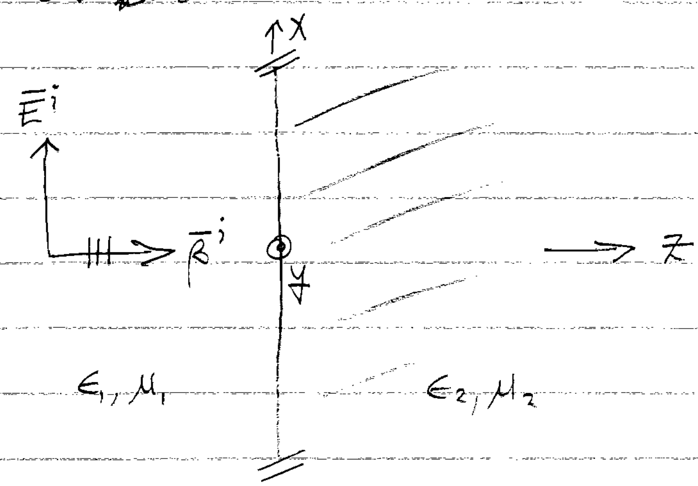
Lecture 8: UPWs Normally Incident on a Lossless Half Space

The past few lectures have covered UPW's propagating in infinite, homogeneous spaces. This work sets the foundation for solving an important set of scattering & radiation problems:

1) UPW impinging on a half space.
2) UPW impinging on planarly layered media.
3) UPW's radiated from infinite current sheets

These types of problems are extraordinarily important to master. Since they are fictitious, they serve primarily as models to understand more complicated scattering & radiation problems.

To begin, consider a UPW incident on a half space as shown.



where $\bar{E}^i = \hat{x} E_0 e^{-j\beta_1 z}$ (5-1a), (1)

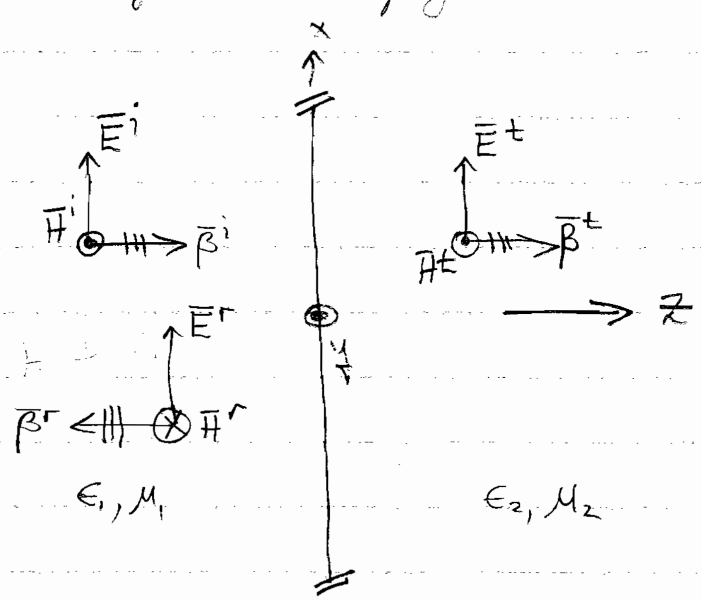
This UPW is produced somewhere at $z < 0$, but the details of this aren't important. Instead, (1) gives all the important information.

Because the UPW is incident perpendicularly to the interface, it is called normal incidence. ("The plane wave is incident normally to the interface.")

We know all the important characteristics of UPWs propagating in an infinite, simple material:

1. $\bar{E}$ & $\bar{H}$ perpendicular to each other
2. $\bar{E}$ & $\bar{H}$ perpendicular to $\bar{\beta}$, the wave vector
   $$\bar{\beta} = \hat{x}\beta_x + \hat{y}\beta_y + \hat{z}\beta_z.$$
   In this problem, $\beta_y = \beta_z = 0$ so that
   $$\bar{\beta}^i = \hat{x}\beta_x^i = \hat{x}\,\beta_1 \text{ rad/m}.$$
3. The ratio of perpendicular components of E & H $= \eta$.
4. The direction of $\bar{E}\times\bar{H}$ is with the direction of wave propagation.

With these characteristics, we can fill in the missing pieces needed to set up the problem shown in the previous figure:



Note that we've assumed all $\bar{E}$'s point in same direction $(+\hat{x})$. (You should always do this, since it's common to use the simple reflection & transmission coeff expressions we'll derive soon.) The "scattered" electric fields are then

$$\bar{E}^r = \hat{x} E^r e^{+j\beta^r z} = \hat{x} \Gamma^b E_o e^{+j\beta_1 z} \qquad (5\text{-}1b), (2)$$

and

$$\bar{E}^t = \hat{x} E^t e^{-j\beta^t z} = \hat{x} T^b E_o e^{-j\beta_2 z} \qquad (5\text{-}1c), (3)$$

Γ^b & T^b are the electric field reflection & transmission coeffs at the interface (or boundary).

From the UPW characteristics listed earlier, we can easily determine $\bar{H}$ in terms of $\bar{E}$:

$$\bar{H}^i = \hat{y} \frac{E_o}{\eta_1} e^{-j\beta_1 z} \qquad (5\text{-}2a), (4)$$

$$\bar{H}^r = -\hat{y} \frac{E_o \Gamma^b}{\eta_1} e^{+j\beta_1 z} \qquad (5\text{-}2b), (5)$$

&

$$\bar{H}^t = \hat{y} \frac{E_o T^b}{\eta_2} e^{-j\beta_2 z} \qquad (5\text{-}2c), (6)$$

All that remains to complete this solution is to determine the coeffs Γ^b & T^b. The factor E_o is a source characteristic. It either needs to be given or more characteristics of the source would have to be provided for us to compute it.

We solve for Γ^b & T^b by applying the boundary conditions that:

(1) E_{tan} continuous $\Rightarrow$ $\hat{n} \times \bar{E}_1(z=0^-) = \hat{n} \times \bar{E}_2(z=0^+)$ (7)
Note that it is the total field that is continuous.
Using (1)-(3) in (7) gives

$$\hat{x}\left[E_0 e^{-j\beta_1 z} + \Gamma_b E_0 e^{+j\beta_1 z}\right]_{z=0^-} = \hat{x}\, T_b E_0 e^{-j\beta_2 z}\Big|_{z=0^+}$$

or
$$1 + \Gamma_b = T_b \qquad (5\text{-}3a),\ (8)$$

(2) $\bar{H}_{tan}$ continuous $\Rightarrow$ $\hat{n} \times \bar{H}_1(z=0^-) = \hat{n} \times \bar{H}_2(z=0^+)$ (9)
Using (4)-(6) in (9) gives

$$\hat{y}\left[\frac{E_0}{\eta_1} e^{-j\beta_1 z} - \frac{E_0 \Gamma^b}{\eta_1}\right]_{z=0^-} = \hat{y}\left[\frac{E_0 T^b}{\eta_2} e^{-j\beta_2 z}\right]_{z=0^+}$$

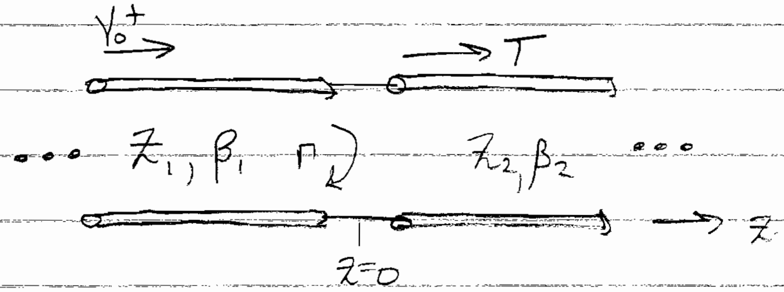
or
$$\frac{1}{\eta_1}\left(1 - \Gamma^b\right) = \frac{T^b}{\eta_2} \qquad (10)$$

Solving these two eqns. (8) & (10) yields

$$\boxed{\Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \qquad (5\text{-}4a),\ (11)$$

$$\boxed{T^b = \frac{2\eta_2}{\eta_2 + \eta_1}} \qquad (5\text{-}4b),\ (12)$$

Notice how similar this UPW problem is to a transmission line problem:



where the voltages are

$$V_1(z) = V_0^+ \left(e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z} \right) \qquad (13)$$

&

$$V_2(z) = V_0^+ T e^{-j\beta_2 z} \qquad (14)$$

and the currents are

$$I_1(z) = \frac{V_0^+}{Z_1} \left(e^{-j\beta_1 z} - \Gamma e^{+j\beta_1 z} \right) \qquad (15)$$

and

$$I_2(z) = \frac{V_0^+}{Z_2} e^{-j\beta_2 z} \qquad (16)$$

where $\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$ & $T = 1 + \Gamma = \frac{2Z_2}{Z_2 + Z_1}$ (17) (18)

A direct analogy between the two problems can be effected if we equate:

(i) V_0^+ w/ E_0

(ii) Z_1 w/ η_1 & Z_2 w/ η_2

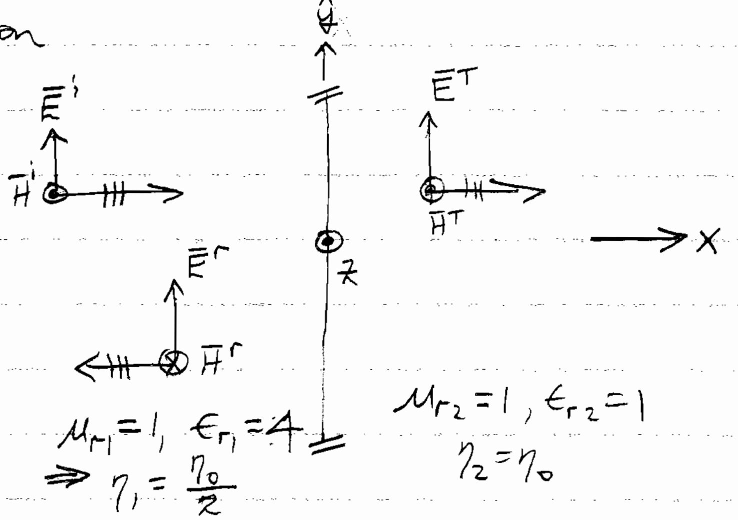
(iii) β_1 w/ β_1 & β_2 w/ β_2

If so, solutions for $V(z)$ are directly analogous ~~for~~ to those for $E_x(z)$ & solutions for $I(z)$ are directly analogous to those for $H_y(z)$!

This analogy is used extensively in practice.

Example N8-1. A UPW w/ $\bar{E}^i = \hat{y}(-3)e^{-j\beta_1 x}$ V/m propagates in a medium w/ $\mu_{r1} = 1$ & $\epsilon_{r1} = 3$ and impinges on a half space filled w/ vacuum. Determine $\bar{E}$ & $\bar{H}$ in both materials, the time-avg. Poynting vector in both materials & the SWR in material 1.

First, make sketch. Don't try to solve these problems w/o one. Assume all E vectors point in the same '+' direction



$$\Gamma^b = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{1/2}{3/2} = \frac{1}{3}$$

$$T^b = 1 + \Gamma^b = 1 + \frac{1}{3} = \frac{4}{3}$$

Hey, this >1! Physically impossible? No, power still conserved as we'll see.

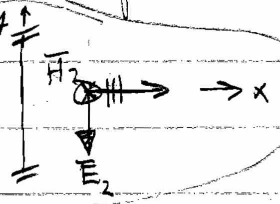
$$\therefore \quad \bar{E}_1(x) = \hat{y}\left(-3e^{-j\beta_1 x} \overbrace{-3\cdot\Gamma^b}^{=1} e^{+j\beta_1 x}\right) \text{ V/m}$$

$$\bar{H}_1(x) = \hat{z}\left(\underbrace{\frac{-3}{\eta_0}\cdot 2}_{=\frac{-6}{\eta_0}} e^{-j\beta x} + \underbrace{\frac{3\Gamma_b}{\eta_0}\,2}_{=\frac{2}{\eta_0}} e^{+j\beta_1 x}\right) \text{ A/m}$$

and in region 2: $\bar{E}_2(x) = \hat{y}\underbrace{(-3)T^b}_{=-4} e^{-j\beta_2 x}$ V/m

$$\bar{H}_2(x) = \hat{z}\,\underbrace{\frac{(-3)T^b}{\eta_0}}_{=-\frac{4}{\eta_0}} e^{-j\beta_2 x} \quad \text{A/m}$$

The minus signs mean E_2 & H_2 are oppositely oriented to what we assumed.



The time-averaged Poynting vector is expressed as

$$\bar{S} = \frac{1}{2}\,\bar{E}\times\bar{H}^* \qquad (19)$$

The incident Poynting vector is

$$\bar{S}^i = \frac{1}{2}\bar{E}^i\times\bar{H}^{i*} = \frac{1}{2}\hat{x}(-3)\cancel{e^{-j\beta_1 x}}\cdot\left(-\frac{6}{\eta_0}\cancel{e^{-j\beta_1 x}}\right)^* = \hat{x}\,\frac{9}{\eta_0}\ \frac{W}{m^2}$$

$$\bar{S}^r = \frac{1}{2}\bar{E}^r\times\bar{H}^{r*} = \frac{1}{2}\hat{x}\left(-1e^{j\beta_1 x}\right)\cdot\left(\frac{2}{\eta_0}e^{j\beta_1 x}\right)^* = -\hat{x}\,\frac{1}{\eta_0}\ \frac{W}{m^2}$$

$$\therefore\ \bar{S}_1 = \bar{S}^i + \bar{S}^r = \hat{x}\,\frac{8}{\eta_0}\ \frac{W}{m^2}$$

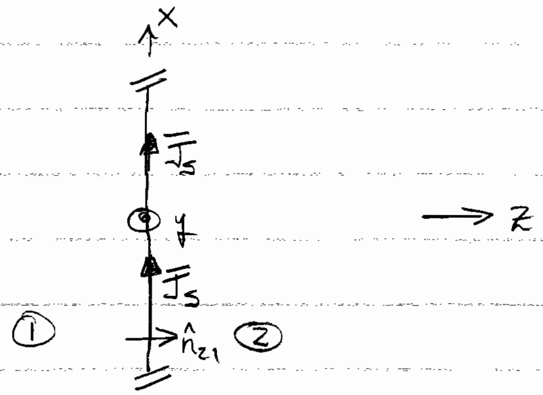
also region 2,

$$\bar{S}_2 = \frac{1}{2}\bar{E}^t\times\bar{H}^{t*} = \frac{1}{2}\hat{x}\left(-4e^{-j\beta_2 x}\right)\cdot\left(-\frac{4}{\eta_0}e^{-j\beta_2 x}\right)^*$$

$$= \hat{x}\,\frac{8}{\eta_0}\ \frac{W}{m^2}$$

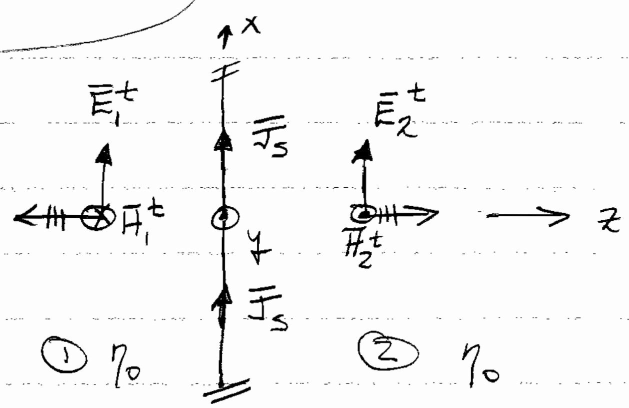
Hey, $\bar{S}_1 = \bar{S}_2$. Power conserved, as expected, even though $|T^b| > 1$.

Example NB-2. A phasor current sheet $\bar{J}_S = \hat{x} J_0$ is immersed in a homogeneous space. Calculate the radiated TEM fields everywhere.



Because $\bar{J}_s$ in $\hat{x}$, then there must be an $\bar{E}$ component in $\hat{x}$ also, because $\bar{E}\cdot\bar{J}$ supplies power to waves.

The source & space do not vary w/ x or y, therefore the radiated fields will not either. Infinite current sheets radiate plane waves. Problem becomes



Hence, $\bar{E}_1(z) = \hat{x} A e^{+j\beta_0 z}$, $\bar{E}_2(z) = \hat{x} B e^{-j\beta_0 z}$

$$\bar{H}_1(z) = -\hat{y}\frac{A}{\eta_0} e^{+j\beta_1 z}, \quad \bar{H}_2(z) = \hat{y}\frac{B}{\eta_0} e^{-j\beta_0 z}$$

where A & B are complex coefficients. These can be evaluated by applying the source boundary conditions:

(*) $\bar{E}_{tan}$ continuous @ $z=0$: $E_{x_1}(z=0^-) = E_{x_2}(z=0^+)$

Sub. for fields gives

$$A e^{j\beta_0 z}\Big|_{z=0^-} = B e^{-j\beta_0 z}\Big|_{z=0^+}$$

or $\underline{A = B}$

(*) $\bar{H}_{tan}$ discontinuous @ $z=0$: $\hat{n}_{21} \times (\bar{H}_2 - \bar{H}_1)\Big|_{z=0} = \bar{J}_S$ "jump condition"

or

$$\hat{z} \times \left[\bar{H}_2(z=0^+) - \bar{H}_1(z=0^-)\right] = \hat{x} J_0$$

Sub for $\bar{H}$'s: $\hat{z} \times \left[\hat{y}\dfrac{B}{\eta_0} - \hat{y}\left(-\dfrac{A}{\eta_0}\right)\right] = \hat{x} J_0$

$$\therefore \quad -\hat{x}\frac{B}{\eta_0} - \hat{x}\frac{A}{\eta_0} = \hat{x} J_0$$

with $A = B$, the $\hat{x}$ equation is

$$-\frac{2A}{\eta_0} = J_0 \quad \text{or} \quad \underline{A = -\frac{\eta_0 J_0}{2} = B}$$

Hence, $\bar{E}_1(z) = -\hat{x}\dfrac{\eta_0 J_0}{2} e^{+j\beta_0 z}$ $\frac{V}{m}$ and

$$\bar{E}_2(z) = -\hat{x}\frac{\eta_0 J_0}{2} e^{-j\beta_0 z} \quad \frac{V}{m}$$

Significance of negative $\bar{E}$? $\bar{E}$ in opposite direction as $\bar{J}_S$. $\bar{E}\cdot\bar{J}$ is power dissipated, hence $-\bar{E}\cdot\bar{J}_S$ is power supplied by the current source to the EM waves.