the energy stored? Stored in the wave that is launched at the input & prop. down the TL to the end. (Once reflected & starts back, energy decreases)

With energy = power · time, then

$$E_{max} = P_+ \cdot t_d = P_+ \frac{L}{v_p} \qquad (3)$$

where P_+ is power in forward wave & t_d is one way delay time (when energy max.)

To determine the average dissipated power, we know for the lossy TL that $V^+ \propto e^{-\alpha L}$, so that $P_+ \propto (e^{-\alpha L})^2 = e^{-2\alpha L}$. Therefore, the one-way dissipated power is

$$P_a = P_+ - P_+ e^{-2\alpha L} \qquad (4)$$

If $2\alpha L$ is small, then $e^{-2\alpha L} \approx 1 - 2\alpha L$, using the truncated series expansion $e^x \approx 1 + x$ if $x \ll 1$. Then (4) becomes

$$P_a \approx P_+\left[1 - (1 - 2\alpha L)\right] = 2\alpha L P_+ \qquad (5)$$

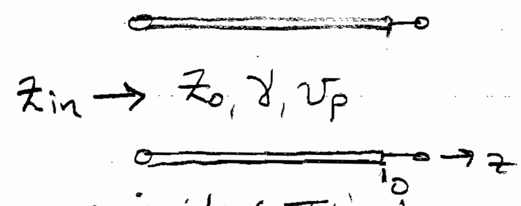
Consequently, using (3) & (5) in (2) gives

$$Q \approx \omega \frac{P_+ \frac{L}{v_p}}{2\alpha L P_+} = \frac{\omega / v_p}{2\alpha}$$

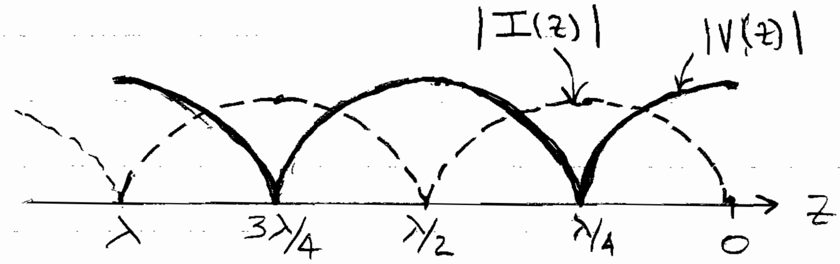
or

$$\underline{\underline{Q \approx \frac{\beta}{2\alpha}}} \qquad (6)$$

- Open-circuited TL –



For a nearly lossless open-circuited TL's:



$\rightarrow$ At $z = \frac{n\lambda}{2}$, $Z_{in} \approx \infty$, $n = 1, 2, 3, \cdots$

$\rightarrow$ At $z = \frac{n\lambda}{4}$, $Z_{in} \approx 0$, $n = 1, 3, 5, \cdots$

Pozar shows that near frequencies where $L \approx \frac{n\lambda}{2}$, $n = 1, 2, 3, \cdots$, $Z_{in}(\omega)$ has the form of a <u>parallel</u> <u>resonant</u> ckt. Conversely, near frequencies where $L \approx \frac{n\lambda}{4}$, $n = 1, 3, 5, \ldots$, the $Z_{in}(\omega)$ has the form of a <u>series</u> <u>resonant</u> ckt.

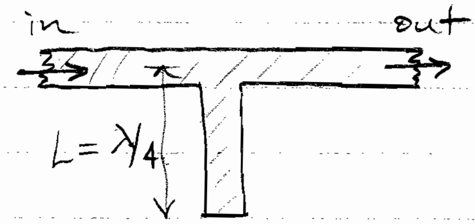
The derivation of Q for the open-ckt termination is identical to s.c. termination. Hence, for a open-ckt terminated TL resonator Q is

$$Q \approx \frac{\beta}{2\alpha}$$

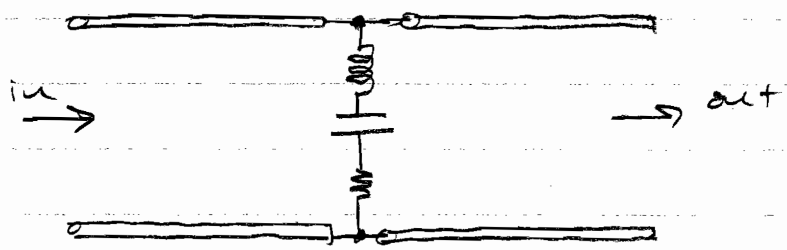
Applications are many in microwave ckts, though few at RF because of length of coax.

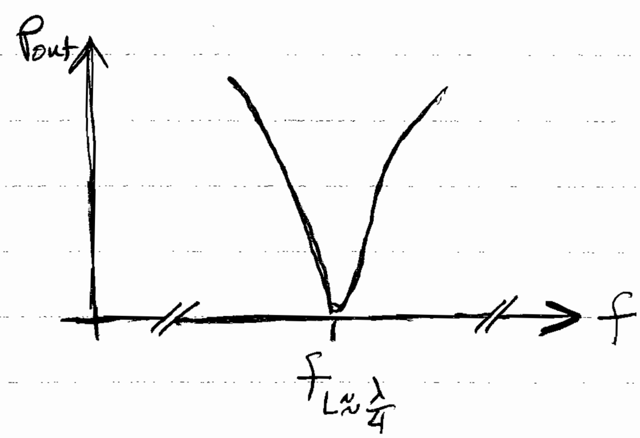
Examples

- Notch filter in microstrip.



Near the frequency where $L \approx \lambda/4$, equivalent input impedance of O.C. TL is a series resonant ckt:





Behaves as a notch filter.