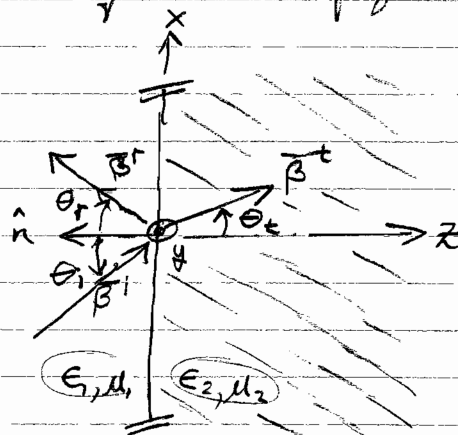
EE692
Whites

Lecture 9: UPWs Obliquely Incident on a Lossless Half Space: E Pol.

Oblique incidence is a very important part of UPW analysis, though it's a bit more complicated than normal incidence analysis. Oblique incidence will unveil interesting phenomenon not present in normal incidence.

There is some important terminology we need to define for oblique incidence. Referring to this figure:



1. Plane of incidence: plane formed by $\hat{n}$ & $\bar{\beta}^i$ (or $\bar{\beta}^r$ or $\bar{\beta}^t$).

2. Perpendicular polarization: UPW w/ $\bar{E}$ $\perp$ to plane of incidence. Also called E pol or horizontal polarization.

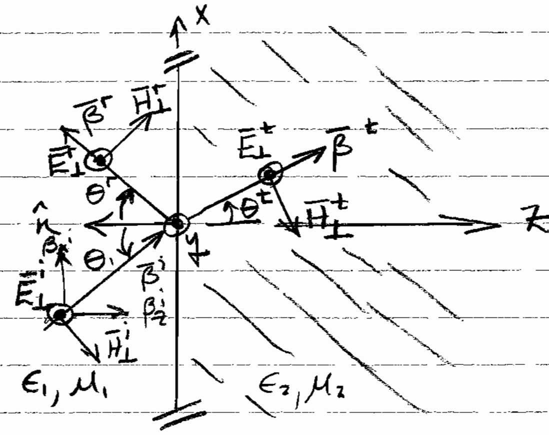
3. Parallel polarization: UPW w/ $\bar{E}$ $\parallel$ to plane of incidence. Also called H pol or vertical polarization.

The last two definitions arise because of this fact: A perpendicular pol UPW will reflect & transmit only $\perp$-pol UPWs when incident on a half space. Similarly, a H pol UPW w/ reflect & transmit only H-pol UPWs. These are called the principal polarizations. (Not to be confused with the polarizations discussed earlier. Here both are linearly polarized.)

Because of this, we will study each of these polarizations separately. For an arbitrarily polarized UPW, decompose into its component polarizations.

Perpendicular Polarization

The geometry for this obliquely incident UPW will be assumed as



As described in the lectures associated w/ Ch. 4 in the text,

$$\bar E_\perp^i = \hat y E^i e^{-j\bar\beta^i\cdot\bar r} = \hat y E_0 e^{-j(\beta_x^i x + \beta_z^i z)} \qquad (5\text{-}10a), (1)$$

& w/ $\nabla\times\bar E = -j\omega\mu\bar H \xrightarrow[\text{w/ } e^{-j\bar\beta\cdot\bar r}]{\text{UPW}} -j\bar\beta^i\times\bar E^i = -j\beta_1\eta_1\bar H^i$

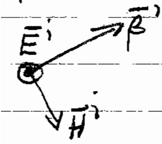
$\omega\sqrt{\mu\epsilon}\frac{\sqrt\mu}{\sqrt\epsilon} = \omega\mu$

See lecture

$$\Rightarrow \bar H^i = \frac{1}{\beta_1\eta_1}\bar\beta^i\times\bar E^i = \frac{1}{\beta_1\eta_1}\begin{vmatrix}\hat x & \hat y & \hat z\\ \beta_x^i & 0 & \beta_z^i\\ 0 & E_y^i & 0\end{vmatrix} = \frac{1}{\beta_1\eta_1}\left(-\hat x\beta_z^i E_y^i + \hat z\beta_x^i E_y^i\right)$$

or $\bar H^i = \frac{1}{\eta_1}\hat\beta^i\times\bar E^i$

This makes sense geometrically



Further, w/ $\beta_x^i = \beta_1\sin\theta_i$ & $\beta_z^i = \beta_1\cos\theta_i$, then

$$\bar H^i = \frac{1}{\beta_1\eta_1}\left(-\hat x\beta_1\cos\theta_i + \hat z\beta_1\sin\theta_i\right)E_y^i$$

or

$$\bar H^i = \left(-\hat x\cos\theta_i + \hat z\sin\theta_i\right)\frac{E_0}{\eta_1}e^{-j(\beta_x^i x + \beta_z^i z)} \qquad (5\text{-}10b), (2)$$

Similarly, the reflected fields are

$$\bar E_\perp^r = \hat y E_\perp^r e^{-j\bar\beta^r\cdot\bar r} = \hat y\Gamma_\perp^b E_0 e^{-j(\beta_x^r x - \beta_z^r z)} \qquad (5\text{-}11a), (3)$$

$$\bar H_\perp^r = \frac{1}{\eta_1}\hat\beta^r\times\bar E^r = \left(\hat x\cos\theta_r + \hat z\sin\theta_r\right)\frac{\Gamma_\perp^b E_0}{\eta_1}e^{-j(\beta_x^r x - \beta_z^r z)} \qquad (5\text{-}11b), (4)$$

where $\quad \Gamma_\perp^b \equiv \left.\frac{E_\perp^r}{E_\perp^i}\right|_{z=0}$, $\beta_x^r = \beta_1 \sin\theta_r$, $\beta_z^r = \beta_1 \cos\theta_r$

and the transmitted fields are

$$\bar{E}_\perp^t = \hat{y}\, E_\perp^t e^{-j\bar{\beta}^t\cdot\bar{r}} = \hat{y}\, T_\perp^b E_0 e^{-j(\beta_x^t x + \beta_z^t z)} \qquad (5\text{-}12a), (5)$$

$$\bar{H}_\perp^t = \frac{1}{\eta_2}\hat{\beta}^t \times \bar{E}^t = (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)\frac{T_\perp^b E_0}{\eta_2} e^{-j(\beta_x^t x + \beta_z^t z)} \qquad (5\text{-}12b), (6)$$

where $\quad T_\perp^b \equiv \left.\frac{E_\perp^t}{E_\perp^i}\right|_{z=0}$, $\beta_x^t = \beta_2 \sin\theta_t$, $\beta_z^t = \beta_2 \cos\theta_t$.

To evaluate the constants $\Gamma_\perp^b$ & $T_\perp^b$, we apply the boundary conditions to the <u>tangential</u> field components.

- <u>Tangential $\bar{E}$</u>: $\quad \hat{n}\times(\bar{E}^i + \bar{E}^r)\big|_{z=0^-} = \hat{n}\times \bar{E}^t\big|_{z=0^+} \qquad (5\text{-}13a), (7)$

or $$(E_y^i + E_y^r)_{z=0^-} = E_y^t\big|_{z=0^+}. \qquad (8)$$

Substituting from (1), (3) & (5) into (8) gives

$$\left[E_0 e^{-j(\beta_x^i x + \beta_z^i z)} + \Gamma_\perp^b E_0 e^{-j(\beta_x^r x - \beta_z^r z)}\right]_{z=0^-} =$$

$$\left[T_\perp^b E_0 e^{-j(\beta_x^t x + \beta_z^t z)}\right]_{z=0^+}$$

or $$\cancel{E_0} e^{-j\beta_x^i x} + \Gamma_\perp^b \cancel{E_0} e^{-j\beta_x^r x} = T_\perp^b \cancel{E_0} e^{-j\beta_x^t x} \qquad (5\text{-}14a), (9)$$

This equation is a fct. of x. However, it is valid ~~at~~ <u>every</u> x. In order for (9) to be enforced at all x requires that

$$\beta_x^i = \beta_x^r = \beta_x^t \qquad (10)$$

This is called *phase matching*. Hence, w/ (10) then (9) becomes

$$1 + \Gamma_\perp^b = T_\perp^b \qquad (11).$$

Tangential $\bar{H}$: $\hat{n} \times (\bar{H}^i + \bar{H}^r)\Big|_{z=0^-} = \hat{n} \times \bar{H}^t\Big|_{z=0^+}$ (5-13b), (12)

Substituting from (2), (4) & (6) into (12) gives

$$\left[-\cos\theta_i \frac{E_0}{\eta_1} e^{-j(\beta_x^i x + \beta_z^i z)} + \cos\theta_r \frac{\Gamma_\perp^b E_0}{\eta_1} e^{-j(\beta_x^r x - \beta_z^r z)}\right]_{z=0^-} =$$

$$\left[-\cos\theta_t \frac{T_\perp^b E_0}{\eta_2} e^{-j(\beta_x^t x + \beta_z^t z)}\right]_{z=0^+}$$

or $$\frac{1}{\eta_1}\left[-\cos\theta_i e^{-j\beta_x^i x} + \cos\theta_r \Gamma_\perp^b e^{-j\beta_x^r x}\right] = -\frac{1}{\eta_2}\cos\theta_t T_\perp^b e^{-j\beta_x^t x} \qquad (5\text{-}14b), (13)$$

By the phase match condition, $\beta_x^i = \beta_x^r = \beta_x^t$ such that (13) becomes

$$-\frac{1}{\eta_1}\cos\theta_i + \Gamma_\perp^b \cos\theta_r = -\frac{T_\perp^b}{\eta_2}\cos\theta_t \qquad (14).$$

Now, there is an interesting phenomenon that occurs because of this phase match condition (10).

$$\beta_x^i = \beta_x^r = \beta_x^t$$

Substituting for these β_x's we find that

$$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r = \beta_2 \sin\theta_t \qquad (15)$$

In particular, we see that $\sin\theta_i = \sin\theta_r$ or

$$\boxed{\theta_i = \theta_r} \qquad (5\text{-}15a), (16)$$

This is called Snell's law of reflection. For a UPW obliquely incident on a half space, the angles of incidence & reflection are the same.

Using (16) in (14), we find that

$$\frac{\cos\theta_i}{\eta_1}\left(-1+\Gamma_\perp^b\right) = -\frac{\cos\theta_t}{\eta_2}T_\perp^b \qquad (5\text{-}16b), (17)$$

Eqns. (11) & (17) can be solved to find $\Gamma_\perp^b$ & $T_\perp^b$ as

$$\frac{\eta_2}{\eta_1} = \frac{\sqrt{\mu_2/\epsilon_2}}{\sqrt{\mu_1/\epsilon_1}} = \frac{\sqrt{\epsilon_1/\epsilon_2}}{\sqrt{\mu_1/\mu_2}}$$

$$\boxed{\Gamma_\perp^b = \left.\frac{E_\perp^r}{E_\perp^i}\right|_{z=0} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}} = \frac{\frac{\eta_2}{\eta_1}\cos\theta_i - \cos\theta_t}{\frac{\eta_2}{\eta_1}\cos\theta_i + \cos\theta_t} \qquad (5\text{-}17a), (18)$$

&

$$\boxed{T_\perp^b = \left.\frac{E_\perp^t}{E_\perp^i}\right|_{z=0} = 1+\Gamma_\perp^b = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}} \qquad (5\text{-}17b), (19)$$

These are usually called the Fresnel reflection & transmission coefficients for perpendicular pol. (First derived by Fresnel in 1823)

The one remaining unknown is θ_t. But, from the second equality in (15)

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t \qquad (5\text{-}15b)$$

or

$$\theta_t = \sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right) \qquad (20)$$

This is called Snell's law of refraction.

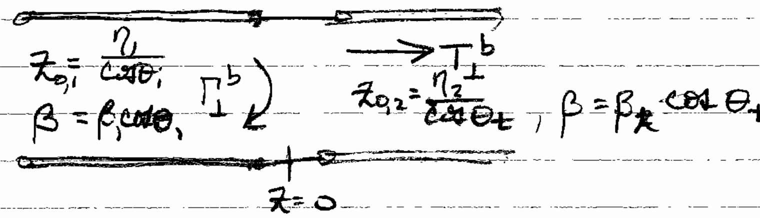
TL Analogy

It is possible to use a TL analogy to model this UPW problem, though it's different than for normal incidence.

We can develop this TL model from (18) & (19).

$$\Gamma_\perp^b = \frac{\frac{\eta_2}{\cos\theta_t} - \frac{\eta_1}{\cos\theta_i}}{\frac{\eta_2}{\cos\theta_t} + \frac{\eta_1}{\cos\theta_i}} \quad \& \quad T_\perp^b = \frac{\frac{2\eta_2}{\cos\theta_t}}{\frac{\eta_2}{\cos\theta_t} + \frac{\eta_1}{\cos\theta_i}} \qquad (21),(22)$$

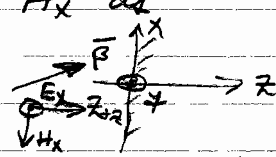
Instead of η_1 & η_2 as the char. impedances of the TLs, we use $\frac{\eta_1}{\cos\theta_i}$ & $\frac{\eta_2}{\cos\theta_t}$, respectively:



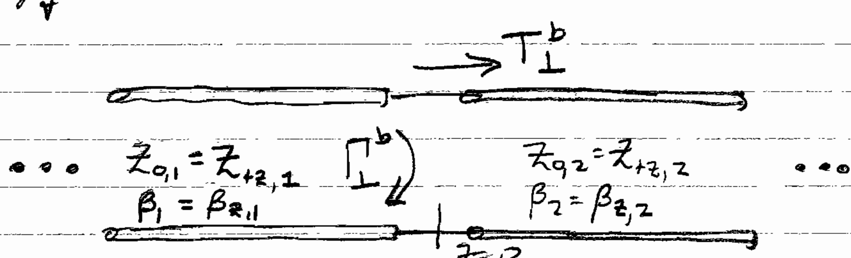
The physical significance of these Z_o's is they are ratios of $\perp$ comps of $\bar{E}$ & $\bar{H}$ that are both tangential to the interface.

In fact, we can define the wave impedance in the z direction as ratio of E_y by H_x as

$$Z_{+z} \equiv -\frac{E_y}{H_x}$$



It is this impedance we use for a transmission line analogy as



(1) & (2)

where $Z_{+z,1} = -\frac{E_y^i}{H_x^i} = -\frac{E_o}{\left(-\cos\theta_i \frac{E_o}{\eta_1}\right)} = \frac{\eta_1}{\cos\theta_i}$

and $Z_{+z,2} = -\frac{E_y^t}{H_x^t} = -\frac{E_o T_\perp^b}{\left(-\cos\theta_t \frac{E_o T_\perp^b}{\eta_2}\right)} = \frac{\eta_2}{\cos\theta_t}$

(2) (4)

Hence, using regular TL analysis,

$$\Gamma_\perp^b = \frac{Z_{o,2} - Z_{o,1}}{Z_{o,2} + Z_{o,1}} = \frac{Z_{+z,2} - Z_{+z,1}}{Z_{+z,2} + Z_{+z,1}}$$

s.t.

$$\Gamma_\perp^b = \frac{\frac{\eta_2}{\cos\theta_t} - \frac{\eta_1}{\cos\theta_i}}{\frac{\eta_2}{\cos\theta_t} + \frac{\eta_1}{\cos\theta_i}}$$

which is (21).

Similar for $T_\perp^b$

Example N9.1

Perpendicularly Polarized UPW Obliquely Incident on a Lossless Half Space - 1

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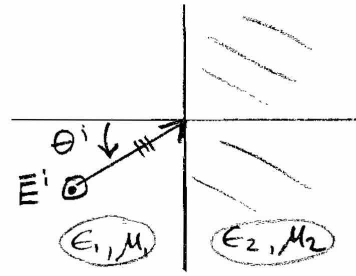


$\frac{\epsilon_1}{\epsilon_2} = \frac{1}{4}$

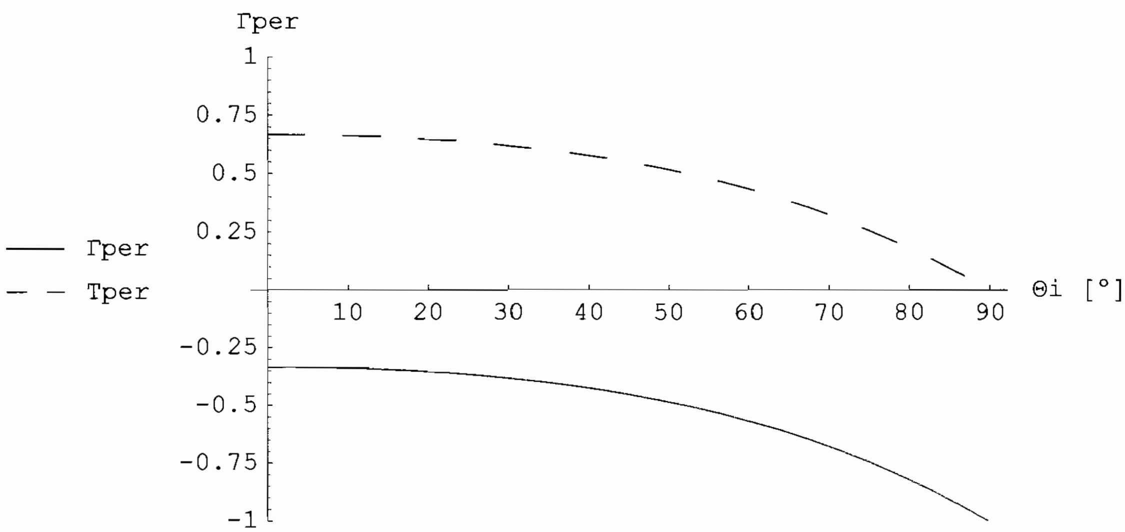
```
<< Graphics`Legend`

ε1overε2 := 1 / 4
μ1overμ2 := 1 / 1
```

$\mu_1 = \mu_2$

```
θt[θi_] := ArcSin[Sqrt[μ1overμ2 * ε1overε2] * Sin[θi]]
Γper[θi_] := (Sqrt[ε1overε2 / μ1overμ2] * Cos[θi] - Cos[θt[θi]]) /
   (Sqrt[ε1overε2 / μ1overμ2] * Cos[θi] + Cos[θt[θi]])
Tper[θi_] := 1 + Γper[θi]
```

```
In[27]:= Plot[{Γper[θi * Degree], Tper[θi * Degree]}, {θi, 0, 90},
  PlotRange → {-1, 1}, Ticks → {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
  PlotStyle → {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
  AxesLabel → {"θi  [°]", "Γper"}, PlotLegend → {"Γper", "Tper"},
  LegendSize → {0.5, 0.2}, LegendShadow → None, LegendPosition → {-1.4, -0.1}]
```




```
Out[27]= - Graphics -
```

Example N9.1 (cont.)

Perpendicularly Polarized UPW Obliquely Incident on a Lossless Half Space - 1

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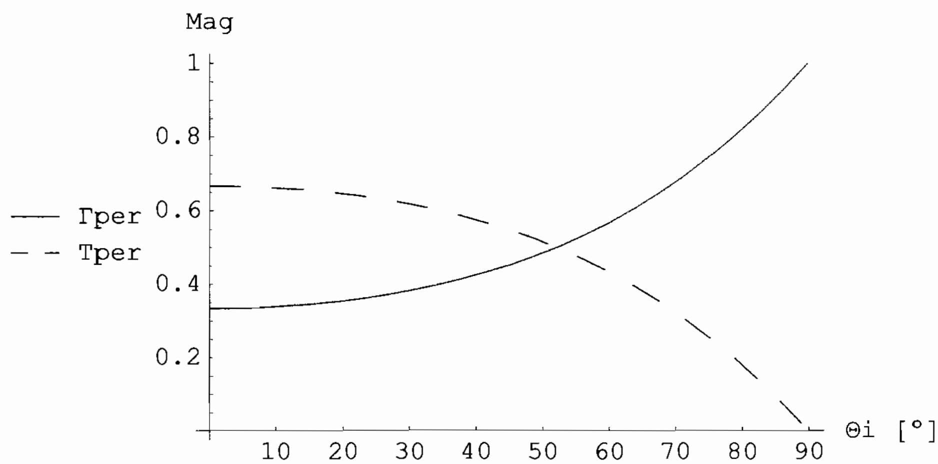
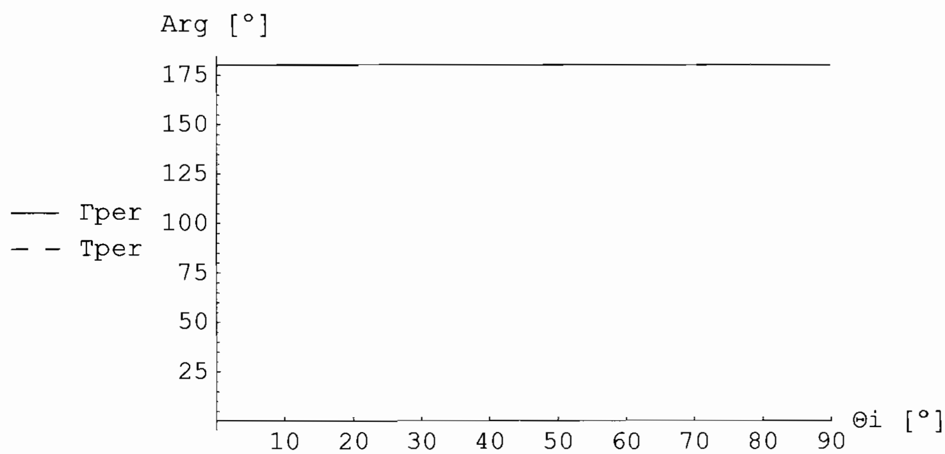
```
In[1]:= << Graphics`Legend`

In[2]:= ε1overε2 := 1 / 4
        μ1overμ2 := 1 / 1

        θt[θi_] := ArcSin[Sqrt[μ1overμ2 * ε1overε2] * Sin[θi]]
        Γper[θi_] := (Sqrt[ε1overε2 / μ1overμ2] * Cos[θi] - Cos[θt[θi]]) /
          (Sqrt[ε1overε2 / μ1overμ2] * Cos[θi] + Cos[θt[θi]])
        Tper[θi_] := 1 + Γper[θi]

In[7]:= Plot[{Abs[Γper[θi * Degree]], Abs[Tper[θi * Degree]]}, {θi, 0, 90},
         PlotRange → Automatic, Ticks → {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
         PlotStyle → {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
         AxesLabel → {"θi  [°]", "Mag"}, PlotLegend → {"Γper", "Tper"},
         LegendSize → {0.5, 0.2}, LegendShadow → None, LegendPosition → {-1.4, -0.1}]
```




```
Out[7]= - Graphics -
```

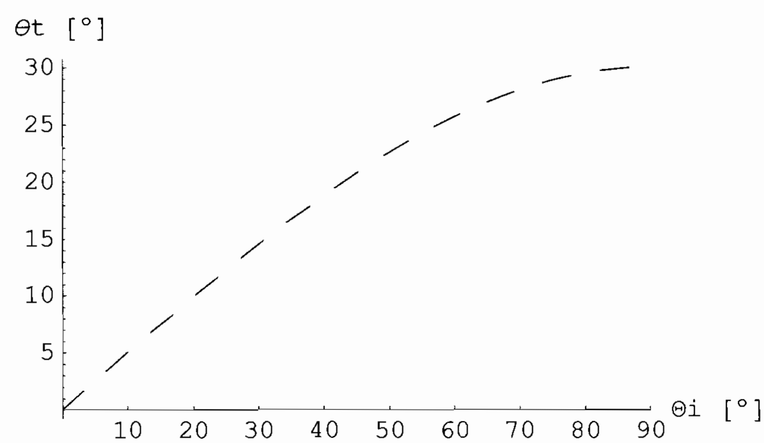
Example N9.1 (cont.)

```
In[8]:= Plot[{Arg[Γper[θi*Degree]] / Degree, Arg[Τper[θi*Degree]] / Degree}, {θi, 0, 90},
    PlotRange → {{0, 90}, Automatic}, Ticks → {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
    PlotStyle → {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
    AxesLabel → {"θi [°]", "Arg [°]"}, PlotLegend → {"Γper", "Τper"},
    LegendSize → {0.5, 0.2}, LegendShadow → None, LegendPosition → {-1.4, -0.1}]
```



Out[8]= - Graphics -

```
In[12]:= Plot[Abs[θt[θi*Degree] / Degree], {θi, 0, 90}, PlotRange → {{0, 90}, Automatic},
    Ticks → {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
    PlotStyle → {{Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
    AxesLabel → {"θi [°]", "θt [°]"}]
```



Out[12]= - Graphics -

Example N9.2

Perpendicularly Polarized UPW Obliquely Incident on a Lossless Half Space - 1

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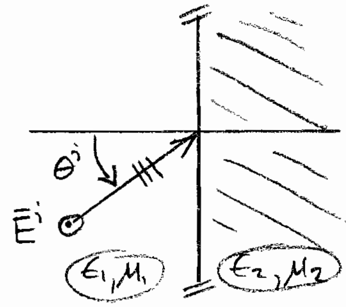



```
<< Graphics`Legend`
```

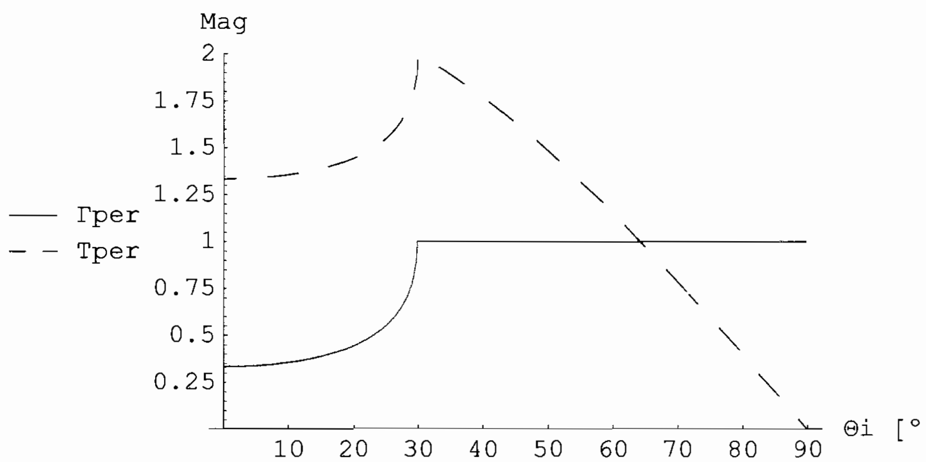
$\frac{\epsilon_1}{\epsilon_2} = 4$

```
In[28]:= ϵloverϵ2 := 4 / 1
         μloverμ2 := 1 / 1
```

$\mu_1 = \mu_2$

```
θt[θi_] := ArcSin[Sqrt[μloverμ2 * ϵloverϵ2] * Sin[θi]]
Γper[θi_] := (Sqrt[ϵloverϵ2 / μloverμ2] * Cos[θi] - Cos[θt[θi]]) /
   (Sqrt[ϵloverϵ2 / μloverμ2] * Cos[θi] + Cos[θt[θi]])
Tper[θi_] := 1 + Γper[θi]
```

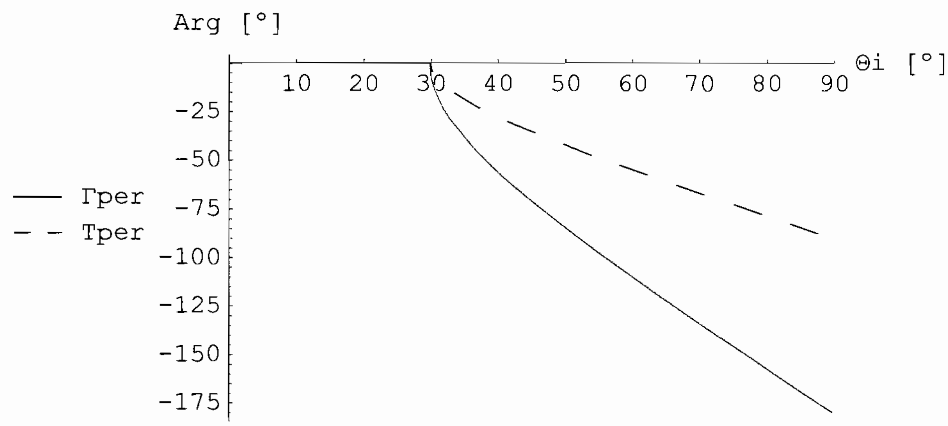
```
In[55]:= Plot[{Abs[Γper[θi * Degree]], Abs[Tper[θi * Degree]]}, {θi, 0, 90},
   PlotRange → {0, 2}, Ticks → {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
   PlotStyle → {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
   AxesLabel → {"θi  [°]", "Mag"}, PlotLegend → {"Γper", "Tper"},
   LegendSize → {0.5, 0.2}, LegendShadow → None, LegendPosition → {-1.4, -0.1}]
```




```
Out[55]= - Graphics -
```

Example N9.2 (cont.)

```
In[59]:= Plot[{Arg[Γper[θi*Degree]] / Degree, Arg[Tper[θi*Degree]] / Degree},
  {θi, 0, 90}, PlotRange → {{0, 90}, Automatic},
  Ticks → {{10, 20, 30, 40, 50, 60, 70, 80, 90}, Automatic},
  PlotStyle → {{RGBColor[1, 0, 0]}, {Dashing[{0.05, 0.05}], RGBColor[0, 0, 1]}},
  AxesLabel → {"θi [°]", "Arg [°]"}, PlotLegend → {"Γper", "Tper"},
  LegendSize → {0.5, 0.2}, LegendShadow → None, LegendPosition → {-1.4, -0.1}]
```

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Out[59]= - Graphics -
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