EE 692
Whites

Lecture 6: Transverse Electromagnetic Waves in Lossy Media.

To some extent, all materials are lossy. Energy will be deposited into the lossy material as the electromagnetic wave travels through it. In the previous couple of lectures, we have chosen to ignore the effects of loss, which can be very accurate for low loss materials

Now we'll consider plane waves propagating in lossy materials. We found in lecture 3 that the wave equations in such media are

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0 \quad (1)$$

$$\nabla^2 \bar{H} - \gamma^2 \bar{H} = 0 \quad (2)$$

To begin, we'll focus once again on the simplistic scenario of E polarized only in the x direction, with propagation only in the $\pm z$ directions.

In such a case, the E field solutions to (1) are

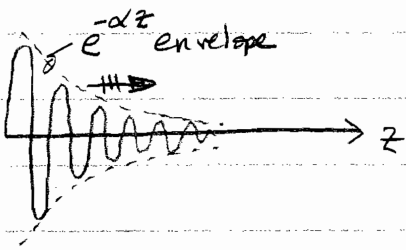
$$\bar{E}(z) = \hat{a}_x \left(E_0^+ e^{-\gamma z} + E_0^- e^{+\gamma z} \right) \qquad \gamma \equiv \alpha + j\beta$$

$$= \hat{a}_x \left(E_0^+ e^{-\alpha z} e^{-j\beta z} + E_0^- e^{+\alpha z} e^{+j\beta z} \right) \quad (3)$$

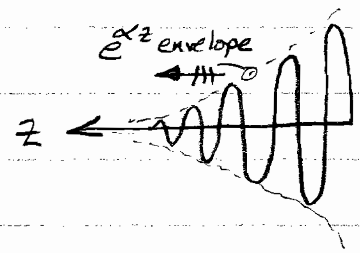
These solutions can be verified by substituting each term separately into (1) and performing the differentiation.

Unlike the lossless case, there is an extra exponential factor in each term:

$e^{-\alpha z} e^{-j\beta z}$:



$e^{+\alpha z} e^{+j\beta z}$:



As the wave propagates in this lossy material, the amplitude of the EM wave decreases. With reference to the time average Poynting vector $S_{av} = \frac{1}{2} \mathrm{Re}(\bar{E} \times \bar{H}^*)$ this decreasing amplitude indicates the power carried by the EM wave is decreasing. Where is it going? It's being deposited in the lossy material and converted to heat through the conduction current process.

From Lecture 3,

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \tag{4}$$

Squaring both sides of (4)

$$(\alpha + j\beta)^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$$

or

$$\alpha^2 - \beta^2 + j2\alpha\beta = -\omega^2\mu\epsilon + j\omega\mu\sigma \tag{5}$$

Equating real and imaginary parts of (5)

Re: $\quad \alpha^2 - \beta^2 = -\omega^2\mu\epsilon \qquad (6)$

Im: $\quad 2\alpha\beta = \omega\mu\sigma \qquad (7)$

We can solve this pair of equations for α & β:

$$\alpha = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]\right\}^{1/2} \quad [\text{Np/m}] \qquad (8)$$

&

$$\beta = \omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]\right\}^{1/2} \quad [\text{rad/m}] \qquad (9)$$

The magnetic fields associated with these electric fields of this plane wave propagating in $\pm z$ can be found from Faraday's law

$$\nabla \times \bar{E} = -j\omega\mu\bar{H}$$

With a spatial dependence $e^{\mp\bar{\gamma}\cdot\bar{r}}$, Faraday's law becomes

$$\mp\bar{\gamma}\times\bar{E} = -j\omega\mu\bar{H} \Rightarrow \bar{H} = \pm\frac{1}{j\omega\mu}\bar{\gamma}\times\bar{E} \qquad (10)$$

The upper sign is associated with the plane wave prop. in $+z$, the lower with prop. in $-z$. Consequently,

$$\bar{H}(z) = \frac{1}{j\omega\mu}\left[\bar{\gamma}\times\bar{E}^{+}(z) - \bar{\gamma}\times\bar{E}^{-}(z)\right]$$

with $\bar{\gamma} = \hat{a}_z\gamma$, then

$$\bar{H}(z) = \frac{\gamma}{j\omega\mu}\left[\hat{a}_z\times\hat{a}_x E_o^{+}e^{-\gamma z} - \hat{a}_z\times\hat{a}_x E_o^{-}e^{+\gamma z}\right]$$

$$= \frac{\gamma}{j\omega\mu}\hat{a}_y\left[E_o^{+}e^{-\gamma z} - E_o^{-}e^{+\gamma z}\right]$$

$$= \frac{1}{Z_W}\, \hat{a}_y \left[E_o^+ e^{-\gamma z} - E_o^- e^{+\gamma z} \right] \quad (11)$$

The wave impedance Z_W is

$$Z_W = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \eta_c \quad (12)$$

This wave impedance is a complex number for lossy media.

Recall for a lossless space, that $Z_W = \eta = \sqrt{\frac{\mu}{\epsilon}}$. We can express η_c in a similar form, but we need to introduce the concept of complex permittivity, ϵ_c, to do so.

We can determine this complex permittivity from Ampère's law in phasor form for a material space that is assumed lossy because of conduction currents:

$$\begin{aligned} \nabla \times \bar{H} &= \bar{J}_c + j\omega\epsilon \bar{E} = \sigma \bar{E} + j\omega\epsilon \bar{E} \\ &= (\sigma + j\omega\epsilon)\, \bar{E} \end{aligned} \quad (13)$$

If we factor out $j\omega$ from the RHS

$$\nabla \times \bar{H} = j\omega \left(\epsilon + \frac{\sigma}{j\omega} \right) \bar{E} \quad (14)$$

we can recognize the RHS as the displacement current in a space with complex permittivity

$$\epsilon_c = \epsilon + \frac{\sigma}{j\omega} \equiv \epsilon' - j\epsilon'' \quad (15)$$

so that $$\nabla \times \bar{E} = j\omega\epsilon_c \bar{E} \qquad (16).$$

The effects of conduction current have been incorporated into Maxwell's equations through this complex permittivity concept, ϵ_c. This is a very common practice. The real part of ϵ_c, ϵ', is the normal permittivity of the material while the imaginary part, ϵ'', represents losses in the space:

$$\epsilon' = \epsilon \quad \text{and} \quad \epsilon'' = \frac{\sigma}{\omega} \qquad (17)$$

Often times, you'll find ϵ' & $\tan\delta$, the so-called loss tangent, given for materials, where

$$\tan\delta \equiv \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} \left(= \frac{\text{loss current}}{\text{charging current}}\right) \qquad (18)$$

Alternatively, the quality factor Q of the material will be specified, where

$$Q \equiv \frac{1}{\tan\delta} = \frac{\omega\epsilon}{\sigma} \left(= \frac{\text{charging current}}{\text{loss current}}\right) \qquad (19).$$

The relative permittivity $\epsilon_r' = \epsilon'/\epsilon_0$ and loss tangent for some materials are shown on the attached pages. Notice in particular Teflon and water. The ϵ_r' & $\tan\delta$ for Teflon are basically not a fct. of freq. up through 25 GHz.

On the other hand, the ϵ_r' & $\tan\delta$ of water is a strong function of frequency. This dependence of frequency

Makes water a <u>dispersive</u> material.

With this complex permittivity, we can define the complex intrinsic impedance of a material as

$$\eta_c \equiv \sqrt{\frac{\mu}{\epsilon_c}} \qquad (20)$$

Substituting (15) into (20) gives

$$\eta_c = \sqrt{\frac{\mu}{\epsilon + j\frac{\sigma}{\omega}}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \qquad (21)$$

This η_c is the same result we obtained for Z_w in (12). Consequently, the wave impedance for a plane wave in a lossy space equals the complex intrinsic impedance of the lossy material.

The complex Poynting vector associated with the +z propagating wave, for example, in the lossy space:

$$\bar{S}^+ = \bar{E}^+ \times \bar{H}^{+*} = \underbrace{\left(\hat{a}_x E_o^+ e^{-\alpha z} e^{-j\beta z}\right)}_{(3)} \times \underbrace{\left(\hat{a}_y \frac{1}{\eta_c} E_o^+ e^{-\alpha z} e^{-j\beta z}\right)^*}_{(11)}$$

$$= \hat{a}_z \frac{|E_o^+|^2}{\eta_c^*} e^{-2\alpha z} \qquad (22)$$

The time-average power carried by this wave is

$$\langle \bar{S}^+ \rangle = \frac{1}{2} \mathrm{Re}\{\bar{S}^+\} = \hat{a}_z \frac{|E_o^+|^2}{2} e^{-2\alpha z}\, \mathrm{Re}\left\{\frac{1}{\eta_c^*}\right\} \quad \left[\frac{W}{m^2}\right] \qquad (23)$$

through the $e^{-2\alpha z}$ factor

This result shows explicitly that the power carried by the wave is decreasing as the wave propagates, which we mentioned earlier in this lecture.

The distance the EM wave must travel in a lossy material for its magnitude to decrease by the factor e^{-1} is called the <u>skin depth</u>. Let's take the +z prop. comp. of $\bar{E}(z)$ in (3), for example:

$$E_x^+(z) = E_0^+ e^{-\alpha z} e^{j\beta z}$$

s.t. $|E_x^+(z)| = |E_0^+| e^{-\alpha z}$

for $\dfrac{|E_x^+(z)|}{|E_0^+|} = e^{-1}$ requires a distance $z = \dfrac{1}{\alpha}$

this distance is called the skin depth given by

$$\delta = \frac{1}{\alpha} \qquad (24)$$

Using (8)

$$\delta = \left(\omega\sqrt{\mu\epsilon} \left\{ \frac{1}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right] \right\}^{1/2} \right)^{-1} \qquad (25)$$

There are two special cases of lossy materials that are sometimes used in applied electromagnetics: good dielectrics and good conductors. We can define each of these types of materials by the predominance of conduction or displacement current in the material

In the situation when displacement current magnitude is much greater than the conduction current magnitude in a material is said to be a "good dielectric." Recall from Ampère's law in (13)

$$\nabla \times \bar{H} = \bar{J}_c + j\omega\epsilon E = \sigma\bar{E} + j\omega\epsilon\bar{E}$$

The ratio of conduction current magnitude to displacement current magnitude is $\sigma/\omega\epsilon$. For $\sigma/\omega\epsilon \ll 1$, the material is called a "good dielectric."

For this material, $\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$, $\beta \approx \omega\sqrt{\mu\epsilon}$

$$\eta_c \approx \sqrt{\frac{\mu}{\epsilon}} \quad \& \quad \delta \approx \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$$

On the other extreme is a material where the conduction current magnitude is much greater than the displacement current magnitude, $\sigma/\omega\epsilon \gg 1$. For this material,

$$\alpha = \beta \approx \sqrt{\frac{\omega\mu\sigma}{2}} \quad , \quad \eta_c \approx (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\& \quad \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

There is a good summary of these results in Table 4-1 in the text.

I. Solids, B. Organic 3. Plastics j. Polyvinyl Resins (cont.) Values for tan δ are multiplied by 10^4; frequency given in c/s.

6) Polytetrafluoro-ethylene (cont.)	T°C		$1x10^2$	$1x10^3$	$1x10^4$	$1x10^5$	$1x10^6$	$1x10^7$	$1x10^8$	$3x10^8$	$3x10^9$	$1x10^{10}$	$2.5x10^{10}$
Dilecto (Teflon Laminate	25	ϵ'/ϵ_o	3.35	3.24	3.18	3.17	3.16	3.15	3.15	3.15**	3.15	3.10	
GB-112T)[a]		tan δ	361	245	169	108	42	23	24	31**	38	47	
(field ∥ laminate)	250	ϵ'/ϵ_o	----	----	----	----	----	----	----	----	----	3.10	
		tan δ	----	----	----	----	----	----	----	----	----	70	
	25*	ϵ'/ϵ_o	3.26	3.25	3.24	3.21							
		tan δ	66	14.3	17.3	27							
Teflon[b]	22	ϵ'/ϵ_o	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.08	2.08
		tan δ	< 5	< 3	< 3	< 3	< 2	< 2	< 2	1.5	1.5	3.7	6
	100	ϵ'/ϵ_o	2.04	2.04	2.04	2.04	2.04	2.04	----	----	----	2.04	
		tan δ	10	4	2	< 3	< 2	< 2	----	----	----	5.1	
Chemelac M1405[c]	25	ϵ'/ϵ_o	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50		
		tan δ	18	5.1	2.7	3	5	7.2	9	8	6.8		
Chemelac M1406[d]	25	ϵ'/ϵ_o	----	----	----	170	72	----	----	----	12.7		
		tan δ***	----	----	>10000	2400	62	----	----	----	.33		
		ρ	----	520	520	440	410	----	----	----	145		
Chemelac M1407[e]	25	ϵ'/ϵ_o	3.42	3.02	2.85	2.74	2.71	2.65	2.63				
		tan δ	917	700	350	180	150	147	158				
Chemelac M1411[f]	25	ϵ'/ϵ_o	2.14	2.14	2.14	2.14	2.14	2.14	2.14				
(field ⊥ sheet)		tan δ	18.5	9.6	7	6.8	7	9.2	10				
(field ∥ sheet)	25	ϵ'/ϵ_o	----	----	----	----	----	----	----	----	2.52	2.50	
		tan δ	----	----	----	----	----	----	----	----	25	28	
Chemelac M1412[g]	25	ϵ'/ϵ_o	----	2.35	----	----	----	----	----	----	2.35		
		tan δ	----	12	----	----	----	----	----	----	15		
Chemelac M1414[h]	25	ϵ'/ϵ_o	----	----	----	----	----	150	43	----	33		
		tan δ***	----	----	>10000	>1000	>100	11.8	4.8	----	3.9		
		ρ	----	----	95	95	95	95	80	----	4.8		

a. 65-68% Teflon, 32-35% continuous-filament glass base (Cont. Diamond). b. Polytetrafluoroethylene (DuPont). c. 75% Teflon, 25% calcium fluoride (U.S.Gasket). d. 80% Teflon, 20% carbon (U.S. Gasket). e. 88% Teflon, 12% ceramic (U.S.Gasket). f. 75% Teflon, 25% Fiberglas (U.S.Gasket). g. 75% Teflon, 25% glass (U.S.Gasket). h. 75% Teflon, 25% graphite (U.S.Gasket).
*After temperature run. **Freq. = 1 x10^9. ***tan δ not multiplied by 10^4.

10/10

II. LIQUIDS

Values for tan δ are multiplied by 10^4; frequency given in c/s.

A. Inorganic	T°C		$1x10^5$	$1x10^6$	$1x10^7$	$1x10^8$	$3x10^8$	$3x10^9$	$1x10^{10}$	$2.5x10^{10}$**
Water, conductivity[a]	1.5	ϵ'/ϵ_o	87.0	87.0	87	87	86.5	80.5	38	15
		tan δ	1900	190	20	70	320	3100	10300	4250
	5	ϵ'/ϵ_o	----	85.5	----	----	85.2	80.2	41	17.5
		tan δ	----	220	----	----	273	2750	9500	3950
	15	ϵ'/ϵ_o	----	81.7	----	----	81.0	78.8	49	25
		tan δ	----	310	----	----	210	2050	7000	3300
	25	ϵ'/ϵ_o	78.2	78.2	78.2	78	77.5	76.7	55	34
		tan δ	4000	400	46	50	160	1570	5400	2650
	35	ϵ'/ϵ_o	----	74.8	----	----	74.0	74.0	58	41
		tan δ	----	485	----	----	125	1270	4400	2150
	45	ϵ'/ϵ_o	----	71.5	----	----	71.0	70.7	59	46
		tan δ	----	590	----	----	105	1060	4000	2750
	55	ϵ'/ϵ_o	----	68.2	----	----	68	67.5	60	49
		tan δ	----	720	----	----	92	890	3600	2450
	65	ϵ'/ϵ_o	----	64.8	----	----	64.5	64.0	59	50.5
		tan δ	----	865	----	----	84	765	3200	1250
	75	ϵ'/ϵ_o	----	61.5	----	----	61	60.5	57	51.5
		tan δ	----	1030	----	----	77	660	2800	1050
	85	ϵ'/ϵ_o	58	58	58	58	57	56.5	54	
		tan δ	12400	1240	125	30	73	547	2600	
	95	ϵ'/ϵ_o	----	55	----	----	52	52		
		tan δ	----	1430	----	----	70	470		
Aqueous sodium chloride[b]										
0.1 molal solution	25	ϵ'/ϵ_o	78.2*	----	----	----	76	75.5	54	
		tan δ	24,000,000	----	----	----	7800	2400	5600	
0.3 molal solution	25	ϵ'/ϵ_o	78.2*	----	----	----	71	69.3	52	
		tan δ	63,000,000	----	----	----	24000	4350	6050	
0.5 molal solution	25	ϵ'/ϵ_o	78.2*	----	----	----	69	67.0	51	
		tan δ	99,000,000	----	----	----	39000	6250	6300	
0.7 molal solution	25	ϵ'/ϵ_o	78.2*	----	----	----	----	----	50	
		tan δ	130,000,000	----	----	----	----	----	6600	

a. Research Laboratory of Physical Chemistry, M.I.T. b. NaCl, Mallinckrodt's Analytical Reagent.

*. ϵ'/ϵ_o of conductivity water assumed for purpose of calculating tan δ from conductivity measurements.

** Data of Collie, Hasted and Ritson, Proc. Phys. Soc. 60, 145 (1948).

Dielectric Data 361

Reference: A. von Hippel, ed. *Dielectric Materials and Applications*. Boston: Artech House, second ed., 1995.