old is common to operate waves in single mode frequency range. For $a > b$, this is the TE$_{10}$ mode in rectangular waveguides as we saw in the last lecture. Consequently, it is worthwhile to study this mode in some detail.

Field Plots

We determined previously the TE$_{20}$ modes that

\[
E_x(x, y) = \frac{j \omega \mu}{\beta_{x0}} \beta_{yn} A_{mn} \sin(\beta_{x0} x) \sin(\beta_{yn} y)
\]

\[
E_y(x, y) = \frac{j \omega \epsilon}{\beta_{x0}} \beta_{yn} A_{mn} \cos(\beta_{x0} x) \cos(\beta_{yn} y)
\]

\[
E_z = 0
\]

\[
h_x(x, y) = \frac{j \beta_{x0}}{\beta_{x0}} \beta_{yn} A_{mn} \sin(\beta_{x0} x) \cos(\beta_{yn} y)
\]

\[
h_y(x, y) = \frac{j \beta_{x0}}{\beta_{x0}} \beta_{yn} A_{mn} \cos(\beta_{x0} x) \sin(\beta_{yn} y)
\]

\[
h_z(x, y) = A_{mn} \cos(\beta_{x0} x) \sin(\beta_{yn} y)
\]

For the TE$_{10}$ mode, $m = 1$ and $n = 0$ so:

\[
e_x = 0 = e_z
\]

\[
e_y = -\frac{j \omega \epsilon}{\beta_{x0}^2} \frac{\pi}{a} A_{10} \sin\left(\frac{\pi x}{a}\right) \frac{\nu}{m}
\]

\[
h_x = -\frac{j \beta_{x10}}{\beta_{x10}^2} \frac{\pi}{a} A_{10} \sin\left(\frac{\pi x}{a}\right) \frac{\nu}{m}
\]

\[
h_y = 0
\]

\[
h_z = A_{10} \cos\left(\frac{\pi x}{a}\right) \frac{\nu}{m}
\]
For $E$, there exists only $E_0$. Field plots for the $TE_{10}$ mode are shown on the following page, along with the other lowest-order modes for $a > b$.

Some things to notice in these plots:

1. Regions of higher density field lines indicate larger field strength.
2. In the top view, where $E$ is larger, so is $H$.
3. The $H$ field lines circulate around (vertical) $E$.

Conduction surface current density on the walls of the waveguide is shown in Fig. 8.8 of Ramo, Whinnery, and Van Duzer. Displacement current forms the connection between converging or diverging surface current regions. Compare $|E|$ to $H$ field lines plot.

**Guide Wavelength**

The guide wavelength is the distance between two equal phase fronts in the $z$ direction (down the waveguide). Specifically,

$$\lambda_g = \lambda_{z_{mn}} = \frac{2\lambda}{f_{z_{mn}}}$$

(12)

Note that this guide wavelength is very different from $\lambda$ for a TM or TE wave, where

$$\lambda = \frac{2\pi}{k}$$

(13)
Figure 3.9 (p. 114)
Field lines for some of the lower order modes of a rectangular waveguide.
Reprinted from Fields and Waves in Communication Electronics, Ramo et al, © Wiley, 1965)

Fig. 8.3 (a) Current flow in walls of rectangular guide with TE \(_{10}\) mode. (b) Guide roughly divided into axial- and transverse-current regions. (c) Path of uniform plane-wave component of TE \(_{10}\) wave in rectangular guide.

To help appreciate this fact, recall that for a smooth, unbuffered model, the wave length is
approximately $\lambda \approx \frac{2\pi}{f}$. As $f \to \infty$, $\lambda \to 0$.

Recalling from the last note sheet

$$\lambda_{\min} = \frac{2\pi}{f_{\text{max}}}$$
...In cross-section, the results of this model are bouncing rays at angle \( \psi \):

\[
\psi = \cos^{-1}\left[\frac{1}{\sqrt{1 - \left(\frac{f_{\text{min}}}{f}\right)^2}}\right]
\]

\( f > f_{\text{min}} \) \hspace{1cm} (16)

...where \( f = f_c \), \( \psi = 90^\circ \) which means wave bouncing back and forth between the side walls, but no propagation down wave, as shown below:

For \( f > f_c^* \), wave bouncing back and forth rapidly per unit length. As \( f \) increases further, less bouncing and very little bouncing per unit length as \( f \gg f_{\text{min}} \), as illustrated above.

This picture of wave prop. in wgst is helpful for understanding dispersion of signals as they prop. down the wgst.
...In particular, as we saw in the previous lecture,...

...The phase velocity is ranging from \( v_\infty \) to \( c_0 \). This velocity is associated with the bouncing rays. The group velocity is associated with how fast signals are transmitted down the wave. At \( f = f_{\text{fin}} \), rays just bouncing back and forth up and down the guide ⇒ \( v_g = 0 \). As \( f >> f_{\text{fin}} \), then signal traveling near \( c_0 \). Hence, \( v_{\text{g}} \approx v_{\text{p}} \approx c_0 \) for \( \frac{f}{f_{\text{fin}}} \to \infty \).

...Add power flow, mode orthogonality?