The past few lectures have covered UPWs propagating in infinite, homogeneous spaces. This work sets the foundation for solving an important set of scattering & radiation problems:

1) UPW impinging on a half space
2) UPW impinging on planarly layered media
3) UPW's radiated from infinite current sheets

These types of problems are extraordinarily important to master. While useful, they serve primarily as models to understand more complicated scattering & radiation problems.

To begin, consider a UPW incident on a half space as shown.

\[ E^i = x E_0 e^{j \beta x} \]  \hspace{1cm} (5-10), (1)

This UPW is produced somewhere at \( z \leq 0 \), but the details of this aren't important. Instead, (1) gives all the important information.
Because the UPW is incident perpendicularly to the interface, it is called normal incidence. ("The plane wave is incident normally to the interface.")

We know all the important characteristics of UPW's propagating in an infinite, simple material:

1. $\mathbf{E} \times \mathbf{H}$ perpendicular to each other
2. $\mathbf{E} \times \mathbf{H}$ perpendicular to $\mathbf{B}$, the wave vector

\[ \mathbf{B} = \hat{x} \beta_x + \hat{y} \beta_y + \hat{z} \beta_z. \]

In this problem, $\beta_y = \beta_z = 0$ so that

\[ \mathbf{B} = \hat{x} \beta_x = \hat{x} \beta \text{ rad/m}. \]

3. The ratio of perpendicular components of $\mathbf{E} \times \mathbf{H}$ is $\eta$.

4. The direction of $\mathbf{E} \times \mathbf{H}$ is with the direction of wave propagation.

With these characteristics, we can fill in the missing pieces needed to set up the problem shown in the previous figure.
Note that we've assumed all $\vec{E}$'s point in same
direction ($+z$). (You should always do this, since
it's common to use the simple reflection-transmission
coefficients we'll derive soon.) The electric
fields are then

$$\vec{E}_r = \hat{x} E_r e^{+j\beta z} = \hat{x} \Gamma^b E_0 e^{+j\beta z} \quad (5.16b)$$

and

$$\vec{E}_t = \hat{x} E_t e^{-j\beta z} = \hat{x} \Gamma^b E_0 e^{-j\beta z} \quad (5.16c)$$

$\Gamma^b$ and $\Gamma^b$ are the reflection-transmission coefficients
at the interface (or boundary).

From the new characteristics listed earlier, we can easily determine $\vec{H}$ in terms of $\vec{E}$:

$$\vec{H}_r = \hat{y} \frac{E_0}{\eta_i} e^{-j\beta z} \quad (5.2a)$$

$$\vec{H}_t = -\hat{y} \frac{E_0 \Gamma^b}{\eta_i} e^{+j\beta z} \quad (5.2b)$$

$$\vec{H}_t = \hat{y} \frac{E_0 \Gamma^b}{\eta_i} e^{-j\beta z} \quad (5.2c)$$

All that remains to complete this solution is to determine
the coefficients $\Gamma^b$ and $\Gamma^b$. The factor $E_0$ is a source
characteristic. It either needs to be given or made
characteristic of the same would have to be
provided for us to compute it.
We seek for $\pi^b$ and $T^b$ by applying the boundary conditions that:

1. $E_{\text{in}}$ continuous $\Rightarrow \hat{n} \times E_1(z=0^-) = \hat{n} \times E_2(z=0^+) \quad (7)$

Note that it is the total field that is continuous. Using (4) - (6) in (7) gives

$$\chi \left[ E_0 e^{-j\beta^b z} + \pi^b E_0 e^{+j\beta^b z} \right] = \hat{n} T_0 E_0 e^{-j\beta^b z} \quad z = 0^+$$

$$\Rightarrow \quad 1 + \pi^b = T_0 \quad (5-3a), \quad (8)$$

2. $\vec{H}_{\text{tan}}$ continuous $\Rightarrow \hat{n} \times \vec{H}_1(z=0^-) = \hat{n} \times \vec{H}_2(z=0^+) \quad (9)$

Using (4) - (6) in (9) gives

$$\frac{\chi}{\eta_1} \left[ \frac{E_0}{\eta_1} e^{-j\beta^b z} - \frac{E_0}{\eta_1} \pi^b \right] = \frac{\chi}{\eta_2} \left[ \frac{E_0}{\eta_2} e^{-j\beta^b z} \right] \quad z = 0^+$$

$$\Rightarrow \quad \frac{1}{\eta_1} (1 - \pi^b) = \frac{T^b}{\eta_2} \quad (10)$$

Solving these two gives, (8) & (10) yields

$$\pi^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (5-4a), \quad (11)$$

$$T^b = \frac{2\eta_2}{\eta_2 + \eta_1} \quad (5-4b), \quad (12)$$
Notice how similar the UFW problem is to a transmission line problem:

\[ \frac{V_0^+}{Z_1} \rightarrow T \rightarrow \frac{Z_2}{\beta_1} \rightarrow Z_2 \frac{\beta_2}{\beta_1} \rightarrow Z_2 \rightarrow \frac{Z_2}{\beta_2} \rightarrow \frac{Z_2}{\beta_2} \]

where the voltages are

\[ V_1(z) = V_0^+ (e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}) \tag{15} \]

and

\[ V_2(z) = V_0^+ \Gamma e^{-j\beta_2 z} \tag{14} \]

and the currents are

\[ I_1(z) = \frac{V_0^+}{Z_1} (e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}) \tag{15} \]

and

\[ I_2(z) = \frac{V_0^+}{Z_2} e^{-j\beta_2 z} \tag{16} \]

where \( \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} \) and \( T = 1 + \Gamma = \frac{Z_2 Z_1}{Z_2 + Z_1} \) \tag{17, 18}

A direct analogy between the two problems can be effected if we equate:

(i) \( V_0^+ \leftrightarrow E_0 \)

(ii) \( Z_1 \leftrightarrow \eta_1 \leftrightarrow \frac{Z_2}{\beta_1} \leftrightarrow \eta_2 \)

(iii) \( \beta_1 \leftrightarrow \beta_2 \leftrightarrow \beta_2 \)

If so, solutions for \( V(z) \) are directly analogous to those for \( E(z) \) and solutions for \( I(z) \) are directly analogous to those for \( H(z) \).
Example NB. 1. A UPW w/ $E^i = \hat{y}(-3)e^{-j\beta x}$ v/m

propagates in a medium w/ $\mu_r = 1$ & $\varepsilon_r = 3$ and impinges
on a half space filled w/ vacuum. Determine $E \perp T$

in both materials, the time-avg. Poynting vector

in both materials & the SWR in material 1.

First, make sketch. Don't try to solve these problems
w/o one. Assume all $E$ vectors point in the same
'+' direction.

\[ E^i \]

$E^T$ \hspace{1cm} $E^c$

$H^i$ \hspace{1cm} $H^T$

$H^c$

$\mu_r = 1, \varepsilon_r = 4 \Rightarrow \eta_r = \eta_0$

$M_2 = 1, \varepsilon_{r_2} = 1 \Rightarrow \eta_2 = \eta_0$

\[ \Gamma^b = \frac{\eta_r - \eta_i}{\eta_r + \eta_i} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{1}{3} \]

\[ T^b = 1 + \Gamma^b = 1 + \frac{1}{3} = \frac{4}{3} \]

Hey, this >1! Physically impossible? No, proof

is contained at well.

\[ E^i(x) = \hat{y}(-3e^{-j\beta x} - 3\eta^b e^{+j\beta x}) \text{ v/m} \]

\[ H^i(x) = \hat{z}\left(\frac{-3}{\eta_0}e^{-j\beta x} + \frac{3\eta^b}{\eta_0}e^{+j\beta x}\right) \text{ A/m} \]
The time-averaged Poynting vector is expressed as
\[ \overline{S} = \frac{1}{2} \overline{E} \times \overline{H}^* \]  (19)

In region 2,
\[ \overline{S}_1 = \frac{1}{2} \overline{E}_1 \times \overline{H}_1^* = \frac{1}{2} \hat{x} \left[ -4 e^{-j \beta_2 x} \right] \left[ \frac{-4}{\eta_0} e^{-j \beta_2 x} \right]^* = \frac{\hat{x}}{\eta_0} \frac{\mathbf{E}}{m^2} \]

\[ \overline{S}_2 = \frac{1}{2} \overline{E}_2 \times \overline{H}_2^* = \frac{1}{2} \hat{x} \left[ -4 e^{-j \beta_2 x} \right] \left[ \frac{-4}{\eta_0} e^{-j \beta_2 x} \right]^* = \frac{\hat{x}}{\eta_0} \frac{\mathbf{E}}{m^2} \]

Therefore, \( \overline{S}_1 = \overline{S}_2 \). Power conserved, as expected, even though \( |T_{12}| > 1 \).
Example. \( \mathbf{N}\mathbf{B} \rightarrow \mathbf{2} \). A current sheet \( \mathbf{T}_s = \times \mathbf{I}_0 \) is immersed in a homogeneous space. Calculate the radiated fields everywhere.

Because \( \mathbf{E} \) is in \( \mathbf{x} \), then there must be an \( \mathbf{E} \) component in \( \mathbf{x} \) also, because \( \mathbf{E} \times \mathbf{J} \) supplies power to waves.

The source is space does not vary with \( \mathbf{x} \) or \( \mathbf{y} \), therefore the radiated fields will not either. Infinite current sheets radiate plane waves. Problem becomes

Hence,

\[
\mathbf{E}_1(z) = \hat{x} \mathbf{A} e^{-j\beta_0 z}, \quad \mathbf{E}_2(z) = \hat{x} \mathbf{B} e^{+j\beta_0 z}
\]

\[
\mathbf{H}_1(z) = -j \frac{\mathbf{A}}{\eta_0} e^{-j\beta_0 z}, \quad \mathbf{H}_2(z) = j \frac{\mathbf{B}}{\eta_0} e^{+j\beta_0 z}
\]

where \( \mathbf{A} \) : \( \mathbf{B} \) are complex coefficients. These can be evaluated by applying the source boundary conditions.
\( E \) \& \( H \) are continuous @ \( z = 0 \):
\[ E_x(\pm) = E_x(\mp) \]
\[ H_z(\pm) = H_z(\mp) \]

Sub. for fields gives
\[ A e^{-j\beta z} = B e^{-j\beta z} \]
\[ z = 0^- \quad z = 0^+ \]

\( A = B \)

\( H \) \& \( E \) are discontinuous @ \( z = 0 \):
\[ \hat{H}_z(\mp) \cdot (\hat{H}_z(\pm) - \hat{H}_z(\mp)) = \hat{J}_s \quad \text{"Jump condition"} \]
\[ \hat{E}_x \left[ \hat{H}_z(\mp) - \hat{H}_z(\pm) \right] = \hat{J}_s \]

Sub. for \( \hat{H}_z \):
\[ \hat{E}_x \left[ \frac{\hat{J}_0 B}{\gamma} - \frac{\hat{J}_0 A}{\gamma} \right] = \hat{J}_s \]

\[ -\hat{E}_x \frac{B}{\gamma} - \hat{E}_x \frac{A}{\gamma} = \hat{J}_s \]

\( A = B \), the \( \hat{E} \) equation is
\[ -\frac{2A}{\gamma} = \hat{J}_s \quad \text{or} \quad A = \frac{\gamma J_0}{2} = B \]

Hence,
\[ E_1(\pm) = -\hat{E}_x \frac{\gamma J_0}{2} e^{\pm j\beta_0 z} \]
\[ E_2(\mp) = -\hat{E}_x \frac{\gamma J_0}{2} e^{-j\beta_0 z} \]

Significance of negative \( E \): \( E \) in opposite direction as \( \hat{J}_s \). \( E \cdot \hat{J}_s \) is power dissipated, hence \(-E \cdot \hat{J}_s\)

is power supplied by the current source to the RF waves.