EE 692
Whites

Lecture 7: Polarization of TEM Waves

As an electromagnetic wave propagates, the tip of the electric field vector may trace out certain geometrical shapes as a fct. of time. It may not, though.

This characteristic of an EM wave is called its *polarization*. If the E field vector moves about randomly as a fct. of time, it is called *random polarization*.

However, if the polarization is not random, it can be classified in one of three categories: *linear*, *circular*, and *elliptical*. These shapes describe the geometrical shape generated by the tip of the E field vector of an EM wave as it propagates.

see Fig 4-8

The polarization can also have one of two senses: right handed and left handed. By IEEE definition, we find the sense by pointing the thumb in the direction of propagation & match the fingers of one hand to the direction that the E field vector rotates as we see as the wave propagates away from us.

See Fig 4-9

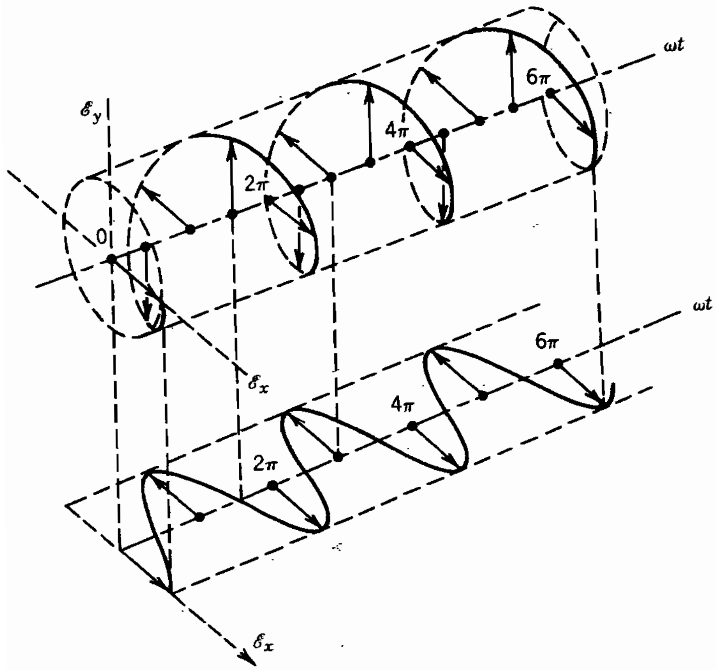


FIGURE 4-8 Rotation of a plane electromagnetic wave at $z = 0$ as a function of time. (*Source:* C. A. Balanis, *Antenna Theory: Analysis and Design*. Copyright © 1982, John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.)

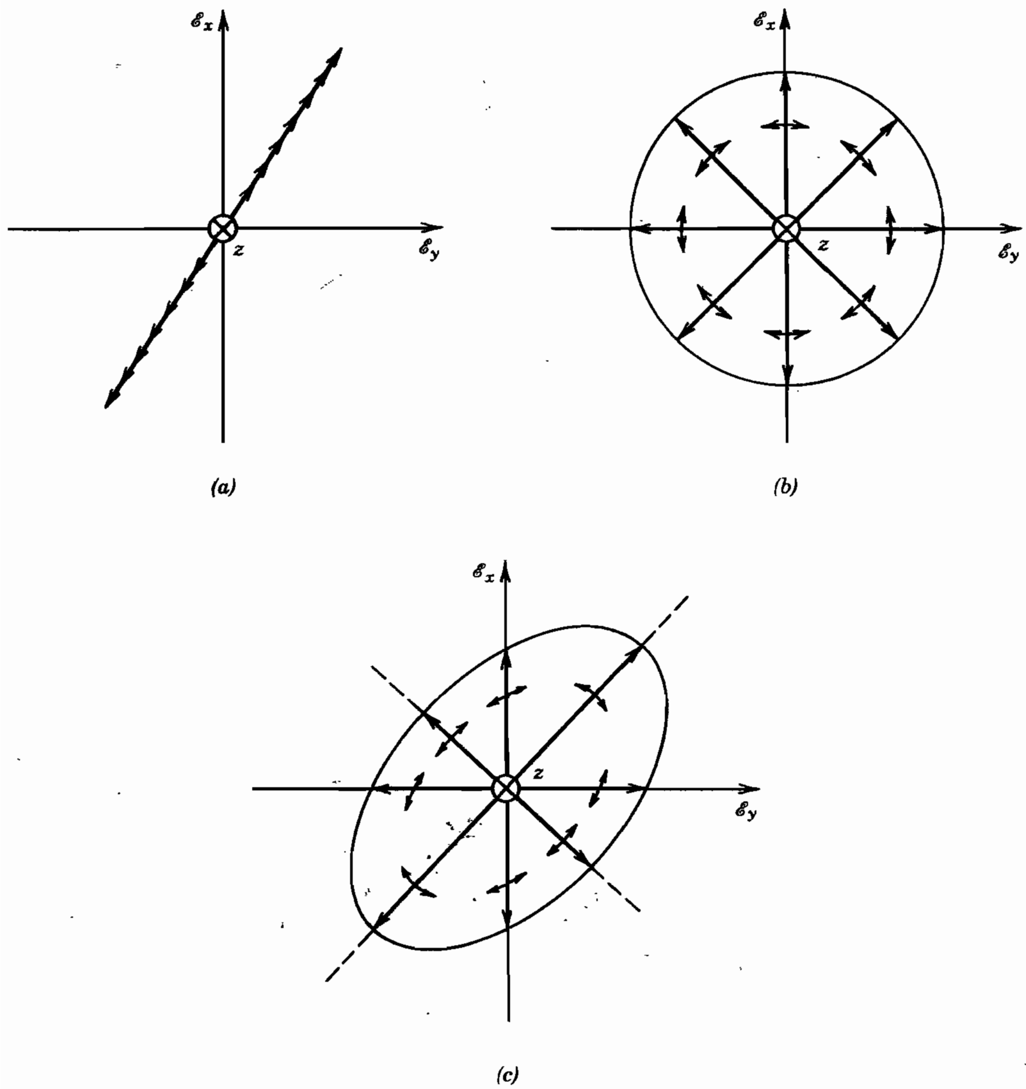


FIGURE 4-9 (a) Linear, (b) circular, and (c) elliptical polarization figure traces of an electric field extremity *as a function of time for a fixed position.*

To examine this polarization phenomenon, we'll consider the special case where we have a TEM wave propagating in the +z direction. This TEM wave has x & y components as

$$E_x = E_x^+ e^{-j\beta z} = E_{x0}^+ e^{j\phi_x} e^{-j\beta z} \quad (1)$$
$$E_y = E_y^+ e^{-j\beta z} = E_{y0}^+ e^{j\phi_y} e^{-j\beta z} \quad (2)$$

In the time domain, electric field is

$$\bar{E}(z,t) = \mathrm{Re}\left\{ \bar{a}_x E_{x0}^+ e^{j(\phi_x - \beta z)} e^{j\omega t} + \bar{a}_y E_{y0}^+ e^{j(\phi_y - \beta z)} e^{j\omega t} \right\}$$
$$= \bar{a}_x E_{x0}^+ \cos(\omega t - \beta z + \phi_x) + \bar{a}_y E_{y0}^+ \cos(\omega t - \beta z + \phi_y) \quad (4\text{-}50a), (3)$$

We'll use (3) to determine the polarization in three special circumstances in the z=0 plane for simplicity.

- $\underline{E_{x0}^+ = E_{y0}^+} = E_0$. Further imagine $\phi_x = 0$ & $\phi_y = -\frac{\pi}{2}$, then

$$E_x(0,t) = E_0 \cos(\omega t) \quad (4)$$
$$E_y(0,t) = E_0 \cos(\omega t - \pi/2) = E_0 \sin(\omega t) \quad (5)$$

$$|\bar{E}(0,t)| = \sqrt{E_0^2 \cos^2(\omega t) + E_0^2 \sin^2(\omega t)} = E_0 \sqrt{\cos^2 \omega t + \sin^2 \omega t}$$
$$= E_0. \quad (6)$$

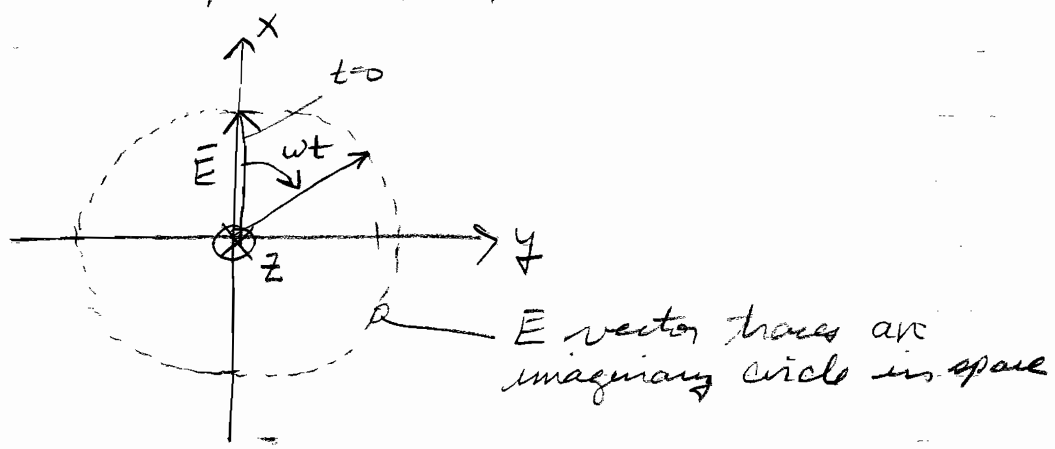
This means the "length" of the electric field vector is constant wrt time.

The angle this electric field vector makes in the x-y plane is

$$\psi = \tan^{-1}\left[\frac{E_y(0,t)}{E_x(0,t)}\right] = \tan^{-1}\left[\frac{E_0 \sin(\omega t)}{E_0 \cos(\omega t)}\right]$$

$$= \tan^{-1}\left[\tan(\omega t)\right] = \omega t. \qquad (4\text{-}52c)\,(7)$$

Collecting these results from (6) & (7)



With thumb in direction of prop (+z), fingers of right hand point in direction that E vector spins in time. Hence this called <u>right handed circular polarization.</u>

For CP, need $|E_x| = |E_y|$ & a 90° difference between them.

- <u>$\phi_x = \phi_y = \phi$</u>. In this case, from (3):

$$E_x(0,t) = E_{x_0}^{+} \cos(\omega t + \phi) \qquad (8)$$
$$E_y(0,t) = E_{y_0}^{+} \cos(\omega t + \phi) \qquad (9)$$

Consequently,

$$|\bar{E}(0,t)| = \sqrt{(E_{x0}^{+})^2 \cos^2(\omega t+\phi) + (E_{y0}^{+})^2 \cos^2(\omega t+\phi)}$$

$$= \sqrt{(E_{x0}^{+})^2 + (E_{y0}^{+})^2}\,|\cos(\omega t+\phi)|$$

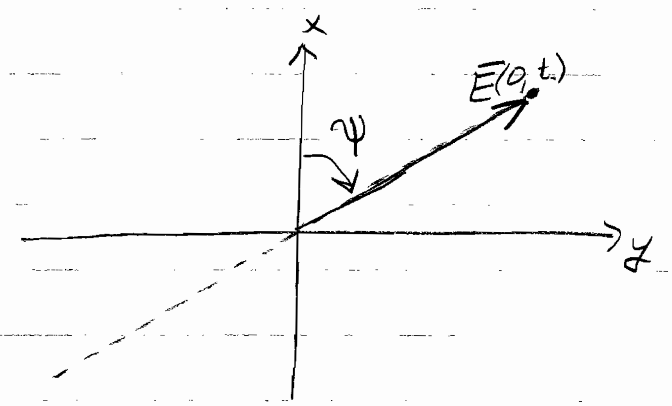
The amplitude of this electric field vector is oscillating w/ time.

The angle $\bar{E}(0,t)$ makes in the xy plane is

$$\psi = \tan^{-1}\left[\frac{E_y(0,t)}{E_x(0,t)}\right] = \tan^{-1}\left[\frac{E_{y0}^{+} \cancel{\cos(\omega t+\phi)}}{E_{x0}^{+} \cancel{\cos(\omega t+\phi)}}\right]$$

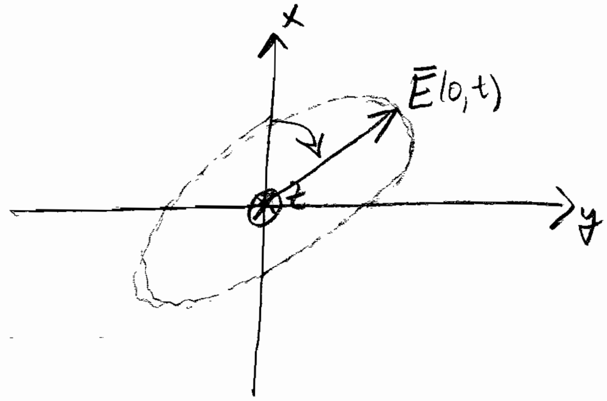
$$= \tan^{-1}\left[\frac{E_{y0}^{+}}{E_{x0}^{+}}\right]$$

The angle is fixed & doesn't vary with time:



This type of field is called <u>linearly polarized</u>.

- If $E_{x0}^+ \neq E_{y0}^+$ and $\phi_x = \pm\phi_y$, then the wave is said to be <u>elliptically</u> <u>polarized</u>. The electric field vector will trace out an ellipse as a fct. of time:



Can be right- or left-handed polarized.