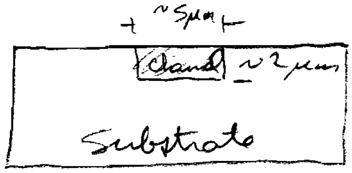
Lecture 30 – Grounded Dielectric Slab Waveguides.

It is possible to guide EM waves by structures that are not metallic. As we saw in a previous lecture, UPWs incident at an angle greater than the critical ∡ will be entirely reflected. Principle is the same for fiber optics & other structures, such as integrated optical waveguides:



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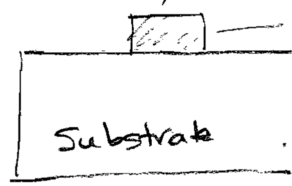
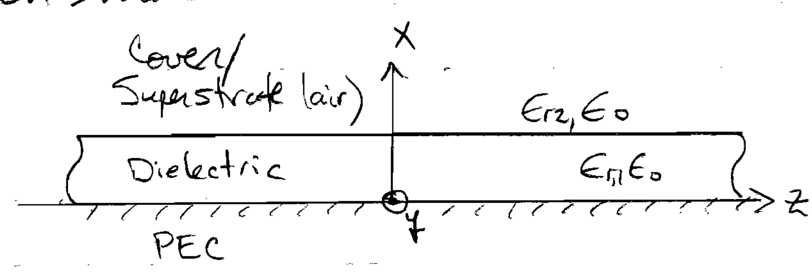


Such structures also find application at microwave & millimeter-wave frequencies.

This study will also lead to surface waves, which we saw associated w/ UPWS incident on half spaces w/ $\theta_i > \theta_c$

Wave equations for Grounded Dielectric Slab

The geometry of the structure we'll be analyzing is shown below



8.5.1 Wave Equations for the Grounded Dielectric Slab Waveguide

The specific electromagnetic field solutions we are searching for in this problem are all families of waves (modes) that are guided by the slab with propagation in the +z direction. As in the previous sections of this chapter, we are not specifically concerned with determining *the* fields that exist inside the waveguide due to a specific source. Instead, we are developing all possible field solutions (modes) that *can* exist inside the waveguide. Therefore, our final field solutions will always involve one arbitrary constant that is left unevaluated because the source boundary condition has not been applied.

Since we are only interested in those fields that behave as waves propagating in the +z direction, the governing equations for $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ are still the reduced wave equations ~~(4)~~ we derived for metallic wvgds. For the slab waveguide in ~~Fig. 8.17~~ the last figure there is no y variation in the structure. Furthermore, we will assume there is also no variation of the source in this direction either. Consequently, we would expect that all resulting fields in the guide should not depend on y. Then, ~~from (4)~~, the reduced wave equations for the axial components of $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ ~~(with $\beta_y = 0$)~~ become

$$\nabla_t^2 H_z + \beta^2 H_z = 0 \quad \& \quad \nabla_t^2 E_z + \beta^2 E_z = 0 \qquad (1),(2)$$

where $\nabla_t^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$,

$$\& \quad H_z(x,y,z) \equiv h_z(x,y)\, e^{-j\beta_z z} \qquad (3)$$

$$\& \quad E_z(x,y,z) \equiv e_z(x,y)\, e^{-j\beta_z z} \qquad (4)$$

Eqns. (1) & (2) are often called reduced wave equations.

Since no variation in y, then $\dfrac{\partial}{\partial y} \to 0$ & (1) & (2) become

$$\frac{d^2 h_z}{dx^2} + \beta_x^2 h_z = 0 \quad \& \quad \frac{d^2 e_z}{dx^2} + \beta_x^2 e_z = 0 \qquad (5),(6)$$

We'll solve these eqns. subject to b.c.'s @ PEC and dielectric interface. Separate into TE^z & TM^z

TE^z modes

For these modes, $E_z = 0$ & $H_z \neq 0$. Use separation of variables for (5), but now two regions versus one w/ hollow metallic guides.

In the slab ($0 \le x \le d$), we expect standing waves in x so the proper solution is

$$H_{z1} = \left[C_1 \cos(\beta_{x1} x) + C_2 \sin(\beta_{x1} x) \right] e^{-j\beta_z z} \qquad (7)$$

where the wavenumbers satisfy the dispersion relation

$$\beta_{x1}^2 + \beta_z^2 = \beta^2 \equiv \omega^2 \mu_0 \epsilon_1 \qquad (8)$$

Note β_z will be the same in regions (1) & (2), by phase match, so we won't subscript here.

If we follow a similar procedure in the cover region ($x \geq d$), the solutions would be in the form of uniform plane waves propagating away from the interface in the $+x$ direction as $\hat{H}_{z2} \propto e^{-j\beta_{x2}x}e^{-j\beta_z z}$. There is a fundamental problem with this type of solution, however. Since it is a plane wave, it will carry energy away from the interface in the $+x$ direction. Imagining that a source somewhere far in the $-z$ direction has provided a signal, the energy being propagated away in the $+x$ direction is wasted. Indeed, for long waveguides none of the signal would reach its intended destination somewhere along the z axis. This plane wave solution in the cover region is much better suited for a dielectric slab *antenna*.

For a dielectric slab wave*guide* we will require another form of the fields in the cover region. If the fields were to decay exponentially away from the slab in the $+x$ direction, and there was no energy carried by this field away from the interface, we could then surmise that all energy must be transported only in the $+z$ direction—which is the intended direction for this waveguide. But the question remains whether such a field is physically realistic. The answer is affirmative, under certain constraints. Recall ~~from Chap. 6~~ that for a uniform plane wave obliquely incident on a dielectric half space when the angle of incidence was greater than the critical angle, the field ~~ceased~~ ceases to propagate as a wave and exponentially decays in a direction perpendicular to the interface. This type of field was called an *evanescent wave*. Evanescent behavior can occur for two dielectric regions, provided the permittivity of the transmission region is less than the permittivity of the incidence region. Such an effect can also occur for this dielectric slab waveguide provided $\epsilon_1 > \epsilon_2$.

For evanescent wave solutions for H_z in region 2, we will require β_x in (5) to become purely imaginary. In this case, the reduced wave equation becomes from (5) w/ $\beta_x^2 \to -\alpha_{x2}^2$

$$\frac{d^2 h_{z2}}{dx^2} - \alpha_{x2}^2 h_{z2} = 0 \qquad (9)$$

which has solutions of the form $h_{z2} \propto e^{-\alpha_{x2}x}, e^{+\alpha_{x2}x}$. Only first is applicable here for $\alpha_{x2} \geq 0$.

Hence, H_{z2} has the solution

$$H_{z2} = C_3 e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \qquad (10)$$

w/ dispersion relation $\beta_z^2 - \alpha_{x2}^2 = \beta_2^2 = \omega^2 \mu_0 \epsilon_2 \qquad (11)$

Now, w/ H_z known in both regions, can determine transverse fields. In particular, $E_x = H_y = 0$ everywhere and

$$E_{y1} = \frac{j\omega\mu_0}{\beta_{x1}^2}\frac{\partial H_{z1}}{\partial x} = \frac{j\omega\mu_0}{\beta_{x1}}\left[-C_1 \sin(\beta_{x1}x) + C_2 \cos(\beta_{x1}x)\right] e^{-j\beta_z z} \qquad (12)$$

and

$$E_{y2} = -\frac{j\omega\mu_0}{\alpha_{x2}^2}\frac{\partial H_{z2}}{\partial x} = \frac{j\omega\mu_0}{\alpha_x^2} C_3 e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \qquad (13)$$

Apply the three b.c.'s to determine two of three constants C_1, C_2, C_3, & unknown phase constant β_z.

(i) $\bar{E}_{tan} = 0$ @ $x = 0$ $\forall y, z$.

From (12): $E_{y1}(x=0) = \frac{j\omega\mu_0}{\beta_{x1}} = \left[-C_1 \sin(0) + C_2 \cos(0)\right] e^{-j\beta_z z}$

$$\Rightarrow C_2 = 0.$$

(ii) $\bar{H}_{tan}$ continuous @ $x = d$: Equating (7) & (10) at $x = d$ w/ $C_2 = 0$ gives

$$C_1 \cos(\beta_{x1} d) e^{-j\beta_z z} = C_3 e^{-\alpha_{x2}(d-d)} e^{-j\beta_z z}$$

or

$$C_1 \cos(\beta_{x1} d) = C_3 \qquad (14)$$

(iii) $\bar{E}_{tan}$ continuous @ $x = d$. Equating (12) & (13) at $x = d$ w/ $C_2 = 0$ gives

$$-\frac{j\omega\mu_0}{\beta_{x1}} C_1 \sin(\beta_{x1} d) e^{-j\beta_z z} = \frac{j\omega\mu_0}{\alpha_{x2}} C_3 e^{-\alpha_{x2}(d-d)} e^{-j\beta_z z}$$

or

$$C_1 \sin(\beta_{x1} d) = -\frac{\beta_{x1}}{\alpha_{x2}} C_3 \qquad (15)$$

Now, substituting C_2 & C_3 into past field expressions gives total fields within dielectric slab ($x \le d$):

$$H_{z1} = C_1 \cos(\beta_{x1} x) e^{-j\beta_z z} \qquad (16)$$

$$E_{y1} = -\frac{j\omega\mu_0}{\beta_{x1}} C_1 \sin(\beta_{x1} x) e^{-j\beta_z z} \qquad (17)$$

&

$$H_{x1} = \frac{j\beta_z}{\beta_{x1}} C_1 \sin(\beta_{x1} x) e^{-j\beta_z z} \qquad (18)$$

& in cover region $x \ge d$:

$$H_{z2} = C_1 \cos(\beta_{x1} d) e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \qquad (19)$$

$$E_{y2} = \frac{j\omega\mu_0}{\alpha_{x2}} C_1 \cos(\beta_{x1} d) e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \qquad (20)$$

&

$$H_{x2} = -\frac{j\beta_z}{\alpha_{x2}} C_1 \cos(\beta_{x1} d) e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \qquad (21)$$

The remaining two unknowns are C_1 & β_z. C_1 depends on source, so will leave it. Need an equation to determine β_z. (Note β_{x1} & α_{x2} related to β_z through D.R.'s.)

An equation for β_z can be determined by dividing (15) by (14):

$$\boxed{\tan(\beta_{x1} d) = -\frac{\beta_{x1}}{\alpha_{x2}}} \qquad (22)$$

Called characteristic equation for TE^z modes in dielectric-slab waveguide. Solve (22) for $\underline{\beta_z}$.

No simple analytical solution. Use graphical or numerical root-finding methods.

TM^z modes

The Method to determine these solutions very similar to TE^z modes. For TM^z, $H_z = 0$ & E_z satisfies wave eqn. (2). Solutions are

$$E_{z1} = \left[C_4 \cos(\beta_{x1} x) + C_5 \sin(\beta_{x1} x)\right] e^{-j\beta_z z} \tag{23}$$

&

$$E_{z2} = C_6 e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \tag{24}$$

& Satisfy same dispersion relations (8) & (11).

Applying b.c.'s @ $x=0$ & d & simplifying gives fields for TM^z modes in the slab ($x \leq 0$):

$$E_{z1} = C_5 \sin(\beta_{x1} x) e^{-j\beta_z z} \tag{25}$$

$$H_{y1} = -\frac{j\omega\epsilon_1}{\beta_{x1}} C_5 \cos(\beta_{x1} x) e^{-j\beta_z z} \tag{26}$$

$$E_{x1} = -\frac{j\beta_z}{\beta_{x1}} C_5 \cos(\beta_{x1} x) e^{-j\beta_z z} \tag{27}$$

While the fields in the cover region are ($x \geq d$):

$$E_{z2} = C_5 \sin(\beta_{x1} d)\, e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \quad (28)$$

$$H_{y2} = \frac{-j\omega\epsilon_2}{\alpha_{x2}} C_5 \sin(\beta_{x1} d)\, e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \quad (29)$$

$$E_{x2} = -\frac{j\beta_z}{\alpha_{x2}} C_5 \sin(\beta_{x1} d)\, e^{-\alpha_{x2}(x-d)} e^{-j\beta_z z} \quad (30)$$

The characteristic equation that determines all allowed β_z values for TM^z modes is

$$\boxed{\tan(\beta_{x1} d) = \frac{\epsilon_1 \alpha_{x2}}{\epsilon_2 \beta_{x1}}} \quad (31)$$

Not same characteristic eqn as TE^z modes.