MM solution of Pocklington's Eqn. Using Pulse Expansion/Pt. Matching

Now that we've derived the integral equation necessary for the analysis of thin, pec wires we will shift our attention to the numerical solution. The first (of three methods) will be to use pulse expansion with point matching.

From our previous discussions, Pocklington's I.E. using the smooth thin-wire Kernal can be written as

$$E_z^i(\bar{r}_s) = \frac{j}{\omega\epsilon}\left(\frac{d^2}{dz^2}+k^2\right)\int_{-\frac{l}{2}}^{\frac{l}{2}} I(z')\, G_{TW}^s(z|z',a)\, dz'$$

Since the integrand is everywhere <u>well-defined</u> (remember the filamentary current approximation), we can <u>interchange</u> the <u>integration</u> and <u>differentiation</u> operations –

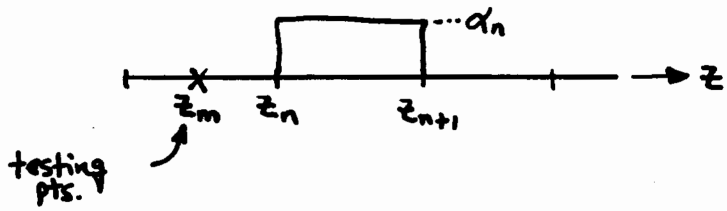
$$E_z^i(\bar{r}_s) = \frac{j}{\omega\epsilon}\int_{-\frac{l}{2}}^{\frac{l}{2}} I(z')\left[\frac{\partial^2}{\partial z^2}+k^2\right] G_{TW}^s(z|z',a)\, dz'$$

$I(z') \neq f(z)$!!

After computing $\left[\frac{\partial^2}{\partial z^2}+k^2\right] G_{TW}^s(z|z',a)$, substituting and simplifying, the EFIE becomes

$$E_z^i(\bar{r}_s) = \frac{j}{4\pi\omega\epsilon}\int_{-l/2}^{l/2} I(z')\left[\left(1+jkR_s^s\right)\left(2(R_s^s)^2-3a^2\right)+\left(kaR_s^s\right)^2\right]\frac{e^{-jkR_s^s}}{(R_s^s)^5}\, dz' \quad ①$$

with a pulse basis expansion for $I(z')$ and point matching @ centers of pulses –



$$I(z') \approx \sum_{n=1}^{N} \alpha_n P_n(z'; z_n, z_{n+1}) \quad , \quad w_m = \delta(z - z_m)$$

the resulting system of algebraic equations is thus –

$$E_z^i(z_m) = \frac{j}{4\pi\omega\epsilon} \sum_{n=1}^{N} \alpha_n \int_{z_n}^{z_{n+1}} \left[(1 + jkR_s^s)(2(R_s^s)^2 - 3a^2) + (kaR_s^s)^2 \right] \frac{e^{-jkR_s^s}}{(R_s^s)^5} \, dz'$$

$$R_s^s = \sqrt{a^2 + (z_m - z')^2} \quad , \quad m = 1, \cdots, N$$

Or in matrix notation –

$$\overset{N\times N}{[Z]} \, \overset{N\times 1}{[\alpha]} = \overset{N\times 1}{[V]}$$

"Impedance" matrix

current

tested E^{inc} – voltage

Note that the integrand is well-defined everywhere and can be computed using <u>numerical</u> <u>integration</u>.