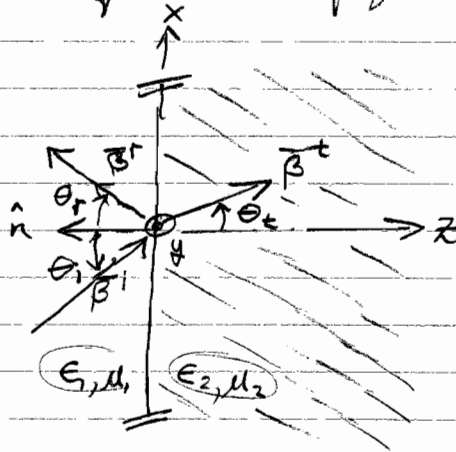


Oblique incidence is a very important part of UPW analysis, though it's a bit more complicated than normal incidence analysis. Oblique incidence will unveil interesting phenomenon not present in normal incidence.

There is some important terminology we need to define for oblique incidence. Referring to this figure:



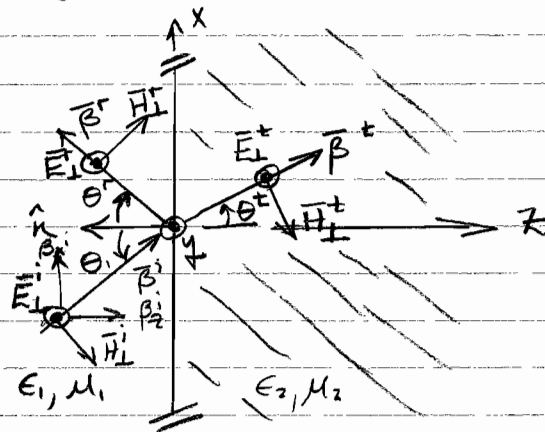
1. Plane of incidence: plane formed by \hat{n} , β^i (or β^r or β^t).
2. Perpendicular polarization: UPW w/ $\vec{E} \perp$ to plane of incidence. Also called E pol or horizontal polarization.
3. Parallel polarization: UPW w/ $\vec{E} \parallel$ to plane of incidence. Also called H pol or vertical polarization.

The last two definitions arise because of this fact: A perpendicular pol UPW will reflect & transmit \perp -pol UPWs when incident on a half space. Similarly, a \parallel -pol UPW will reflect & transmit only \parallel -pol UPWs. These are called the principal polarizations. (Not to be confused with the polarizations discussed earlier. Here both are linearly polarized.)

Because of this, we will study each of these polarizations separately. For an arbitrarily polarized UPW, decompose into its component polarizations.

Perpendicular Polarization

The geometry for this obliquely incident UPW will be assumed as



As described in the lectures associated w/ Ch. 4 in the text,

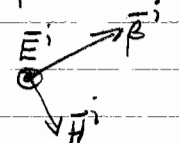
$$\vec{E}_\perp^i = \hat{y} E^i e^{-j\beta^i \cdot \vec{r}} = \hat{y} E_0 e^{-j(\beta_x^i x + \beta_z^i z)} \quad (5-10a), (1)$$

∴ w/ $\nabla \times \vec{E} = -j\omega \mu \vec{H}$ $\xrightarrow[\text{w/ } e^{-j\beta \cdot \vec{r}}]{\text{UPW}}$ $-j\beta^i \times \vec{E}^i = -j\beta_1 \eta_1 \vec{H}^i$
where $\frac{\omega}{v} = \omega \mu$

See sketch

$$\Rightarrow \vec{H}^i = \frac{1}{\beta_1 \eta_1} \beta^i \times \vec{E}^i = \frac{1}{\beta_1 \eta_1} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \beta_x^i & 0 & \beta_z^i \\ 0 & E_y^i & 0 \end{vmatrix} = \frac{1}{\beta_1 \eta_1} (-\hat{x} \beta_z^i E_y^i + \hat{z} \beta_x^i E_y^i)$$

or $\vec{H}^i = \frac{1}{\eta_1} \hat{\beta}^i \times \vec{E}^i$
 This makes sense geometrically



Further, w/ $\beta_x^i = \beta_1 \sin \theta_i$; $\beta_z^i = \beta_1 \cos \theta_i$, then

$$\vec{H}^i = \frac{1}{\beta_1 \eta_1} (-\hat{x} \beta_1 \cos \theta_i + \hat{z} \beta_1 \sin \theta_i) E_y^i$$

$$\vec{H}^i = (-\hat{x} \cos \theta_i + \hat{z} \sin \theta_i) \frac{E_0}{\eta_1} e^{-j(\beta_x^i x + \beta_z^i z)} \quad (5-10b), (2)$$

Similarly, the reflected fields are

$$\vec{E}_\perp^r = \hat{y} E_r^i e^{-j\beta^r \cdot \vec{r}} = \hat{y} \Gamma_\perp^b E_0 e^{-j(\beta_x^r x - \beta_z^r z)} \quad (5-11a), (3)$$

$$\vec{H}_\perp^r = \frac{1}{\eta_1} \hat{\beta}^r \times \vec{E}^r = (\hat{x} \cos \theta_r + \hat{z} \sin \theta_r) \frac{\Gamma_\perp^b E_0}{\eta_1} e^{-j(\beta_x^r x - \beta_z^r z)} \quad (5-11b), (4)$$

where $\Gamma_{\perp}^b \equiv \left. \frac{E_{\perp}^r}{E_{\perp}^i} \right|_{z=0}$, $\beta_x^r = \beta_1 \sin \theta_r$, $\beta_z^r = \beta_1 \cos \theta_r$

and the transmitted fields are

$$\vec{E}_{\perp}^t = \hat{y} E_{\perp}^t e^{-j\vec{\beta}^t \cdot \vec{r}} = \hat{y} T_{\perp}^b E_0 e^{-j(\beta_x^t x + \beta_z^t z)} \quad (5-12a), (5)$$

$$\vec{H}_{\perp}^t = \frac{1}{\eta_2} \hat{\beta}^t \times \vec{E}^t = (-\hat{x} \cos \theta_t + \hat{z} \sin \theta_t) \frac{T_{\perp}^b E_0}{\eta_2} e^{-j(\beta_x^t x + \beta_z^t z)} \quad (6), (5-12b),$$

where $T_{\perp}^b \equiv \left. \frac{E_{\perp}^t}{E_{\perp}^i} \right|_{z=0}$, $\beta_x^t = \beta_2 \sin \theta_t$, $\beta_z^t = \beta_2 \cos \theta_t$.

To evaluate the constants Γ_{\perp}^b & T_{\perp}^b , we apply the boundary conditions to the tangential field components.

• Tangential \vec{E} : $\hat{n} \times (\vec{E}^i + \vec{E}^r) \Big|_{z=0^-} = \hat{n} \times \vec{E}^t \Big|_{z=0^+} \quad (5-13a), (7)$

or $(E_y^i + E_y^r)_{z=0^-} = E_y^t \Big|_{z=0^+} \quad (8)$

Substituting from (1), (3) & (5) into (8) gives

$$\left[E_0 e^{-j(\beta_x^i x + \beta_z^i z)} + \Gamma_{\perp}^b E_0 e^{-j(\beta_x^r x - \beta_z^r z)} \right]_{z=0^-} =$$

$$\left[T_{\perp}^b E_0 e^{-j(\beta_x^t x + \beta_z^t z)} \right]_{z=0^+}$$

or $E_0 e^{-j\beta_x^i x} + \Gamma_{\perp}^b E_0 e^{-j\beta_x^r x} = T_{\perp}^b E_0 e^{-j\beta_x^t x} \quad (5-14a), (9)$

This equation is a fct. of x . However, it is valid for every x . In order for (9) to be enforced at all x requires that

$$\beta_x^i = \beta_x^r = \beta_x^t \quad (10)$$

This is called phase matching. Hence, w/ (10) then (9) becomes

$$1 + \Gamma_{\perp}^b = T_{\perp}^b \quad (11)$$

• Tangential \vec{H} : $\hat{n} \times (\vec{H}^i + \vec{H}^r) \Big|_{z=0^-} = \hat{n} \times \vec{H}^t \Big|_{z=0^+}$ (5-13b), (12)

Substituting from (2), (4) & (6) into (12) gives

$$\left[-\cos \theta_i \frac{E_0}{\eta_1} e^{-j(\beta_x^i x + \beta_z^i z)} + \cos \theta_r \frac{\Gamma_{\perp}^b E_0}{\eta_1} e^{-j(\beta_x^r x - \beta_z^r z)} \right]_{z=0^-} =$$

$$\left[-\cos \theta_t \frac{T_{\perp}^b E_0}{\eta_2} e^{-j(\beta_x^t x + \beta_z^t z)} \right]_{z=0^+}$$

$$\text{or } \frac{1}{\eta_1} \left[-\cos \theta_i e^{-j\beta_x^i x} + \cos \theta_r \Gamma_{\perp}^b e^{-j\beta_x^r x} \right] = -\frac{1}{\eta_2} \cos \theta_t T_{\perp}^b e^{-j\beta_x^t x} \quad (5-14b), (13)$$

By ^{the} phase match condition, $\beta_x^i = \beta_x^r = \beta_x^t$ such that (13) becomes

$$-\frac{1}{\eta_1} \cos \theta_i + \Gamma_{\perp}^b \cos \theta_r = -\frac{T_{\perp}^b}{\eta_2} \cos \theta_t \quad (14)$$

Now, there is an interesting phenomenon that occurs because of this phase match condition (10).

$$\beta_x^i = \beta_x^r = \beta_x^t$$

Substituting for these β_x 's we find that

$$\beta_1 \sin \theta_i = \beta_1 \sin \theta_r = \beta_2 \sin \theta_t \quad (15)$$

In particular, we see that $\sin \theta_i = \sin \theta_r$ or

$$\boxed{\theta_i = \theta_r} \quad (5-15a), (16)$$

This is called Snell's law of reflection. For a UPW obliquely incident on a half space, the angles of incidence & reflection are the same.

Using (16) in (14), we find that

$$\frac{\cos \theta_i}{\eta_1} (-1 + \Gamma_{\perp}^b) = -\frac{\cos \theta_t}{\eta_2} T_{\perp}^b \quad (5-16b), (17)$$

Eqs. (11) & (17) can be solved to find Γ_{\perp}^b & T_{\perp}^b as $\eta_2 = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$

$$\Gamma_{\perp}^b = \left. \frac{E_{\perp}^r}{E_{\perp}^i} \right|_{z=0} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} = \frac{\frac{\eta_2}{\eta_1} \cos \theta_i - \cos \theta_t}{\frac{\eta_2}{\eta_1} \cos \theta_i + \cos \theta_t} \quad (5-17a), (18)$$

$$T_{\perp}^b = \left. \frac{E_{\perp}^r}{E_{\perp}^i} \right|_{z=0} = 1 + \Gamma_{\perp}^b = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \quad (5-17b), (19)$$

These are usually called the Fresnel reflection & transmission coefficients for perpendicular pol. (First derived by Fresnel in 1823)

The one remaining unknown is θ_t . But, from the second equality in (15)

$$\beta_1 \sin \theta_i = \beta_2 \sin \theta_t \quad (5-15b)$$

$$\text{or } \theta_t = \sin^{-1} \left(\frac{\beta_1}{\beta_2} \sin \theta_i \right) \quad (20)$$

This is called Snell's law of refraction.

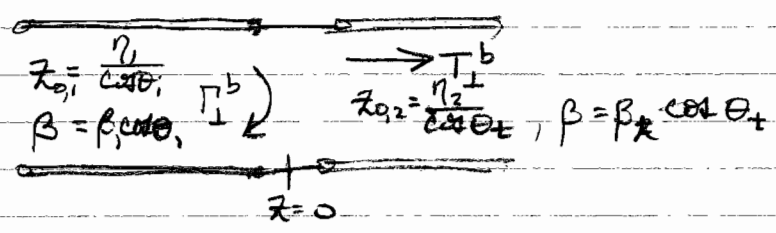
The Analogy

It is possible to use a TH analogy to model this UPW problem, though it's different than for normal incidence.

We can develop this TL model from (18) & (19).

$$\Gamma_{\perp}^b = \frac{\frac{\eta_2}{\cos \theta_t} - \frac{\eta_1}{\cos \theta_i}}{\frac{\eta_2}{\cos \theta_t} + \frac{\eta_1}{\cos \theta_i}} \quad ; \quad T_{\perp}^b = \frac{\frac{2\eta_2}{\cos \theta_t}}{\frac{\eta_2}{\cos \theta_t} + \frac{\eta_1}{\cos \theta_i}} \quad (21), (22)$$

instead of η_1 & η_2 as the char. impedances of the TLS, we use $\frac{\eta_1}{\cos \theta_i}$ & $\frac{\eta_2}{\cos \theta_t}$, respectively:

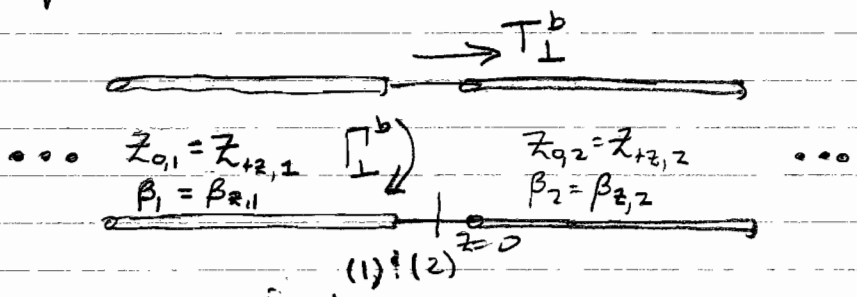


The physical significance of these Z_0 's is they are ratios of \perp comps of $E \neq H$ that are both tangential to the interface.

In fact, we can define the wave impedance in the z direction as ratio of E_y by H_x as

$$Z_{+z} \equiv -\frac{E_y}{H_x}$$

It is this impedance we use for a transmission line analogy as



Hence, using regular TL analysis,

$$\Gamma_{\perp}^b = \frac{Z_{0,2} - Z_{0,1}}{Z_{0,2} + Z_{0,1}} = \frac{Z_{+z,2} - Z_{+z,1}}{Z_{+z,2} + Z_{+z,1}}$$

s.t.

$$\Gamma_{\perp}^b = \frac{\frac{\eta_2}{\cos \theta_t} - \frac{\eta_1}{\cos \theta_i}}{\frac{\eta_2}{\cos \theta_t} + \frac{\eta_1}{\cos \theta_i}}$$

which is (21). Similar for T_{\perp}^b

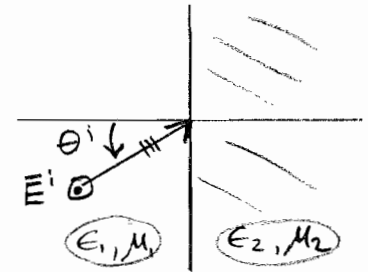
where $Z_{+z,1} = -\frac{E_y^i}{H_x^i} = -\frac{E_0}{(-\cos \theta_i \frac{E_0}{\eta_1})} = \frac{\eta_1}{\cos \theta_i}$

and $Z_{+z,2} = -\frac{E_y^t}{H_x^t} = -\frac{E_0 T_{\perp}^b}{(+\cos \theta_t \frac{E_0 T_{\perp}^b}{\eta_2})} = \frac{\eta_2}{\cos \theta_t}$

Example N9.1

Perpendicularly Polarized UPW Obliquely Incident on a Lossless Half Space - 1

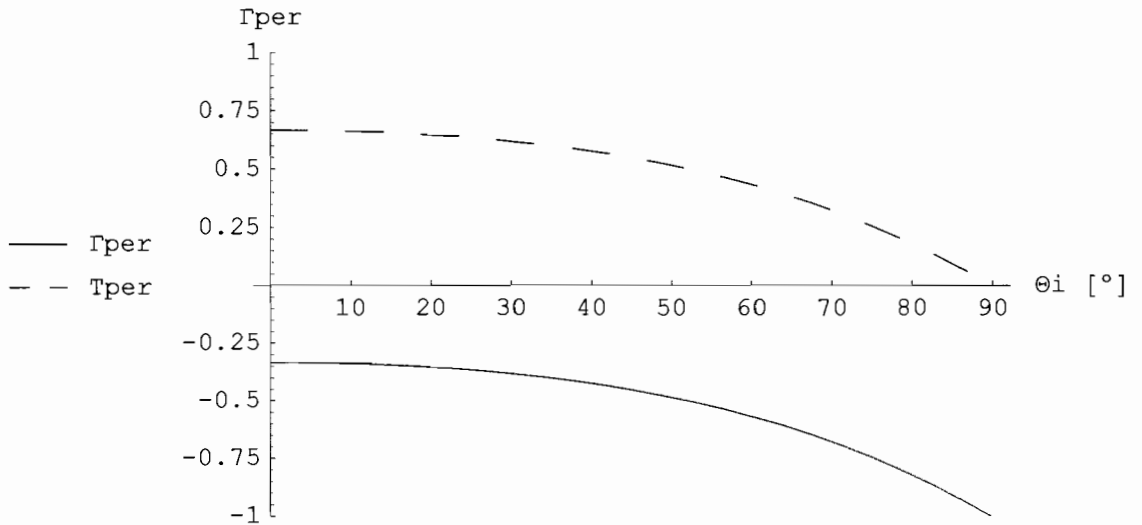
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```
<< Graphics`Legend`
E1/E2 = 1/4
epsilon2 := 1/4
mu2 := 1/1
mu1 = mu2
```

```
theta[theta_i_] := ArcSin[Sqrt[mu2/epsilon2] * Sin[theta_i]]
Gamma_per[theta_i_] := (Sqrt[epsilon2/mu2] * Cos[theta_i] - Cos[theta[theta_i]]) /
(Sqrt[epsilon2/mu2] * Cos[theta_i] + Cos[theta[theta_i]])
T_per[theta_i_] := 1 + Gamma_per[theta_i]
```

```
In[27]:= Plot[{Gamma_per[theta_i * Degree], T_per[theta_i * Degree]}, {theta_i, 0, 90},
PlotRange -> {-1, 1}, Ticks -> {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
PlotStyle -> {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
AxesLabel -> {"theta_i [°]", "Gamma_per"}, PlotLegend -> {"Gamma_per", "T_per"},
LegendSize -> {0.5, 0.2}, LegendShadow -> None, LegendPosition -> {-1.4, -0.1}]
```



Out[27]= - Graphics -

Example N9.1 (cont.)

Perpendicularly Polarized UPW Obliquely Incident on a Lossless Half Space - 1

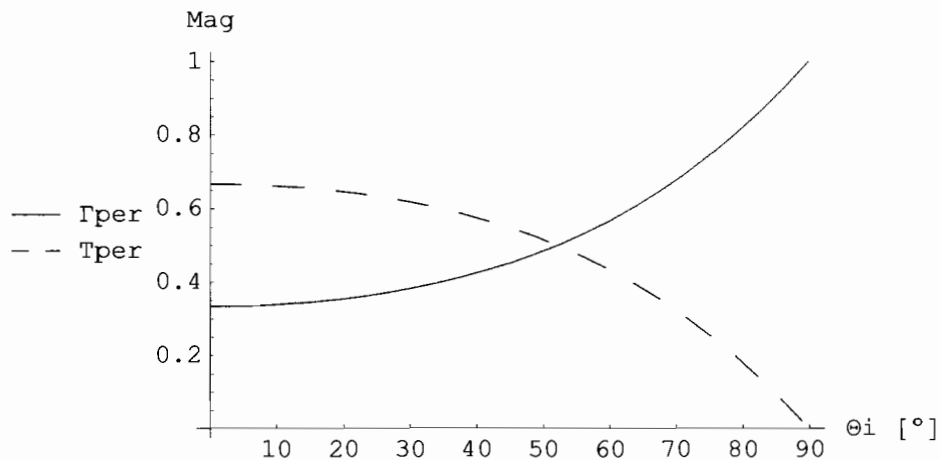
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```
In[1]:= << Graphics`Legend`
```

```
In[2]:=  $\epsilon$ lover $\epsilon$ 2 := 1 / 4  
 $\mu$ lover $\mu$ 2 := 1 / 1
```

```
 $\theta$ t[ $\theta$ i_] := ArcSin[Sqrt[ $\mu$ lover $\mu$ 2 *  $\epsilon$ lover $\epsilon$ 2] * Sin[ $\theta$ i]]  
 $\Gamma$ per[ $\theta$ i_] := (Sqrt[ $\epsilon$ lover $\epsilon$ 2 /  $\mu$ lover $\mu$ 2] * Cos[ $\theta$ i] - Cos[ $\theta$ t[ $\theta$ i]]) /  
  (Sqrt[ $\epsilon$ lover $\epsilon$ 2 /  $\mu$ lover $\mu$ 2] * Cos[ $\theta$ i] + Cos[ $\theta$ t[ $\theta$ i]])  
 $T$ per[ $\theta$ i_] := 1 +  $\Gamma$ per[ $\theta$ i]
```

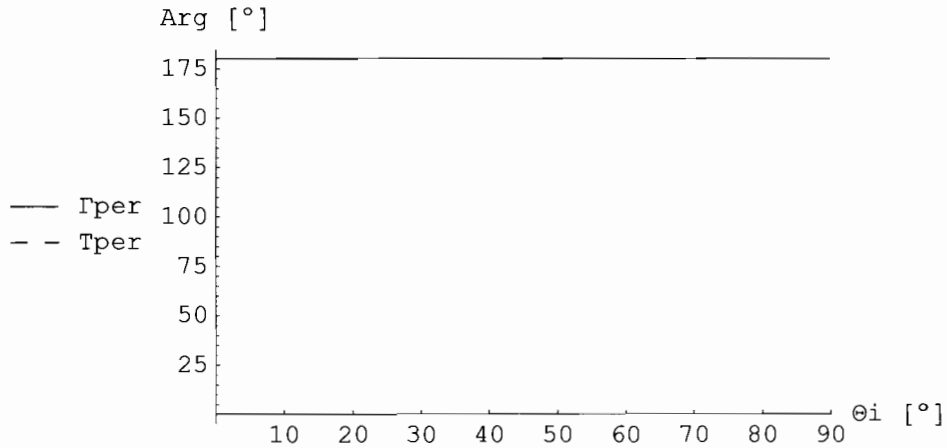
```
In[7]:= Plot[{Abs[ $\Gamma$ per[ $\theta$ i * Degree]], Abs[ $T$ per[ $\theta$ i * Degree]]}, { $\theta$ i, 0, 90},  
  PlotRange -> Automatic, Ticks -> {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},  
  PlotStyle -> {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},  
  AxesLabel -> {" $\theta$ i [°]", "Mag"}, PlotLegend -> {" $\Gamma$ per", " $T$ per"},  
  LegendSize -> {0.5, 0.2}, LegendShadow -> None, LegendPosition -> {-1.4, -0.1}]
```



```
Out[7]= - Graphics -
```

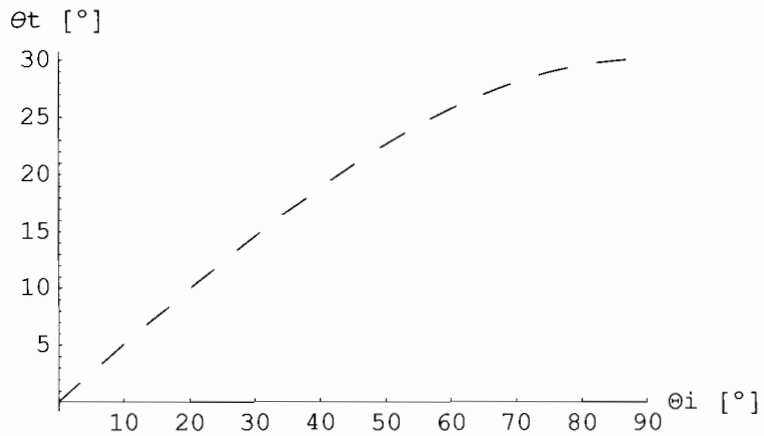

Example N9.1 (cont.)

```
In[8]:= Plot[{Arg[Gamma_per[theta_i * Degree]] / Degree, Arg[T_per[theta_i * Degree]] / Degree}, {theta_i, 0, 90},
  PlotRange -> {{0, 90}, Automatic}, Ticks -> {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
  PlotStyle -> {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
  AxesLabel -> {"theta_i [degrees]", "Arg [degrees]"}, PlotLegend -> {"Gamma_per", "T_per"},
  LegendSize -> {0.5, 0.2}, LegendShadow -> None, LegendPosition -> {-1.4, -0.1}]
```



Out[8]= - Graphics -

```
In[12]:= Plot[Abs[theta_t[theta_i * Degree]] / Degree, {theta_i, 0, 90}, PlotRange -> {{0, 90}, Automatic},
  Ticks -> {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
  PlotStyle -> {{Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
  AxesLabel -> {"theta_i [degrees]", "theta_t [degrees]"}]
```

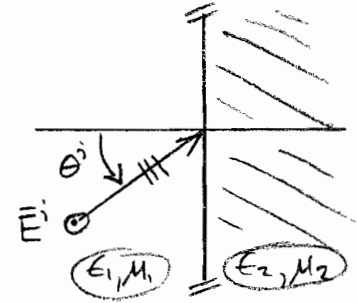


Out[12]= - Graphics -

Example N9.2

Perpendicularly Polarized UPW Obliquely Incident on a Lossless Half Space - 1

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<< Graphics`Legend`

$$\frac{\epsilon_1}{\epsilon_2} = 4$$

In[28]:= $\epsilon \text{ over } \epsilon_2 := 4 / 1$ ←

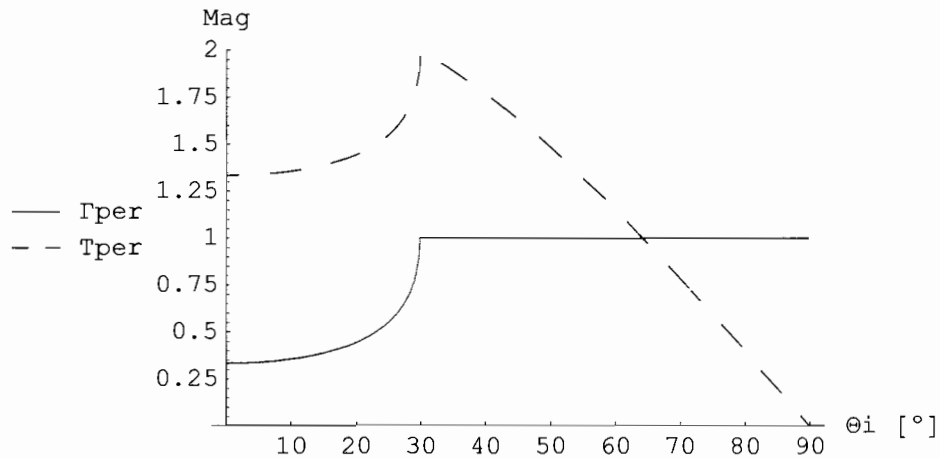
$\mu \text{ over } \mu_2 := 1 / 1$ ← $\mu_1 = \mu_2$

$\theta_t[\theta_i] := \text{ArcSin}[\text{Sqrt}[\mu \text{ over } \mu_2 + \epsilon \text{ over } \epsilon_2] * \text{Sin}[\theta_i]]$

$\Gamma_{\text{per}}[\theta_i] := (\text{Sqrt}[\epsilon \text{ over } \epsilon_2 / \mu \text{ over } \mu_2] * \text{Cos}[\theta_i] - \text{Cos}[\theta_t[\theta_i]]) /$
 $(\text{Sqrt}[\epsilon \text{ over } \epsilon_2 / \mu \text{ over } \mu_2] * \text{Cos}[\theta_i] + \text{Cos}[\theta_t[\theta_i]])$

$T_{\text{per}}[\theta_i] := 1 + \Gamma_{\text{per}}[\theta_i]$

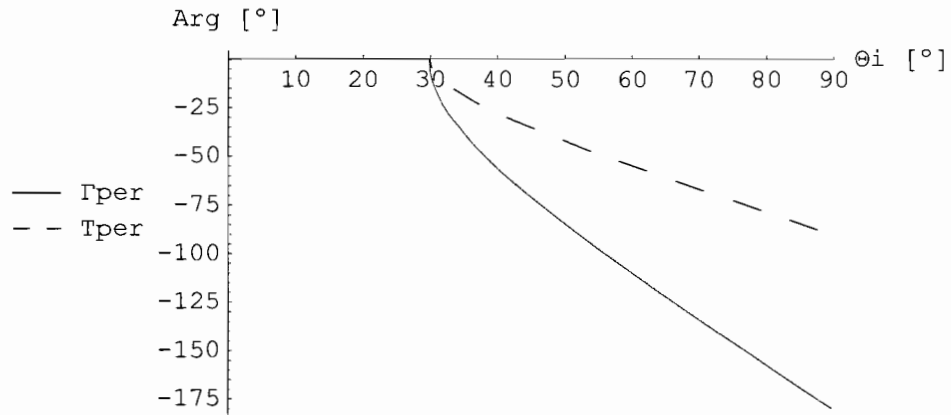
```
In[55]:= Plot[{Abs[Γper[θi * Degree]], Abs[Tper[θi * Degree]]}, {θi, 0, 90},
  PlotRange -> {0, 2}, Ticks -> {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
  PlotStyle -> {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
  AxesLabel -> {"θi [°]", "Mag"}, PlotLegend -> {"Γper", "Tper"},
  LegendSize -> {0.5, 0.2}, LegendShadow -> None, LegendPosition -> {-1.4, -0.1}]
```



Out[55]= - Graphics -

Example N9.2 (cont.)

```
In[59]:= Plot[{Arg[ $\Gamma_{per}$ [ $\theta_i$  * Degree]] / Degree, Arg[ $T_{per}$ [ $\theta_i$  * Degree]] / Degree},  
  { $\theta_i$ , 0, 90}, PlotRange -> {{0, 90}, Automatic},  
  Ticks -> {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},  
  PlotStyle -> {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},  
  AxesLabel -> {" $\theta_i$  [°]", "Arg [°]"}, PlotLegend -> {" $\Gamma_{per}$ ", " $T_{per}$ "},  
  LegendSize -> {0.5, 0.2}, LegendShadow -> None, LegendPosition -> {-1.4, -0.1}]
```



Out[59]= - Graphics -