

Purpose of this course is to study <sup>EM</sup> wave propagation in waveguides and how these devices can be used to measure the EM properties of materials. Not a cobbling of disparate topics! Wgds used for other applications, for sure, but materials characterization is an important use.

We will use sinusoidal plane waves almost exclusively in this course, with  $e^{j\omega t}$  time convention. Maxwell's eqns are then

$$\nabla \times \bar{E} = -j\omega \bar{B} - \bar{M}^i \quad (1)$$

$$\nabla \times \bar{H} = j\omega \bar{D} + \bar{J}^i \quad (2)$$

$$\nabla \cdot \bar{D} = \rho_e \quad (3)$$

$$\nabla \cdot \bar{B} = \rho_m \quad (4)$$

$\bar{J}^i$  &  $\bar{M}^i$  are impressed volume sources of electric & magnetic current densities, respectively. Likewise,  $\rho_e$  &  $\rho_m$  are volume sources of electric and magnetic charge densities.

The sources  $\bar{M}^i$  &  $\rho_m$  are fictitious in the sense that there are no "magnetic charges" to form a magnetic charge density or magnetic current. However, these magnetic sources are a valuable tool in EM for solving or modeling certain types of geometrical features; often apertures in closed metallic objects, or perfect magnetic conductors (PMC).

waveguides:  
 very nice characterization system: closed system  
 • can model well  
 • high precision construction  
 • excellent calibration

The physically fundamental EM fields are  $\vec{E}$  &  $\vec{B}$ , because of the Lorentz force equation:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (5)$$

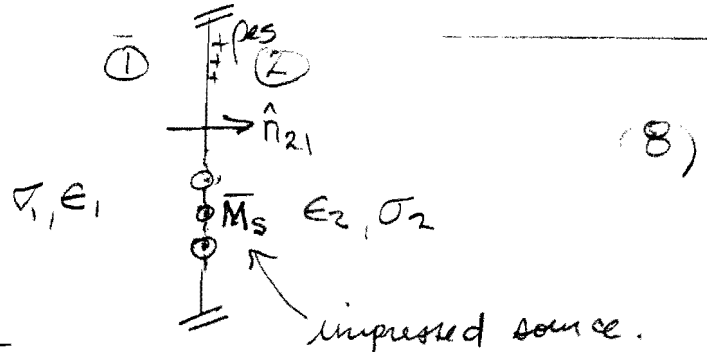
The fields  $\vec{D}$  &  $\vec{H}$  are "derived" quantities that are related to  $\vec{E}$  &  $\vec{B}$  through the constitutive equations:

$$\vec{D} = \epsilon \vec{E} \quad \& \quad \vec{B} = \mu \vec{H} \quad (6), (7)$$

The integral form of Maxwell's equations can be used to derive conditions the fields must satisfy at interfaces between different materials, and at sheets of impressed electric & magnetic current densities.

At a dielectric interface:

$$\hat{n}_{21} \times (\vec{E}_2 - \vec{E}_1) = -\vec{M}_s$$



Because of '-' on RHS, this b.c. follows the left-hand rule.

If  $\vec{M}_s^i = 0$ , then tang comp. of  $\vec{E}$  are continuous at a material interface.  $\vec{M}_s$  either source or induced at surface of a perfect magnetic conductor (PMC).

$$\text{At this same interface, } \hat{n}_{21} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_{es} \quad (9)$$

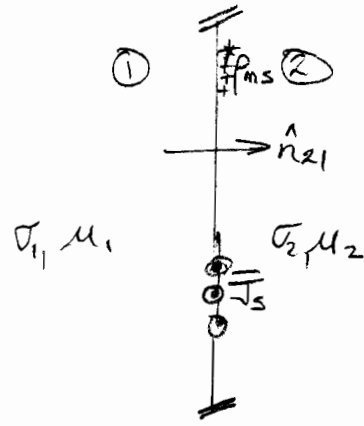
Here,  $\rho_{es}$  may be an impressed source, or at the

interface w/ PEC (aka an electric wall) it can be an induced surface charge density.

At a magnetic interface:

$$\hat{n}_{21} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s \quad (10)$$

If no impressed surface current density at the interface  
 s.t.  $\vec{J}_s = 0$ , then tan.  $\vec{H}$  cont.  
 across this interface.



However, if one region is PEC, then  $\vec{J}_s$  will be a non-zero induced surface current density.

At this same interface,

$$\hat{n}_{21} \cdot (\vec{B}_2 - \vec{B}_1) = \rho_{ms} \quad (11)$$

Here,  $\rho_{ms}$  may be an impressed source of surface magnetic charge density, or an induced magnetic surface charge density at a PMC interface. (aka a magnetic wall).

**NIST**

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**Measuring the Permittivity  
and Permeability of Lossy  
Materials: Solids, Liquids,  
Metals, Building Materials,  
and Negative-Index Materials**

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Table 3. Dielectric measurement categories for medium-high-loss materials compared with typical uncertainties for each method [49].

Technique	Field	Advantages	$\Delta\epsilon'_r/\epsilon'_r$	$\Delta \tan \delta_d$
Coaxial line, waveguide	TEM, TE <sub>10</sub>	Broadband	$\pm 1$ to 10	$\pm 0.005$
Slot in waveguide	TE <sub>10</sub>	Broadband	$\pm 1$ to 10	$\pm 0.005$
Capacitor	Normal E-field	Low frequency	$\pm 1$	$\pm 10^{-4}$
Cavity	TE <sub>01</sub>	Very accurate	$\pm 0.2\%$	$\pm 5 \times 10^{-5}$
Cavity	TM <sub>0m</sub>	$\epsilon'_{rz}$	$\pm 0.5$	$\pm 5 \times 10^{-4}$
Dielectric resonator	TE <sub>01</sub>	Very accurate	$\pm 0.2\%$	$\pm 1 \times 10^{-5}$
Coaxial Probe	TEM, TM <sub>01</sub>	Nondestructive	$\pm 2$ to 10	$\pm 0.02$
Fabry-Perot	TEM	High frequency	$\pm 2$	$\pm 0.0005$

Table 4. Magnetic measurement methods compared with typical uncertainties.

Technique	Field	Advantages	$\Delta\mu'_r/\mu'_r, \%$	$\Delta \tan \delta_d$
Coaxial line, waveguide or waveguide	TEM, TE <sub>10</sub>	Broadband	$\pm 2$	$\pm 0.01$
Cavity	TE <sub>011</sub>	Very accurate	$\pm 0.5$	$\pm 5 \times 10^{-4}$
Cavity	TM <sub>110</sub>	$\mu'_{rz}$	$\pm 0.5$	$\pm 5 \times 10^{-4}$
Dielectric resonator	TE <sub>011</sub>	Very accurate	$\pm 0.5$	$\pm 1 \times 10^{-5}$
Whispering-gallery	Hybrid	Very accurate	$\pm 1$	$\pm 5 \times 10^{-6}$
Courtney	TE <sub>01</sub>	Very accurate	$\pm 1$	$\pm 5 \times 10^{-5}$

materials can be measured in coaxial lines and waveguides, split-post magnetic resonators, TM<sub>110</sub> [50] or TE<sub>011</sub> cavities, whispering-gallery modes, or other dielectric resonators (see Table 4).

Over the years certain methods have been identified as particularly good for various classes of measurements, and these have been incorporated as standards of the ASTM (American Society for Testing and Materials) [51] and the European Committee for Electrotechnical Standardization (CENELEC). However, the list is rather dated and does not include some of the more highly-accurate, recently developed methods. The ASTM standard techniques applicable to thin materials are summarized in Table 6.

Measurement fixtures where the electromagnetic fields are tangential to the air-material interfaces, such as in TE<sub>01</sub> cavities and dielectric resonators, generally yield more accurate results than fixtures where the fields are normal to the interface. This is because the tan-