TE_{10} mode in Rectangular Waveguide

It is common to operate wvgds. in single mode frequency range. For $a>b$, this is the TE_{10} mode in rect. wvgds as we saw in the last lecture. Consequently, it is worthwhile to study this mode in some detail.

Field Plots

We determined previously for TE^z modes that

$$e_x(x,y) = \frac{j\omega\mu}{\beta_{cmn}^2}\beta_{yn}A_{mn}\cos(\beta_{xm}x)\sin(\beta_{yn}y) \tag{1}$$

$$e_y(x,y) = \frac{-j\omega\mu}{\beta_{cmn}^2}\beta_{xm}A_{mn}\sin(\beta_{xm}x)\cos(\beta_{yn}y) \tag{2}$$

$$e_z = 0 \tag{3}$$

$$h_x(x,y) = \frac{j\beta_{zmn}}{\beta_{cmn}^2}\cdot\beta_{xm}A_{mn}\sin(\beta_{xm}x)\cos(\beta_{yn}y) \tag{4}$$

$$h_y(x,y) = \frac{j\beta_{zmn}}{\beta_{cmn}^2}\beta_{yn}A_{mn}\cos(\beta_{xm}x)\sin(\beta_{yn}y) \tag{5}$$

$$h_z(x,y) = A_{mn}\cos(\beta_{xm}x)\cos(\beta_{yn}y) \tag{6}$$

For the TE_{10} mode, $m=1$ & $n=0$ s.t.

$$e_x = 0 = e_z \tag{7}$$

$$e_y = \frac{-j\omega\mu}{\beta_{c10}^2}\frac{\pi}{a}A_{10}\sin\left(\frac{\pi x}{a}\right) \quad \frac{V}{m} \tag{8}$$

$$h_x = \frac{j\beta_{z10}}{\beta_{c10}^2}\frac{\pi}{a}A_{10}\sin\left(\frac{\pi x}{a}\right) \quad \frac{A}{m} \tag{9}$$

$$h_y = 0 \tag{10}$$

$$h_z = A_{10}\cos\left(\frac{\pi x}{a}\right) \quad \frac{A}{m} \tag{11}$$

For $\bar{E}$, there exists only E_y. Field plots for the TE_{10} mode are shown on the following page, along with five other lowest-ordered modes for $a > b$.

Some things to notice in these plots:
1.) Regions of higher density field lines indicate larger field strengths.
2.) In the top view, where $\bar{E}$ is larger, so is $\bar{H}$.
3.) The $\bar{H}$ field lines circulate around (vertical) $\bar{E}$.

Conduction surface current density on the pec walls of the waveguide is shown in Fig 8.8 of Ramo, Whinnery, and Van Duzer for the TE_{10} mode. Displacement current forms the connection between converging or diverging surface current regions. Compare w/ $\bar{E}$ & $\bar{H}$ field lines plot.

Guide Wavelength

The guide wavelength is the distance between two equal phase fronts in the z direction (down the waveguide). Specifically

$$\lambda_g = \lambda_{zmn} = \frac{2\pi}{\beta_{zmn}} \tag{12}$$

Note that this guide wavelength is very different from that for a TEM wave where

$$\lambda = \frac{2\pi}{\beta} \tag{13}$$

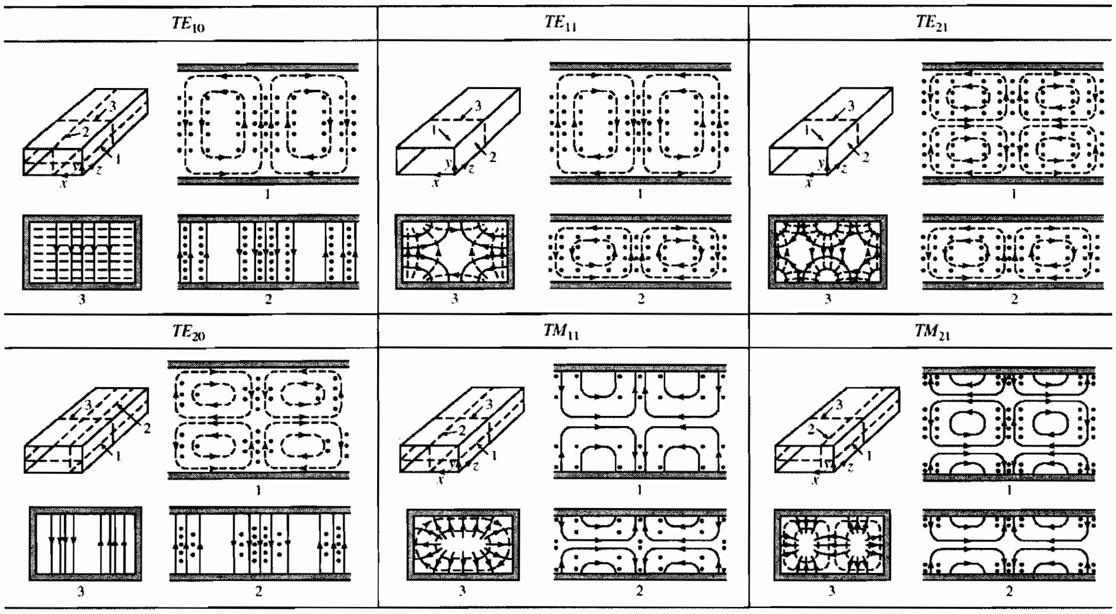


Figure 3.9 (p. 114)

Field lines for some of the lower order modes of a rectangular waveguide.

Reference: D. M. Pozar, *Microwave Engineering*. Hoboken, NJ: John Wiley & Sons, third ed., 2005.

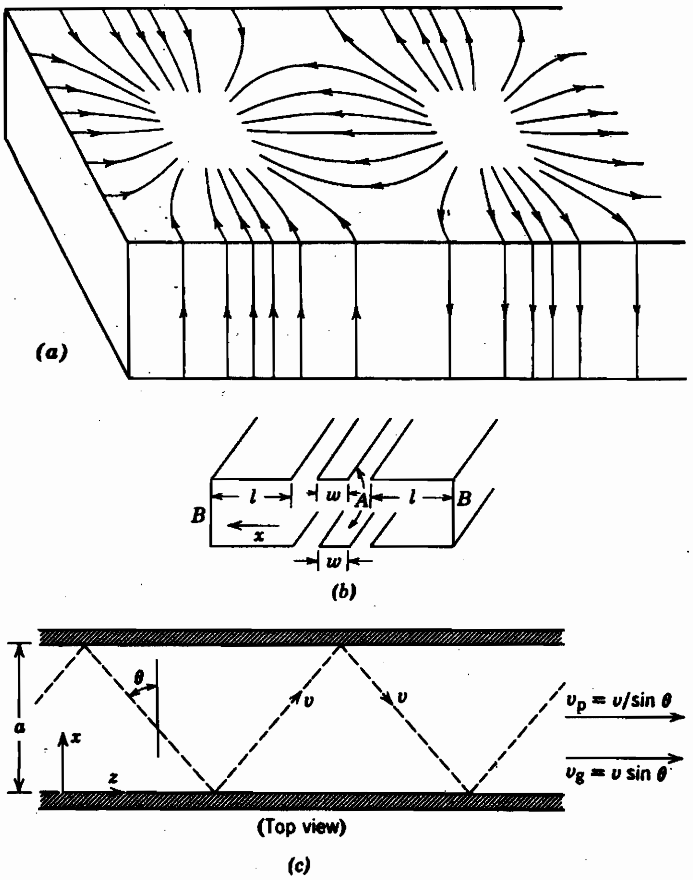


FIG. 8.8 (*a*) Current flow in walls of rectangular guide with TE_{10} mode. (*b*) Guide roughly divided into axial- and transverse-current regions. (*c*) Path of uniform plane-wave component of TE_{10} wave in rectangular guide.

Reference: S. Ramo, J. R. Whinnery and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: John Wiley & Sons, third ed., 1994.

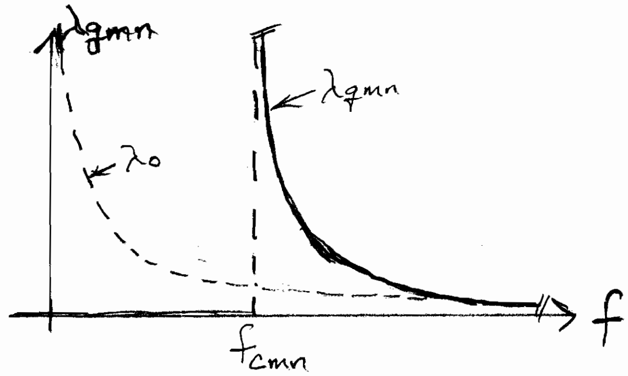
Recalling from the last lecture that

$$\beta_{zmn} = \sqrt{\beta^2 - \beta_{cmn}^2} \qquad (14)$$

then sub → (12)

$$\lambda_{gmn} = \frac{2\pi}{\sqrt{\beta^2 - \beta_{cmn}^2}} \qquad (15)$$

Plot for $\beta^2 > \beta_{cmn}^2$ (for $\beta^2 < \beta_{cmn}^2$, no wave prop., just attenuation.)



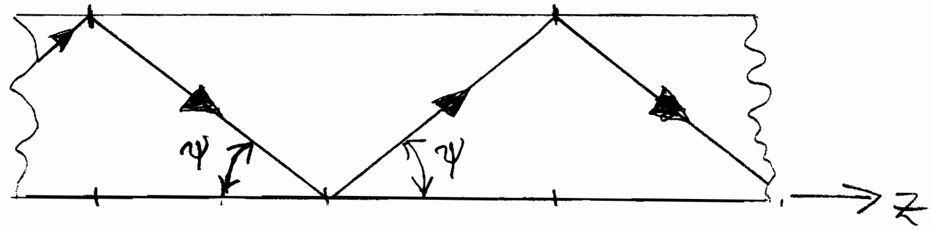
Guide wavelength approaching ∞ as $f \to f_{cmn}^{+}$ and approaches λ_o as $f >> f_{cmn}$.

Bouncing Ray Model

To help appreciate this freq. variation of λ_g, as well as provide an insightful interpretation for phase & group velocity in waveguides, it is possible to mathematically construct a bouncing ray model for wave prop. in the wvgd.

This model is rigorous and begins with the field equations for a mode type as well as the dispersion relation.

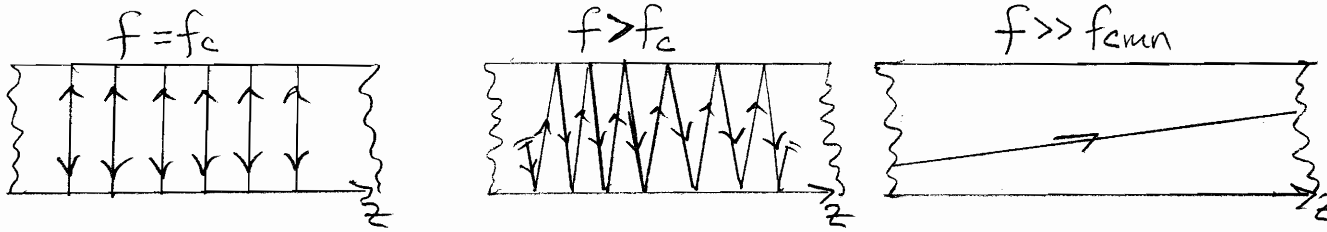
In cross-section, the results of this model are bouncing rays w/ angle Ψ:



where

$$\Psi = \cos^{-1}\left[\sqrt{1-\left(\frac{f_{cmn}}{f}\right)^2}\right] \qquad f \geq f_{cmn} \qquad (16)$$

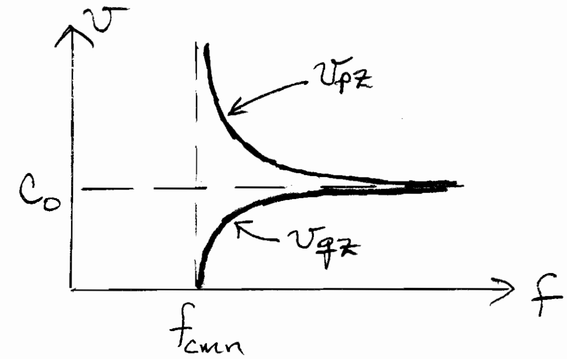
when $f=f_c$, $\Psi = 90°$ which means wave bouncing back in forth between the side walls, but no propagation down wgd, as shown below:



For $f > f_c^+$, wave bouncing back & forth rapidly per unit length. As f increases further, less bouncing and very little bouncing per unit length as $f >> f_{cmn}$, as illustrated above.

This picture of wave prop. in wgd is helpful for understanding dispersion of signals as they prop. down the wgd.

In particular, as we saw in the previous lecture,



The phase velocity is ranging from ∞ to c_o. This velocity is associated with the bouncing rays. The group velocity is associated with how fast signals are transmitted down the wvgd. At $f = f_{cmn}$, rays just bouncing back & forth w/ no prop down guide $\Rightarrow$ $v_g = 0$. As $f >> f_{cmn}$, then signal traveling near c_o, Hence, $v_{gz} \approx v_{pz} \approx c_o$ for $\left(\frac{f}{f_{cmn}}\right)^2 >> 1$.

Add power flow, mode orthogonality?