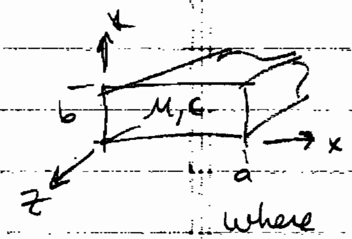
General Behavior of Rectangular Waveguides

The dispersion relation for a rectangular waveguide is very similar for TE^z & TM^z modes. We found in the last lecture, for wave prop in $+z$, that



$$\beta_{xm}^2 + \beta_{yn}^2 + \beta_{zmn}^2 = \beta^2 = \omega^2 \mu \epsilon \tag{1}$$

where $$\beta_{xm} = \frac{m\pi}{a} \qquad m = 0, 1, 2, \ldots \quad \begin{cases} m = n \neq 0 & TE^z \\ m \neq 0 & TM^z \end{cases} \tag{2}$$

$$\beta_{yn} = \frac{n\pi}{b} \qquad n = 0, 1, 2, \ldots \tag{3}$$

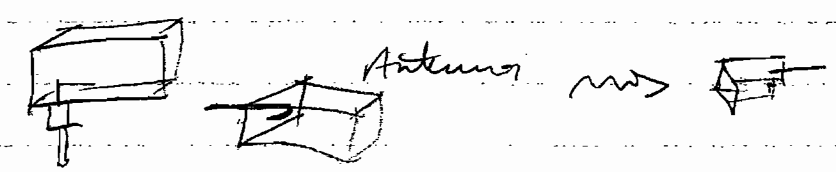
and for TE^z modes $m = n \neq 0$ while for TM^z modes $m \neq 0$ & $n \neq 0$.

The form of the wave propagation in $+z$ is $e^{-j\beta_{zmn} z}$ and the longitudinal wave number is from (1)

$$\beta_{zmn} = \pm\sqrt{\beta^2 - \beta_{xm}^2 - \beta_{yn}^2} \tag{4}$$

There are an infinite # of these modes mn that are generally "excited" when a source is applied to the waveguide. Each of these modes is identified by it's indices m & n as TE_{mn}^z & TM_{mn}^z. How "much" of each mode contributes to the total field depends on the specific excitation.

Launcher



We will study the general solutions to the waveguide (the eigenspectrum) & not concern ourselves w/ the general excitation. Very difficult analysis. (See R.E. Collin, "Field Theory of Guided Waves.")

These modes in the waveguide have a very unusual characteristic we can see from (4). We'll define

$$\beta_{cmn}^2 = \beta_{xm}^2 + \beta_{yn}^2 \tag{5}$$

So that (4) reads

$$\beta_{zmn} = \pm\sqrt{\beta^2 - \beta_{cmn}^2} \tag{6}$$

Notice in (6) that when $\beta^2 < \beta_{cmn}^2$ the wave is purely attenuated while when $\beta^2 > \beta_{cmn}^2$ the wave is purely propagating. β^2 is a function of frequency (& μ & ϵ) while β_{cmn}^2 is dependent on dimensions a & b as well as indices m, n.

When $\beta^2 = \beta_{cmn}^2$ the wave is neither propagating nor attenuating. It is called <u>cut off</u>. For this reason, β_{cmn} is called the <u>cutoff wavenumber</u>.

Hence, from (5)

$$\beta_{cmn} = \pm\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{7}$$

Also, from (6)

$$\beta_{cmn} = \beta\Big|_{\beta_{zmn}=0} = \omega_{cmn}\sqrt{\mu\epsilon} = 2\pi f_{c_{mn}}\sqrt{\mu\epsilon} \tag{8}$$

where f_{cmn} is the <u>cutoff frequency</u> of mode mn.

Using (8) in (7), $f_{cmn} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$ (3.84), (9)

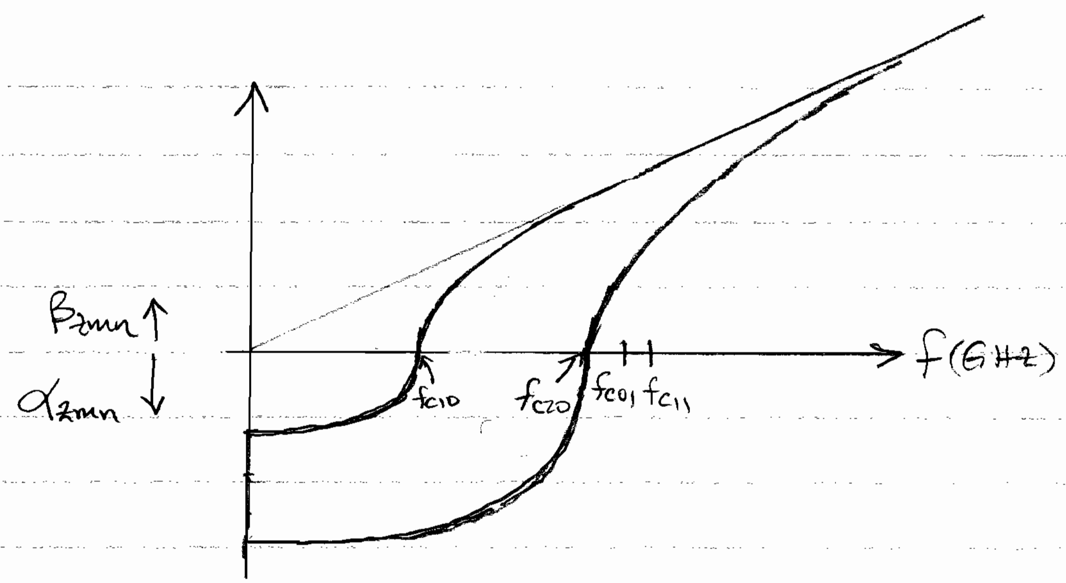
Let's take an X-band rectangular waveguide as an example. This is called a WR-90 wvgd.

From Appendix I, Inside dimensions $a = 0.90''$ (2.286 cm) & b $b = 0.40''$ (1.016 cm). Assuming air-filled, The first 10 modes with the lowest cutoff frequencies are

1. TE_{10} $f_{c10} = 6.562$ GHz
2. TE_{20} $f_{c20} = 13.124$ GHz
3. TE_{01} $f_{c01} = 14.764$ GHz
4. TE_{11} $f_{c11} = 16.56$ GHz } degenerate modes
5. TM_{11} "
6. TE_{30} $f_{c30} = 19.685$ GHz
7. TE_{21} $f_{c21} = 19.754$ GHz } degenerate
8. TM_{21} "
9. TE_{31} $f_{c31} = 24.607$ GHz } degenerate
10. TM_{31} "

The mode with the smallest f_c is called the dominant mode. For this waveguide having $a > b$, the dominant mode is TE_{10}.

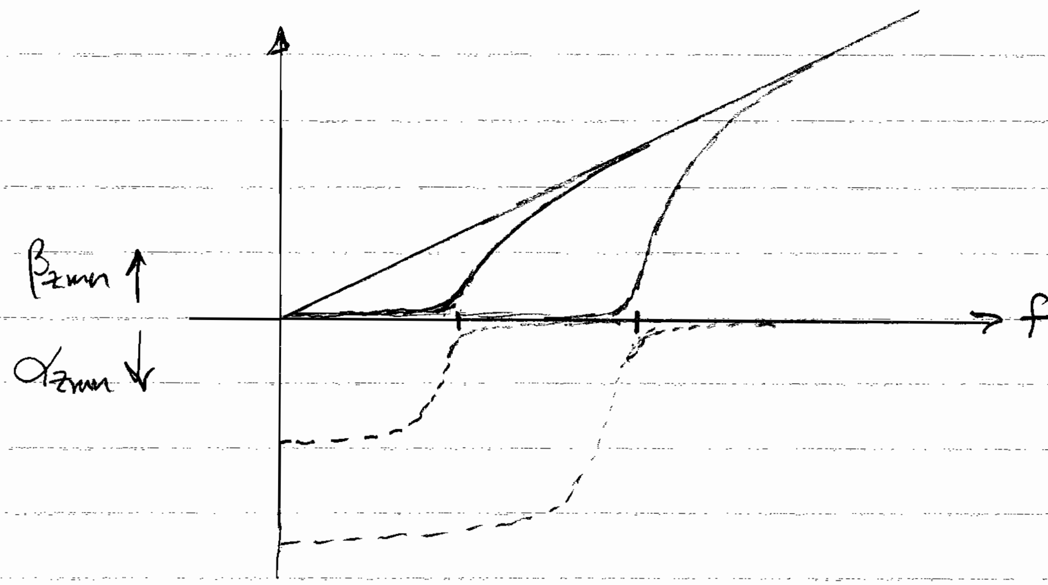
Let's plot β_{z10}



For $f < f_{c10}$, no modes prop. They are purely attenuated. (Fields near source are not zero, however.
They are reactive type fields of this mode yielding reactive input impedance)

For $f_{c10} < f < f_{c20}$ only one mode will propagate. All others purely attenuated. Most often will operate wvgds in single mode. For this particular wvgd, single mode operation is ideally possible from 6.562 GHz to 13.124 GHz.

This is assuming no loss. Metal losses, if small, tend to "smear out" the response such as



There is another characteristic of wave propagation in hollow metallic waveguides that is very apparent from these plots of β_{zmn}. These wvgds. are dispersive, even when there are no losses! (Metallic losses would add further dispersion.)

That is, w/ $e^{-j\beta_{zmn}z}$ propagation, the phase velocity is dependent on frequency. By definition, the phase velocity v_p is given as

$$v_p = \frac{\omega}{\beta}$$

For prop in $+z$, the phase velocity in that direction is

$$v_{pz} = \frac{\omega}{\beta_{zmn}} = \frac{\omega}{\sqrt{\beta^2 - \beta_{cmn}^2}} \quad \beta^2 > \beta_{cmn}^2 \quad (9)$$

Unlike plane waves in lossless media, v_{pz} is a fct. of frequency.

Similarly, the group velocity v_p is also a function of f. This velocity is defined as the inverse slope of the dispersion curve (β_z–ω):

$$v_{gz} = \frac{\partial \omega}{\partial \beta_{zmn}}$$

Group velocity is speed of (energy) information transmission

Interesting to plot the ~~phase~~ phase & group velocities: