

Another approach to numerically solving the scattering/radiation problem for thin wires is to first start with the mixed potential form of the source/field relationship.

$$\vec{E}^s(\vec{r}) = -jk\eta \vec{A}(\vec{r}) - \nabla\Phi_e(\vec{r})$$

Employing the same approximations as before, namely:

$$a \ll l, \quad a \ll \lambda \quad (a = \text{radius})$$

such that (1) the equivalent surface currents on the endcaps can be neglected,

(2) J_ϕ is small wrt J_z

and (3) the smooth thin-wire kernel approximation is valid,

then the EFIE can be written as -

$$\hat{n} \times \vec{E}^i(\vec{r}_s) = -\hat{n} \times \vec{E}^s(\vec{r}_s) \quad \vec{r}_s \in \{\text{pts. on periphery}\}$$

or

$$\vec{E}_z^i = jk\eta A_z + \hat{z} \cdot \nabla\Phi_e = jk\eta A_z + \frac{\partial\Phi_e}{\partial z} \quad (1)$$

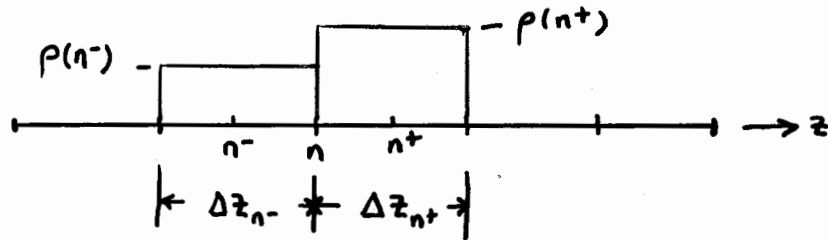
where

$$\bullet A_z = I(z') * G_{TW}^s, \quad G_{TW}^s = \frac{e^{-jkR_s^s}}{4\pi R_s^s}, \quad R_s^s = \sqrt{a^2 + (z-z')^2}$$

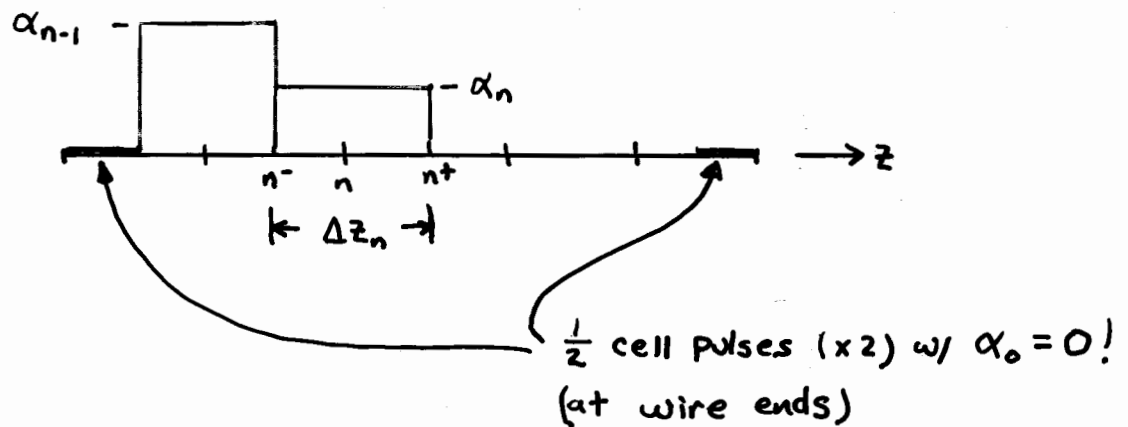
$$\bullet \Phi_e = \frac{\rho(z')}{\epsilon} * G_{TW}^s$$

$$\bullet \frac{dI(z)}{dz} = -j\omega\rho(z) \quad (\text{continuity eqn - current ; charge})$$

Using Harrington's approach (little book, ch. 4), we will subdivide the wire into $(N+1)$ segments - assuming the charge is constant over these segments (pulse basis).

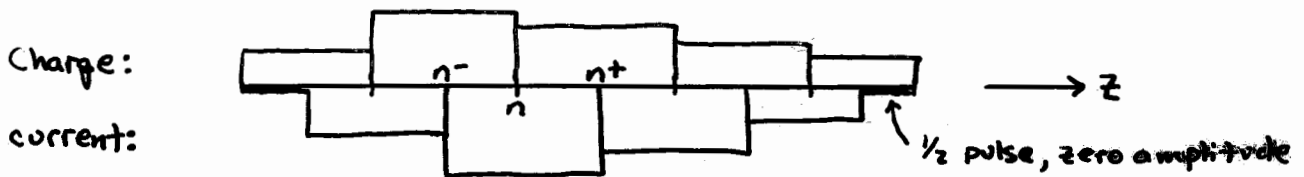


In a similar manner, the current will also be approximated as pulses over domains which are shifted $\frac{1}{2}$ a cell w.r.t. the charge domains - (N segments)



with a ^{half} pulse of amplitude 0 extending from each end of the wire.

Superimposed:



From the continuity equation, the derivative on the current in z can be approximated by a finite difference as -

$$\rho(n^+) \approx -\frac{1}{j\omega} \frac{\alpha_{n+1} - \alpha_n}{\Delta z_{n^+}}$$

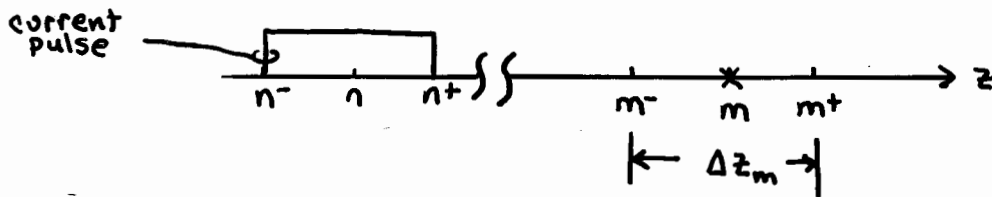
$$\rho(n^-) \approx -\frac{1}{j\omega} \frac{\alpha_n - \alpha_{n-1}}{\Delta z_{n^-}}$$

α 's are current coeffs.

Likewise, the $\frac{d\Phi_e}{dz}$ term in ① can be approximated by a finite difference as -

$$\frac{d\Phi_e}{dz} \approx \frac{\Phi_e(m^+) - \Phi_e(m^-)}{\Delta z_m}$$

where 'm' are observation locations as -



By testing ① using point matching at the centroids of the current pulses (m) gives:

$$E_z^i(m) = jk\eta A_z(m) + \frac{\Phi_e(m^+) - \Phi_e(m^-)}{\Delta z_m} \quad \text{②}$$

But,

$$\Phi_e(m^\pm) \approx \frac{1}{\epsilon} \sum_{n=1}^{N+1} \rho(n) \int_{\Delta z_{n^+}} \frac{e^{-jk(R_s^s)_{m^\pm}}}{4\pi (R_s^s)_{m^\pm}} dz'$$

← evaluated @ $z = m^\pm$

and $A_z(m) \approx \sum_{n=1}^N \alpha_n \int_{\Delta z_n} \frac{e^{-jk(R_s^s)_m}}{4\pi(R_s^s)_m} dz'$

Substituting for $p(n) \doteq E_e$ in (2) gives -

$$E_z^i(m) = jk\eta A_z(m) - \frac{1}{\Delta z_m} \frac{1}{j\omega\epsilon} \sum_{n=1}^{N+1} \left\{ \frac{\alpha_{n+1} - \alpha_n}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m+}}}{4\pi(R_s^s)_{m+}} dz' - \frac{\alpha_{n+1} - \alpha_n}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m-}}}{4\pi(R_s^s)_{m-}} dz' \right\} \quad (3)$$

We need to simplify & rewrite (3) in order that a matrix eqn of the form

$$[V] = [Z][I]$$

\uparrow tested incident E_z \uparrow mm "Impedance Matrix" \leftarrow current coeffs.

can be written.

This is accomplished by expanding (3) and examining a few terms. For example, w/ $n=3 \doteq 4$ gives for the summation term in (3) -

$$\frac{\alpha_4 - \alpha_3}{\Delta z_{3+}} \int_{\Delta z_{3+}} (m^+) dz' - \frac{\alpha_4 - \alpha_3}{\Delta z_{3+}} \int_{\Delta z_{3+}} (m^-) dz' +$$

$$\frac{\alpha_5 - \alpha_4}{\Delta z_{4+}} \int_{\Delta z_{4+}} (m^+) dz' - \frac{\alpha_5 - \alpha_4}{\Delta z_{4+}} \int_{\Delta z_{4+}} (m^-) dz'$$

Notice, however, that $\Delta z_{3+} = \Delta z_{4-}$.

With this substitution & that for A_z , the matrix eqn can be written as

$$\Delta z_m E_z^i = z_{mn} \alpha_n \quad m, n = 1, \dots, N$$

where, $z_{mn} = jk\eta \Delta z_m \int_{\Delta z_n} \frac{e^{-jk(R_s^s)_m}}{4\pi(R_s^s)_m} dz'$ ④

$$+ \frac{1}{j\omega\epsilon} \left\{ \frac{1}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m+}}}{4\pi(R_s^s)_{m+}} dz' - \frac{1}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m-}}}{4\pi(R_s^s)_{m-}} dz' - \right.$$

$$\left. \frac{1}{\Delta z_{n-}} \int_{\Delta z_{n-}} \frac{e^{-jk(R_s^s)_{m+}}}{4\pi(R_s^s)_{m+}} dz' + \frac{1}{\Delta z_{n-}} \int_{\Delta z_{n-}} \frac{e^{-jk(R_s^s)_{m-}}}{4\pi(R_s^s)_{m-}} dz' \right\}$$

↑ compare w/ Harrington, eqn. (4-20).

Furthermore, $V_m = \Delta z_m E_z^i(m)$ ⑤

Egns. ④ & ⑤ are the final equations necessary for the MM solution of the thin, pec wire problem. (what about z_{mm} ??)

Note that the integrals must be evaluated using numerical integration methods. However, all of these integrals have identical integrands & forms! Therefore, only need one subroutine.

The question remains as to why the charge and current subdomains are shifted w.r.t. one another.

Is this necessary?

The primary reason for this is the finite difference approximation used for the charge \rightarrow

$$\rho(n^+) \approx -\frac{1}{j\omega} \frac{\alpha_{n+1} - \alpha_n}{\Delta z_{nt}}$$

Without the shifted subdomains, this approximation would not be well-defined.