

As an electromagnetic wave propagates, the tip of the electric field vector may trace out certain geometrical shapes. It may not, though.
as a fct. of time

This characteristic of an EM wave is called its polarization. If the \vec{E} field vector moves about randomly as a fct. of time, it is called random polarization.

See
Fig 4-8

However, if the polarization is not random, it can be classified in one of three categories: linear, circular, and elliptical. These shapes describe the geometrical shape generated by the tip of the \vec{E} field vector of an EM wave as it propagates.

See
Fig 4-9

The polarization can also have one of two senses, right handed and left handed. By IEEE definition, we find the sense by putting the thumb in the direction of propagation; match the fingers of one hand to the direction that the \vec{E} field vector rotates as we see as the wave propagates away from us.

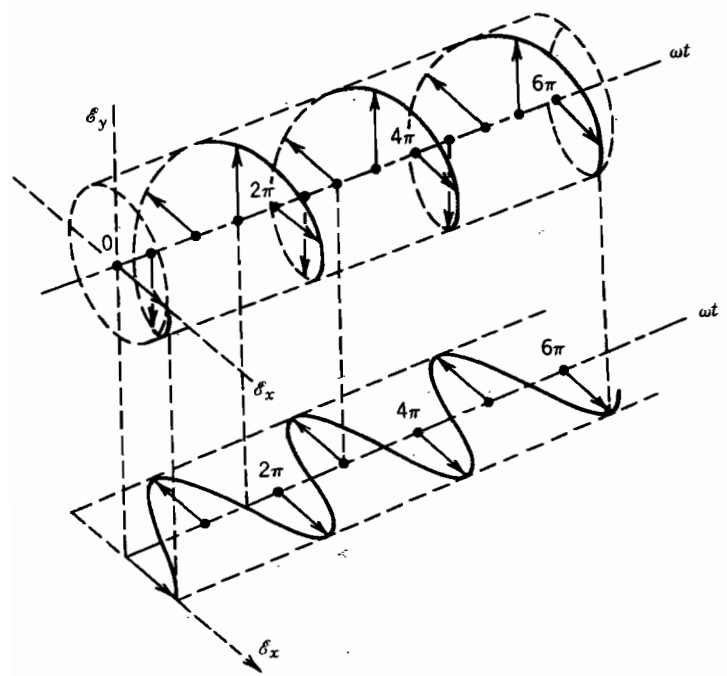


FIGURE 4-8 Rotation of a plane electromagnetic wave at $z = 0$ as a function of time. (Source: C. A. Balanis, *Antenna Theory: Analysis and Design*. Copyright © 1982, John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.)

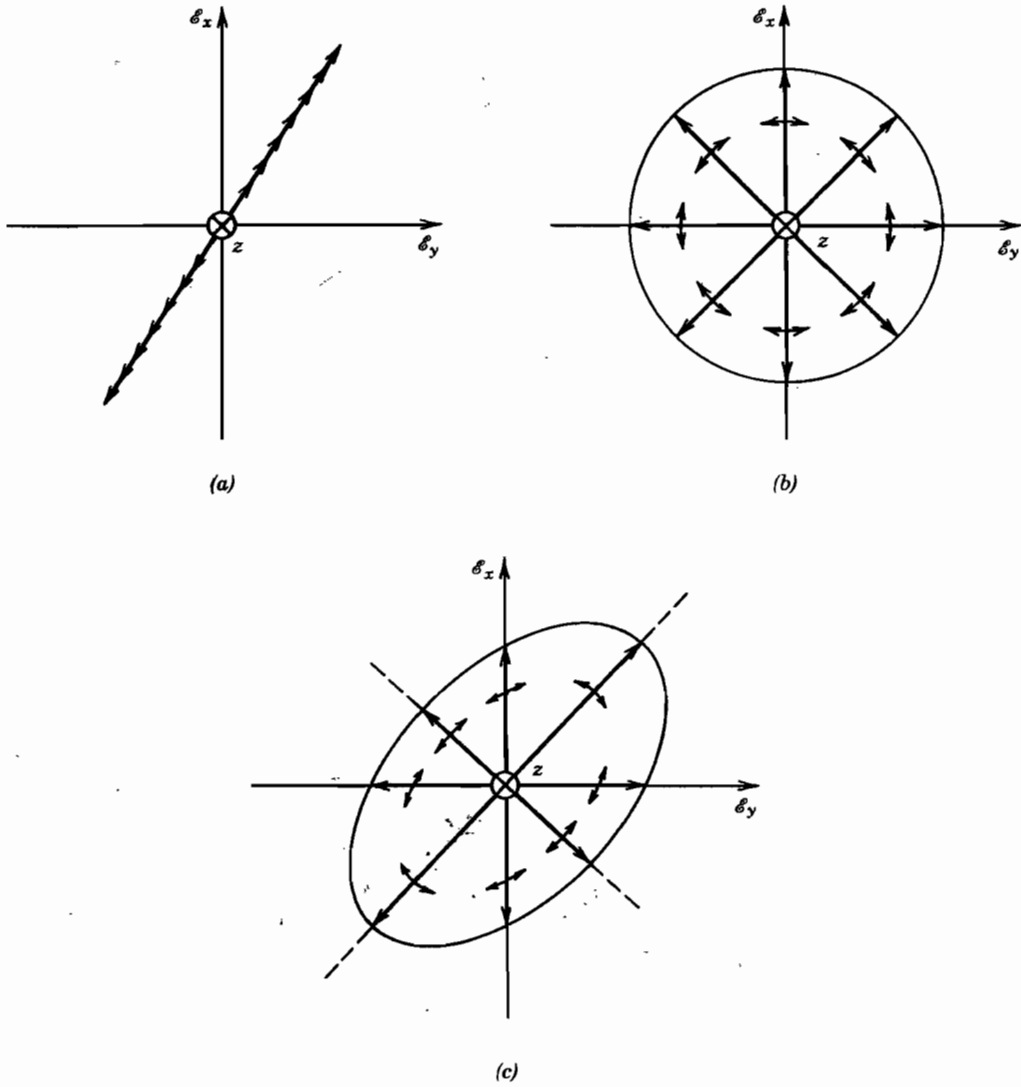


FIGURE 4-9 (a) Linear, (b) circular, and (c) elliptical polarization figure traces of an electric field extremity as a function of time for a fixed position.

To examine this polarization phenomenon, we'll consider the special case where we have a TEM wave propagating in the $+z$ direction. This TEM wave has x, y components as

$$E_x = E_x^+ e^{-j\beta z} = E_{x0}^+ e^{j\phi_x} e^{-j\beta z} \quad (1)$$

$$E_y = E_y^+ e^{-j\beta z} = E_{y0}^+ e^{j\phi_y} e^{-j\beta z} \quad (2)$$

in the time domain, electric field is

$$\begin{aligned} \vec{E}(z, t) &= \text{Re} \left\{ \bar{a}_x E_{x0}^+ e^{j(\phi_x - \beta z)} e^{j\omega t} + \bar{a}_y E_{y0}^+ e^{j(\phi_y - \beta z)} e^{j\omega t} \right\} \\ &= \bar{a}_x E_{x0}^+ \cos(\omega t - \beta z + \phi_x) + \bar{a}_y E_{y0}^+ \cos(\omega t - \beta z + \phi_y) \end{aligned} \quad \begin{matrix} (4-50a) \\ (3) \end{matrix}$$

We'll use (3) to determine the polarization in three special circumstances in the $z=0$ plane for simplicity.

- $E_{x0}^+ = E_{y0}^+ = E_0$. Further imagine $\phi_x = 0$; $\phi_y = -\frac{\pi}{2}$, then

$$E_x(0, t) = E_0 \cos(\omega t) \quad (4)$$

$$E_y(0, t) = E_0 \cos(\omega t - \pi/2) = E_0 \sin(\omega t) \quad (5)$$

$$\begin{aligned} |\vec{E}(0, t)| &= \sqrt{E_0^2 \cos^2(\omega t) + E_0^2 \sin^2(\omega t)} = E_0 \sqrt{\cos^2 \omega t + \sin^2 \omega t} \\ &= E_0. \end{aligned} \quad (6)$$

This means the "length" of the electric field vector is constant w/rt time.

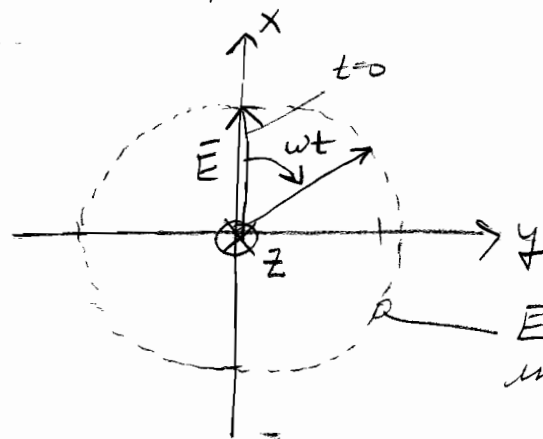
... The angle this electric field vector makes in the $x-y$ plane
... is

$$\Psi = \tan^{-1} \left[\frac{E_y(0,t)}{E_x(0,t)} \right] = \tan^{-1} \left[\frac{E_0 \sin(\omega t)}{E_0 \cos(\omega t)} \right]$$

$$= \tan^{-1} [\tan(\omega t)] = \omega t.$$

(4-52c)(7)

... Collecting these results from (6) & (7)



... With thumb in direction of prop (+z), fingers of right
... hand point in direction that E vector spins in time.
... Hence this called right handed circular polarization.

... For CP, need $|E_x| = |E_y|$ & a 90° difference between
... them.

• $\phi_x = \phi_y = \phi$. In this case, from (3):

$$E_x(0,t) = E_{x0}^+ \cos(\omega t + \phi) \quad (8)$$

$$E_y(0,t) = E_{y0}^+ \cos(\omega t + \phi) \quad (9)$$

Consequently,

$$|\vec{E}(0,t)| = \sqrt{(E_{x_0}^+)^2 \cos^2(\omega t + \phi) + (E_{y_0}^+)^2 \cos^2(\omega t + \phi)}$$

$$= \sqrt{(E_{x_0}^+)^2 + (E_{y_0}^+)^2} |\cos(\omega t + \phi)|$$

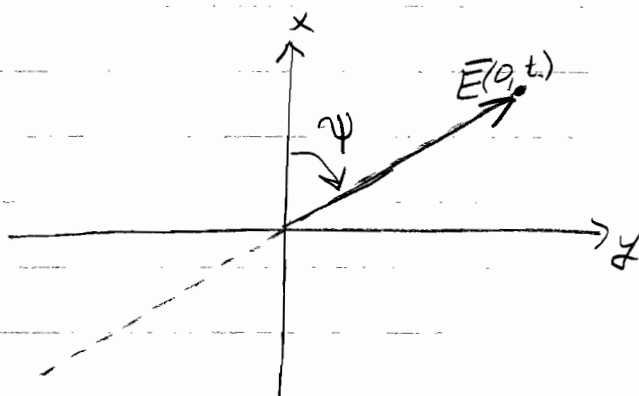
The amplitude of this electric field vector is oscillating w/ time.

The angle $\vec{E}(0,t)$ makes in the xy plane is

$$\psi = \tan^{-1} \left[\frac{E_y(0,t)}{E_x(0,t)} \right] = \tan^{-1} \left[\frac{E_{y_0}^+ \cos(\omega t + \phi)}{E_{x_0}^+ \cos(\omega t + \phi)} \right]$$

$$= \tan^{-1} \left[\frac{E_{y_0}^+}{E_{x_0}^+} \right]$$

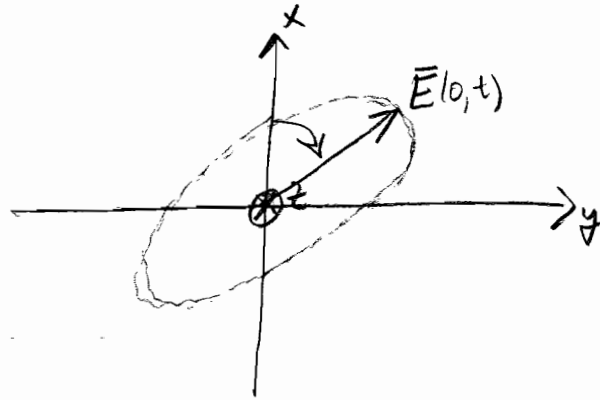
The angle is fixed & doesn't vary with time:



This type of field is called linearly polarized.

7/7

- If $E_{x0} \neq E_{y0}$ and $\phi_x = \pm \phi_y$, then the wave is said to be elliptically polarized. The electric field vector will trace out an ellipse as a fct. of time:



Can be right- or left-handed polarized.