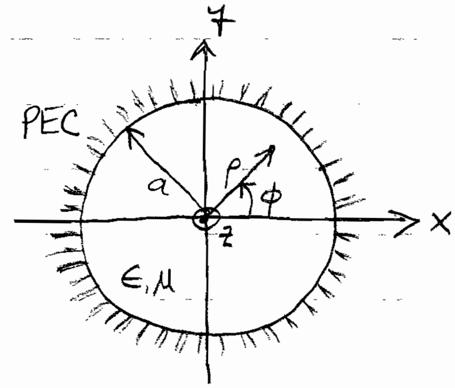
Lecture 32: Circular Metallic Waveguides.

Other than the rectangular cross-section, another popular type of hollow metallic waveguide is one with a circular cross-section:



Because the interior of this waveguide is separable in the circular cylindrical coordinate system, it will be necessary for us to solve the wave equation in that coord. system.

As in Lecture 21, we'll denote coordinates as ρ, ϕ, z and expand the electric & magnetic fields as

$$\bar{E} = \hat{\rho}\, E_\rho(\rho,\phi,z) + \hat{\phi}\, E_\phi(\rho,\phi,z) + \hat{z}\, E_z(\rho,\phi,z) \tag{1}$$

&

$$\bar{H} = \hat{\rho}\, H_\rho(\rho,\phi,z) + \hat{\phi}\, H_\phi(\rho,\phi,z) + \hat{z}\, H_z(\rho,\phi,z) \tag{2}$$

As with the rectangular waveguide, if we are searching for wave solutions propagating in $\pm z$, it is possible to express the transverse fields as fcts. only of the field components E_z and H_z.

For example, with propagation in $+z$ as $e^{-j\beta_z z}$, then in a homogeneous space (Pozar, 3rd ed., p. 118):

$$E_\rho = \frac{-j}{\beta_c^2}\left(\beta_z \frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi}\right) \quad (3)$$

$$E_\phi = \frac{-j}{\beta_c^2}\left(\frac{\beta_z}{\rho}\frac{\partial E_z}{\partial \phi} - \omega\mu \frac{\partial H_z}{\partial \rho}\right) \quad (4)$$

$$H_\rho = \frac{j}{\beta_c^2}\left(\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial \phi} - \beta_z \frac{\partial H_z}{\partial \rho}\right) \quad (5)$$

$$H_\phi = \frac{-j}{\beta_c^2}\left(\omega\epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta_z}{\rho}\frac{\partial H_z}{\partial \phi}\right) \quad (6)$$

Once we have analytical solutions for E_z and H_z, we can determine all remaining field components. Consequently, it makes sense to separate the field solutions into TE^z & TM^z modes in a homogeneous waveguide, as in a rectangular waveguide.

We saw earlier in Lecture 21 that the vector wave equation for $\bar{T} = \bar{E}$ or $\bar{H}$ is

$$\nabla^2 \bar{T} + \beta^2 \bar{T} = 0 \quad (7)$$

In cylindrical coordinates, this vector equation can be separated into 3 scalar wave equations

$$\hat{\rho}: \quad \nabla^2 T_\rho + \left(\frac{-T_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial T_\phi}{\partial \phi}\right) + \beta^2 T_\rho = 0 \quad (8)$$

$$\hat{\phi}: \quad \nabla^2 T_\phi + \left(\frac{-T_\phi}{\rho^2} + \frac{2}{\rho^2}\frac{\partial T_\rho}{\partial \phi}\right) + \beta^2 T_\phi = 0 \quad (9)$$

$$\hat{z}: \quad \nabla^2 T_z + \beta^2 T_z = 0 \quad (10)$$

We can solve (10) for the axial components of $\bar{E}$ & $\bar{H}$. Using (3)-(6), we can determine all the transverse field components from the axial component E_z & H_z.

These solutions for E_z & H_z we wrote in lecture 21 as the product

$$T_z = f(\rho)\, g(\phi)\, h(z) \tag{11}$$

The specific linear combinations used for f, g, & h depend a great deal on the type of problem which is being solved. For the hollow metallic circular waveguide, useful choices are:

$$f(\rho) = C_1 J_n(\beta_c \rho) + C_2 Y_n(\beta_c \rho) \tag{12}$$

where $\beta_c^2 \equiv \beta^2 - \beta_z^2$ (I called $\beta_c = k_\rho$ in Lecture 21) and

$$g(\phi) = C_3 \sin(n\phi) + C_4 \cos(n\phi) \tag{13}$$