MM Solution of Thin-Wire Problem Using Harrington's Method: Pulse Expansion / Finite Difference

Another approach to numerically solving the scattering/radiation problem for thin wires is to first start with the mixed potential form of the source/field relationship.

$$\bar{E}^s(\bar{r}) = -jk\eta\,\bar{A}(\bar{r}) - \nabla\Phi_e(\bar{r})$$

Employing the same approximations as before, namely:

$a \ll l$, $a \ll \lambda$ (a = radius)

such that (1) the equivalent surface currents on the endcaps can be neglected,

(2) J_ϕ is small wrt J_z

and (3) the smooth thin-wire Kernal approximation is valid,

then the EFIE can be written as -

$$\hat{n}\times\bar{E}^i(\bar{r}_s) = -\hat{n}\times\bar{E}^s(\bar{r}_s) \qquad \bar{r}_s \in \{\text{pts. on periphery}\}$$

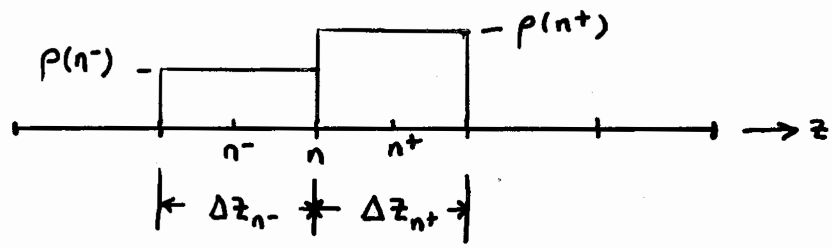
or

$$E_z^i = jk\eta A_z + \hat{z}\cdot\nabla\Phi_e = jk\eta A_z + \frac{\partial\Phi_e}{\partial z} \qquad (1)$$

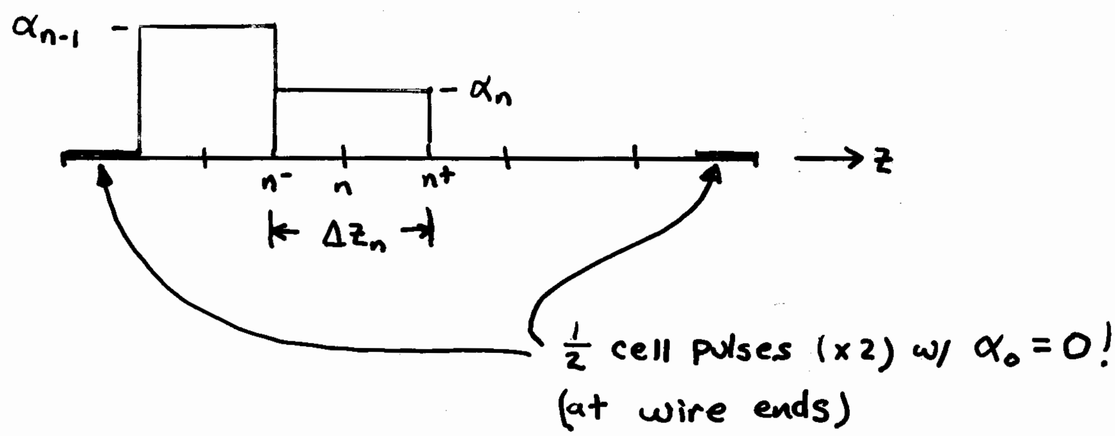
where

- $A_z = I(z') * G^s_{TW}$, $G^s_{TW} = \dfrac{e^{-jkR^s_s}}{4\pi R^s_s}$, $R^s_s = \sqrt{a^2+(z-z')^2}$
- $\Phi_e = \dfrac{\rho(z')}{\epsilon} * G^s_{TW}$
- $\dfrac{dI(z)}{dz} = -j\omega\rho(z)$ (continuity eqn - current & charge)

Using Harrington's approach (little book, Ch. 4), we will subdivide the wire into (N+1) segments — assuming the charge is constant over these segments (pulse basis).

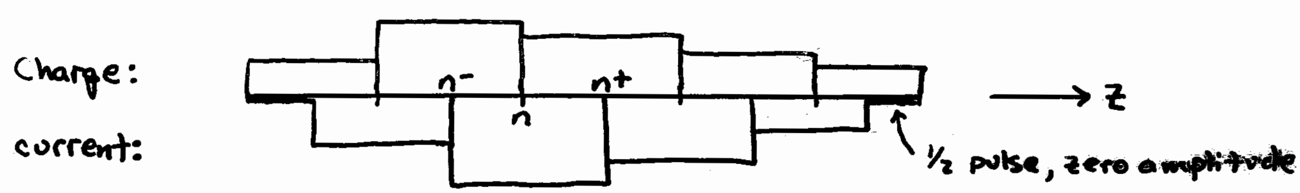


In a similar manner, the current will also be approximated as pulses over domains which are shifted ½ a cell w.r.t. the charge domains — (N segments)



with a half pulse of amplitude 0 extending from each end of the wire.

Superimposed:



From the continuity equation, the derivative on the current in z can be approximated by a finite difference as –

$$\rho(n^+) \approx -\frac{1}{j\omega}\,\frac{\alpha_{n+1}-\alpha_n}{\Delta z_{n+}}$$

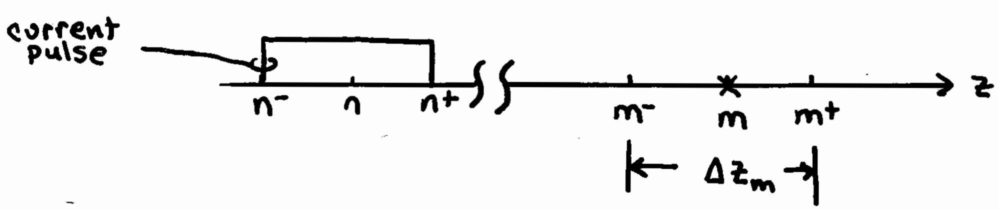
α's are <u>current</u> coeffs.

$$\rho(n^-) \approx -\frac{1}{j\omega}\,\frac{\alpha_n-\alpha_{n-1}}{\Delta z_{n-}}$$

Likewise, the $\frac{d\Phi_e}{dz}$ term in ① can be approximated by a finite difference as –

$$\frac{d\Phi_e}{dz} \approx \frac{\Phi_e(m^+)-\Phi_e(m^-)}{\Delta z_m}$$

where 'm' are observation locations as –



By testing ① using <u>point matching</u> at the centroids of the <u>current pulses</u> (m) gives:

$$E_z^i(m) = jk\eta\, A_z(m) + \frac{\Phi_e(m^+)-\Phi_e(m^-)}{\Delta z_m} \qquad ②$$

But,

evaluated @ $z = m^\pm$

$$\Phi_e(m^\pm) \approx \frac{1}{\epsilon}\sum_{n=1}^{N+1} \rho(n^+) \int_{\Delta z_{n^+}} \frac{e^{-jk(R_s^s)_{m\pm}}}{4\pi\,(R_s^s)_{m\pm}}\, dz'$$

and $$A_z(m) \approx \sum_{n=1}^{N} \alpha_n \int_{\Delta z_n} \frac{e^{-jk(R_s^s)_m}}{4\pi (R_s^s)_m} dz'$$

Substituting for $\rho(n^+)$ & Φ_e in (2) gives –

$$E_z^i(m) = jk\eta\, A_z(m) - \frac{1}{\Delta z_m} \frac{1}{j\omega\epsilon} \sum_{n=1}^{N+1} \left\{ \frac{\alpha_{n+1} - \alpha_n}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m+}}}{4\pi (R_s^s)_{m+}} dz' - \right.$$

$$\left. \frac{\alpha_{n+1} - \alpha_n}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m-}}}{4\pi (R_s^s)_{m-}} dz' \right\} \quad (3)$$

We need to simplify & rewrite (3) in order that a matrix eqn of the form

$$\underset{\substack{\uparrow \\ \text{tested} \\ \text{incident } E_z}}{[V]} = \underset{\substack{\uparrow \\ \text{mm "Impedance} \\ \text{Matrix"}}}{[Z]} \underset{\text{current coeffs.}}{[I]}$$

can be written.

This is accomplished by expanding (3) and examining a few terms. For example, w/ $n = 3$ & 4 gives for the summation term in (3) –

$$\frac{\alpha_4 - \alpha_3}{\Delta z_{3+}} \int_{\Delta z_3^+} (m^+)\, dz' - \frac{\alpha_4 - \alpha_3}{\Delta z_{3+}} \int_{\Delta z_{3+}} (m^-)\, dz' \quad +$$

$$\frac{\alpha_5 - \alpha_4}{\Delta z_{4+}} \int_{\Delta z_{4+}} (m^+)\, dz' - \frac{\alpha_5 - \alpha_4}{\Delta z_{4+}} \int_{\Delta z_{4+}} (m^-)\, dz'$$

Notice, however, that $\Delta z_{3+} = \Delta z_{4-}$.

With this substitution & that for A_z , the matrix eqn can be written as

$$\Delta z_m E_z^i = Z_{mn} \alpha_n \qquad m,n = 1,\cdots,N$$

where,

$$Z_{mn} = jk\eta \Delta z_m \int_{\Delta z_n} \frac{e^{-jk(R_s^s)_m}}{4\pi (R_s^s)_m} dz' \qquad (4)$$

$$+ \frac{1}{j\omega\epsilon}\left\{ \frac{1}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m+}}}{4\pi (R_s^s)_{m+}} dz' - \frac{1}{\Delta z_{n+}} \int_{\Delta z_{n+}} \frac{e^{-jk(R_s^s)_{m-}}}{4\pi (R_s^s)_{m-}} dz' - \right.$$

$$\left. \frac{1}{\Delta z_{n-}} \int_{\Delta z_{n-}} \frac{e^{-jk(R_s^s)_{m+}}}{4\pi (R_s^s)_{m+}} dz' + \frac{1}{\Delta z_{n-}} \int_{\Delta z_{n-}} \frac{e^{-jk(R_s^s)_{m-}}}{4\pi (R_s^s)_{m-}} dz' \right\}$$

Compare w/ Harrington , eqn. (4-20).

Furthermore,

$$V_m = \Delta z_m E_z^i(m) \qquad (5)$$

Eqns. (4) & (5) are the final equations necessary for the MM solution of the thin, pec wire problem. (what about Z_{mm} ??)

Note that the integrals must be evaluated using numerical integration methods. However, all of these integrals have identical integrands & forms! Therefore, only need one subroutine.

The question remains as to why the charge and current subdomains are shifted w.r.t. one another.

<u>Is this necessary?</u>

The primary reason for this is the finite difference approximation used for the charge →

$$\rho(n^+) \approx -\frac{1}{j\omega}\,\frac{\alpha_{n+1}-\alpha_n}{\Delta z_{n+}}$$

Without the shifted subdomains, this approximation would not be well-defined.