Equations (4) and (5) from the last lecture can be used to extract the material parameters of the specimen. We'll discuss two approaches for extracting material parameters, using reflection and transmission data.

Nicolson-Ross-Weir (NRW) algorithm, also this algorithm, the measured $S$ parameters of the specimen faces are used to determine $\epsilon$ and $\mu$. From (4) and (5) in the previous lecture,

$$ S_{11} = \frac{P_p (1-P_p^2)}{1-P_p^2 P_p^2} = S_{22} \tag{1} $$

$$ S_{21} = \frac{P (1-P_p^2)}{1-P_p^2 P_p^2} = S_{12} \tag{2} $$

These $S$ parameters are equal to those measured at the sample faces. That is,

$$ S_{11}^{m} = \frac{P_p (1-P_p^2)}{1-P_p^2 P_p^2} = S_{22}^{m} \tag{3} $$

and

$$ S_{21}^{m} = \frac{P (1-P_p^2)}{1-P_p^2 P_p^2} = S_{12}^{m} \tag{4} $$

Following Nicolson-Ross and Weir, these two equations can be combined to yield

$$ P_p = X \pm \sqrt{X^2 - 1} \tag{5} $$

where

$$ X = \frac{(S_{11}^{m})^2 - (S_{21})^2 + 1}{2 S_{11}^{m}} \tag{6} $$
... The sign in (5) is chosen so that \( |\Gamma_p| \leq 1 \).

... Further, from (3) - (5) an expression for \( P \) can be found as

\[
\rho = \frac{S_{11} + S_{22} - \Gamma_p}{1 - (S_{11} + S_{22}) \Gamma_p}
\]  

(7)

... The RHS's of (5) \& (7) are numbers computed from measured data. The LHS's of (6) \& (7) are directly related to the specimen's material parameters. We defined in the last lecture that

\[
\frac{\rho - 1}{\rho + 1} = \Gamma_p
\]  

(8)

... and

\[
e^{-\lambda L} = \rho
\]  

(9)

... where \( \rho = \sqrt{\frac{\varepsilon_r}{\mu_r}} \) and \( \lambda = j \omega \sqrt{\mu_r \varepsilon_r} = \gamma_0 \sqrt{\mu_r \varepsilon_r} \) (10),(11)

... with \( \gamma_0 = j \omega \sqrt{\mu_0 \varepsilon_0} \) (12)

... Using (10) \& (11) in (8) \& (9) the relative material parameters of the FUT are

\[
\frac{1 + \Gamma_p}{(1 - \Gamma_p) \Lambda / \lambda_0} = \frac{\lambda_0}{\Lambda} \frac{1 + \Gamma_p}{1 - \Gamma_p}
\]  

(13)

... where

\[
\varepsilon_r = \frac{\lambda^2}{\mu_r \Lambda^2}
\]  

(14)

... Text equation is incorrect, even for \( \lambda_0 \to \infty. \)
where \( \lambda_0 \) = free space wavelength and

\[
\frac{1}{\lambda^2} = \frac{\varepsilon_r \mu_r}{\lambda_0^2} = -\left[ \frac{1}{2\pi L} \ln \left( \frac{1}{p} \right) \right]^2
\]  

(15)

Because \( P \) is a complex number, we must be careful when computing its natural logarithm. In general,

\[
\ln \frac{1}{p} = \ln \frac{1}{|p|} + j [\text{Arg}(\frac{1}{p}) + n 2\pi]
\]

(16)

for \( n = 0, \pm 1, \pm 2, \ldots \)

and \( \text{Arg}(\frac{1}{p}) \) is the principal value s.t. \(-\pi < \text{Arg}(\frac{1}{p}) \leq \pi\).

The proper choice for \( n \) in (16) is determined by equating the predicted line delay through the specimen to the line delay computed from measured data. This ambiguity arises because the phase angle of the propagation factor \( P \) is then changed when the sample length is increased by a multiple of a wavelength.

Another technique to determine the proper \( n \) in (16) is to use a technique called "phase-unwrapping," as discussed in the text (pp. 174-178).

Example from: Baker, Jarvis. PTFE, 1981. WEIR - PTFE
Fig. 2. Measured dielectric constant and permeability of Teflon in the X-band region. (a) Relative dielectric constant ($\varepsilon_r$). (b) Relative permeability ($\mu_r$).

Fig. 3. Measured dielectric constant and permeability of Ecosorb SF-5.5 in the X-band region. (a) Relative dielectric constant ($\varepsilon_r$). (b) Relative permeability ($\mu_r$).

Fig. 2. The determination of the permittivity of a PTFE sample as a function of frequency using the Nicolson and Ross equations (solid line) and the iteration procedure (dashed line) (eq. (21)).

Fig. 3. The permittivities obtained from (24) for various values of $\beta$ for a sample of PTFE. The dashed line is for $\beta \to \infty$, the solid line for $\beta = 0$, and the dotted line for $\beta = 1$.

...Weir's results for Teflon show more freq. variation and
...even then unexpected. He reports this was likely due to the
...lack of freq. stabilization in the network analyzer, which
...was present for the Agrow-L SF-5.3 measurements.

... Teflon

...The results from Baker-Jarvis show many peaks
...and large errors. These occur when specimen is an integer
...# of half-wavelengths long in the specimen. Why not
...peaks in Weir data? Specimen was 1" long. \( \lambda = 0.0234 \mu \)
...\( \Rightarrow f_{\lambda} = 4.14 \text{ GHz} \) for TEM. But his measurements in
...rectangular waveguide, \( \lambda \) different.) Baker-Jarvis'3
...measurements in air-die fixture (TEM) with a relatively
...long specimen = 3.279 cm \( \lambda/2 = 0.03279 \Rightarrow f_{\lambda} \approx 3.16 \text{ GHz} \)

...Boughnet, et al., attributes this error in the \( H_{21} \)
...algorithm to the term \( \frac{1+P_0}{1-P_0} \) in (13) and \( \frac{1-P_0}{1+P_0} \) in (14).

...Otherwise, all calc. in \( H_{21} \) in (5), (6), (7) are
...not prone to this error.
2. Baker-Javis Method. In this algorithm, a set of equations is solved numerically using a root-finding or optimization method. Not an explicit solution for \( \epsilon = M \)

but doesn't suffer from numerical instabilities at \( f \),

where specimen length is multiple number of half-wavelengths.

In a typical Baker-Javis algorithm application, the material parameters are unknown, as well as the specimen length \( L \), and the specimen location in the holder, \( R_1 \) and \( R_2 \).

That's seven unknowns, where \( R_2 \equiv \epsilon_2 L_2 \)

Baker-Javis, et al. show a number of equations that are not dependent on the location of the specimen, i.e., so-called reference plane independent expressions.

For example, from \( S_{il}^m = R_1 \frac{\Gamma_p (1-p^2)}{1-p^2 p^2} \) then,

\[
|S_{il}^m| = \left| \frac{\Gamma_p (1-p^2)}{1-p^2 p^2} \right|
\]

(17)

Which is not dependent on \( R_1 \).

Other reference plane independent expressions can be found in [18]-(21) of Baker-Javis, et al.

As mentioned previously, grab as many of these reference plane independent expressions as # of unknowns. Use numerical methods to solve for unknowns.
...A technique of combining linear combinations of the reference-plane independent expressions proposed by Baker-Jarvis et al. has become quite popular. In particular, (3) is added to $\beta \cdot (4)$, where $\beta$ is a real number as:

$$\frac{1}{2} \left[ S_{12}^{m'} + S_{21}^{m'} + \beta (S_{11}^{m'} + S_{22}^{m'}) \right] = \frac{P(1-P^2) + \beta P(1-P^2)}{1-P^2 P^2} \quad (18)$$

...The parameter $\beta$ is chosen to weight the reflection measurement more if the reflect is small, or large to weight reflected more if Transmission small.

[Show Fig 3, Baker-Jarvis]
To further illustrate the usefulness of the Baker-Javis paper, imagine a situation where measuring the "effective" permittivity of particulate media, such as this configuration:

![Diagram of particulate medium with thicknesses L1 and L2 and permittivity εr, εreff, εreff.]

The specimen thickness and position in the fixture may not be known, or known to a necessary precision. Other examples include:

- Liquids - can't see into fixture, or measure "nondestructively"
- Compressible specimens - rubber-like materials, etc., with a composite material such as it tends in polymers,
  epoxy will change when compressed. For solids, ε won't change, but still need accurate specimen dimensions
  for accurate εr.

In such cases where L may not be known as well as perhaps L2, Baker-Javis et al. Suggest combining

\[
S_{11}S_{22} - S_{12}S_{21} = \epsilon \frac{-2\epsilon_0 (\kappa_1 - \kappa) \frac{L_2^2 - \rho^2}{1 - 2\rho^2}}{\epsilon_0} = \frac{L_2^2 - \rho^2}{1 - 2\rho^2}, \quad \epsilon = \frac{L_2}{\epsilon_0}
\]

by their (17) - (20) to treat the measurement as independent of reference plane and specimen thickness. Next!