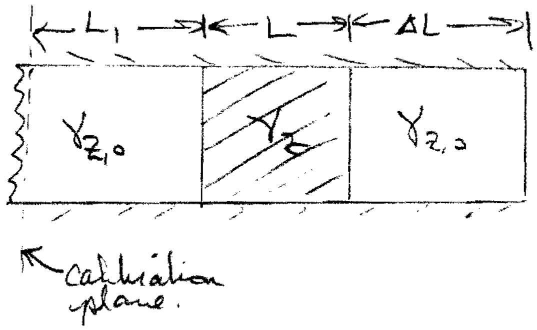
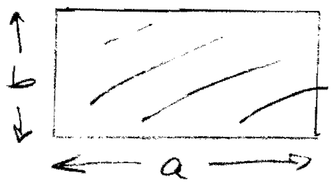
Extracting Material Parameters from Rectangular Waveguide Measurements

Equipped with the TL equivalent model for homogeneous waveguides, we are able to extract ϵ_r & μ_r from specimen measurements using all of the methods we have developed in this course. To accomplish this, we need only to substitute into the equations we developed for TEM waves the TL analogous quantity, such as $\gamma \to \gamma_z$.

We'll quickly review the changes.

- **Short circuit measurements:**





$$S_{11} = \frac{-2\tilde{\beta}\delta + [(\delta+1) + (\delta-1)\tilde{\beta}^2]\tanh(\gamma_z L)}{2\tilde{\beta} + [(\delta+1) - (\delta-1)\tilde{\beta}^2]\tanh(\gamma_z L)} \qquad (1)$$

where $\tilde{\beta} = \dfrac{\gamma_z \mu_0}{\gamma_{z,0}\mu}$ and $\delta = e^{-2\gamma_{z,0}\Delta L}$ (2), (3)

The waves prop. as $e^{\mp\gamma_0 z}$ in air regions and as $e^{\mp\gamma_z z}$ in the specimen.

For a rectangular waveguide (most likely TE_{10} mode)

$$\lambda_z\Big|_{f=f_c} = \infty$$

$$\gamma_z = j\sqrt{\beta^2 - \beta_c^2} \qquad (4)$$

$$\beta_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

We can express transverse wavenumber @cutoff $\beta_c = \dfrac{2\pi}{\lambda_c}$, (5)

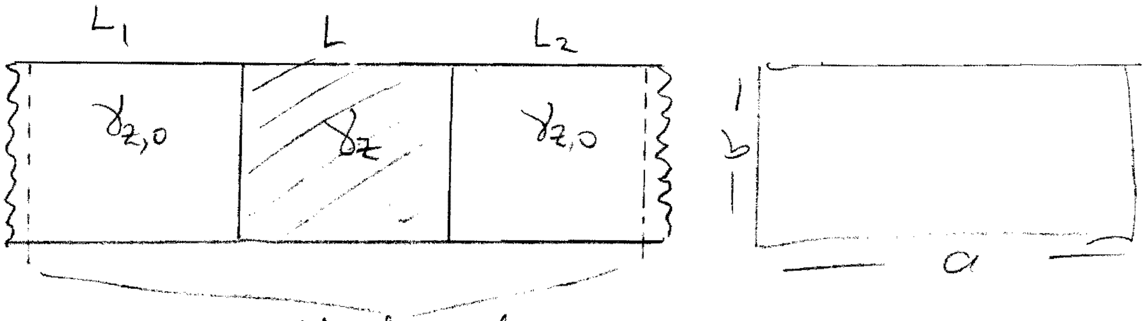
where λ_c is called the cutoff wavelength, to be consistent w/ the literature. Sub this into (4) & setting $\beta^2 = \omega^2 \mu \epsilon$ gives

$$\gamma_z = j\sqrt{\frac{\omega^2 \mu_r \epsilon_r}{c_0^2} - \left(\frac{2\pi}{\lambda_c}\right)^2} \qquad (6)$$

In the air regions, $\mu_r = \epsilon_r = 1$ so that

$$\gamma_{z,0} = j\sqrt{\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{2\pi}{\lambda_c}\right)^2} \qquad (7)$$

Transmission - Reflection Measurements:



calibration planes.

With reference to the faces of the specimen, we found that

$$S_{11} = \frac{\Gamma_p (1 - P^2)}{1 - \Gamma_p^2 P^2} = S_{22} \qquad (8)$$

and $$S_{21} = \frac{P(1-\Gamma_p)^2}{1-\Gamma_p^2 P^2} = S_{12} \tag{9}$$

In the case of a hollow metallic waveguide,

$$P \equiv e^{-\gamma_z L} \tag{10}$$

where γ_z defined in (6), and Γ_p is the partial ref. coeff at the face of the sample for a TE wave incident from the air region.

In a rectangular waveguide, the TE^z mode wave impedance is

$$Z_{TE} = \frac{\omega\mu}{\beta_z} = \frac{\omega\mu}{j\gamma_z} \tag{11}$$

Consequently,

$$\Gamma_p = \frac{Z_{TE} - Z_{TE,0}}{Z_{TE} + Z_{TE,0}}$$

or

$$\Gamma_p = \frac{\frac{\omega\mu}{j\gamma_z} - \frac{\omega\mu_0}{j\gamma_{z,0}}}{\frac{\omega\mu}{j\gamma_z} + \frac{\omega\mu_0}{j\gamma_{z,0}}} = \frac{\frac{\mu}{\gamma_z} - \frac{\mu_0}{\gamma_{z,0}}}{\frac{\mu}{\gamma_z} + \frac{\mu_0}{\gamma_{z,0}}} \left(\frac{\frac{\gamma_{z,0}}{\mu_0} \cdot \frac{\gamma_z}{\mu}}{\frac{\gamma_{z,0}}{\mu_0} \cdot \frac{\gamma_z}{\mu}} \right)$$

s.t.

$$\Gamma_p = \frac{\frac{\gamma_{z,0}}{\mu_0} - \frac{\gamma_z}{\mu}}{\frac{\gamma_{z,0}}{\mu_0} + \frac{\gamma_z}{\mu}} \quad \checkmark \quad \underline{\underline{TE^z}} \quad \underline{\underline{TE_{10}}} \tag{12}$$

which is used by Baker Jarvis