Lecture 27 – Fields of Rectangular Waveguides

For particular m & n indices, it is possible to plot the normalized $\vec{E}$ & $\vec{H}$ fields inside the wvgd. A vector plot is simple to make, while plots of the field lines is a bit more difficult.

C.S. Lee, et al. published extensive plots of xverse fields in rectangular wvgd for a/b = 2. One thing we can deduce from these plots is that the index m (or n) indicates the number of periodic variations in the xverse fields in the x (or y) direction.

Dominant TE_{10} Mode

It is common to operate wvgds. in single mode. For a > b, this is the TE_{10} mode, as we saw in the last lecture. Hence, worthwhile to study this mode in detail.

From Lecture 25 & using m=1 & n=0, the cartesian components of $\vec{E}$ & $\vec{H}$ for the TE_{10} mode are

$$e_x = 0 = e_z \tag{1}$$

$$e_y = -\frac{j\omega\mu}{\beta_{c10}^2}\frac{\pi}{a}A_{10}\sin\left(\frac{\pi x}{a}\right) \quad \frac{V}{m} \tag{2}$$

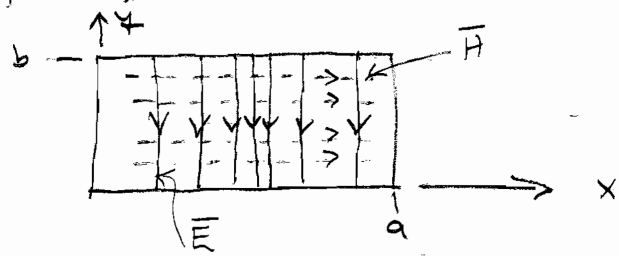
and

$$h_x = \frac{j\beta_{z10}}{\beta_{c10}^2}\frac{\pi}{a} A_{10} \sin\left(\frac{\pi x}{a}\right) \quad A/m \qquad (3)$$

$$h_y = 0 \qquad (4)$$

$$h_z = A_{10} \cos\left(\frac{\pi x}{a}\right) \quad A/m \qquad (5)$$

For $\bar{E}$, there exists only E_y. A sketch of the transverse fields is (Fig 8-5)



A few things to notice in these field plots of Fig 8-5:

(i) Regions of high density of field lines indicate larger field strengths

(ii) in top view, where $\bar{E}$ is large, so is $\bar{H}$.

(iii) The $\bar{H}$ field lines circulate around (vertical) E

[Time domain field variations in Visual EM.]

Conduction Current density on the walls of wvgd are shown in Fig 8-6. Displacement current forms the connection between conduction currents.

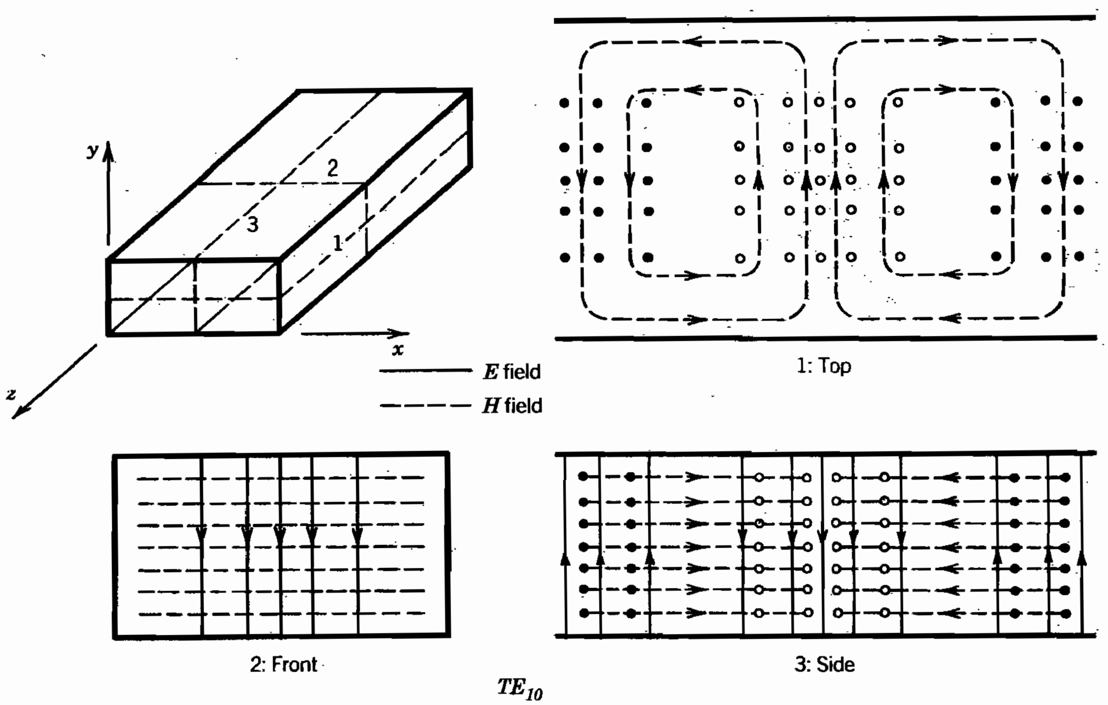


FIGURE 8-5 Electric field patterns for TE_{10} mode in a rectangular waveguide (*Source:* S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*, 1984. Reprinted with permission of John Wiley & Sons, Inc.)

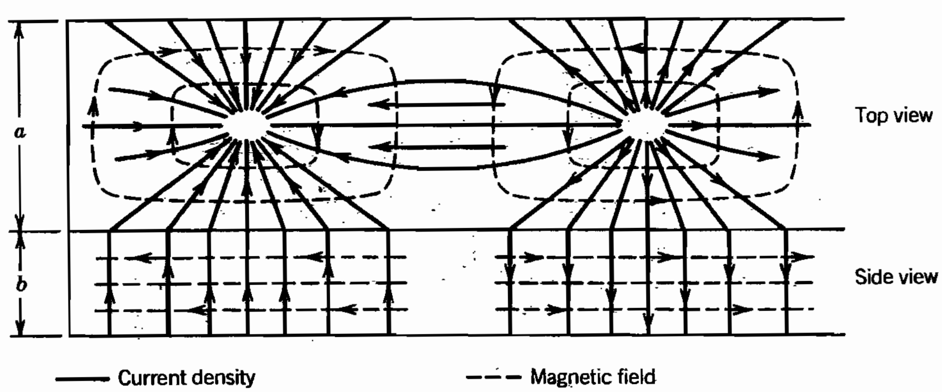


FIGURE 8-6 Magnetic field and electric current density patterns for the TE_{10} mode in a rectangular waveguide. (*Source:* S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*, 1984. Reprinted with permission of John Wiley & Sons, Inc.)

Bouncing Plane Waves

The total electric field for the TE_{10} mode is (prop in $+z$)

$$\bar{E} = -\hat{y}\,\frac{j\omega\mu}{\beta_{c10}^2}\,\frac{\pi}{a}\,A_{10}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta_{z10}z}\ \frac{V}{m} \qquad (6)$$

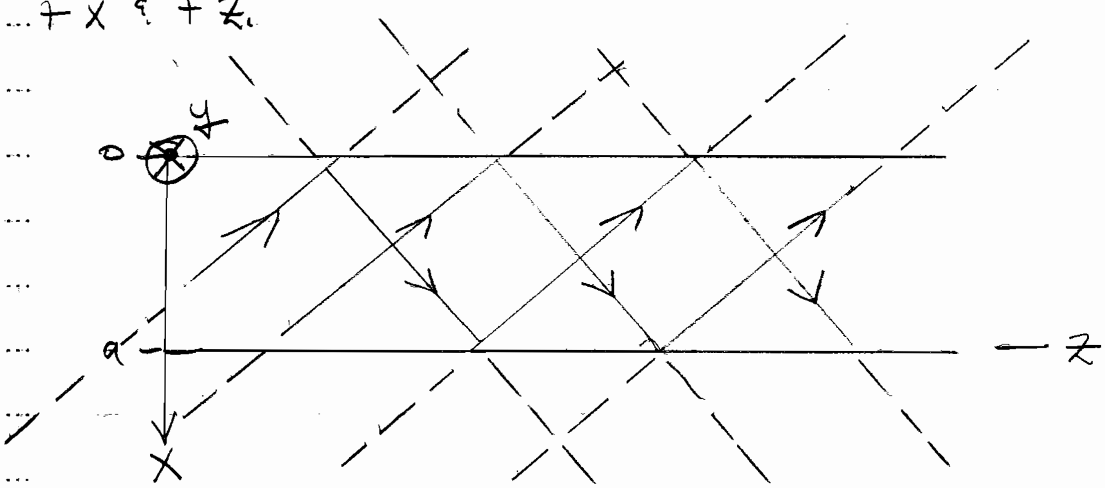
Using the id. $\sin\left(\frac{\pi x}{a}\right) = \dfrac{e^{j\frac{\pi x}{a}} - e^{-j\frac{\pi x}{a}}}{2j}$, (6) becomes

$$\bar{E} = -\hat{y}\,\frac{\omega\mu}{2\beta_{c10}^2}\,\frac{\pi}{a}\,A_{10}\left[e^{+j\frac{\pi x}{a}} - e^{-j\frac{\pi x}{a}}\right]e^{-j\beta_{z10}z} \qquad (7)$$

with $\beta_{x1} = \dfrac{\pi}{a}$, (7) can be expressed as

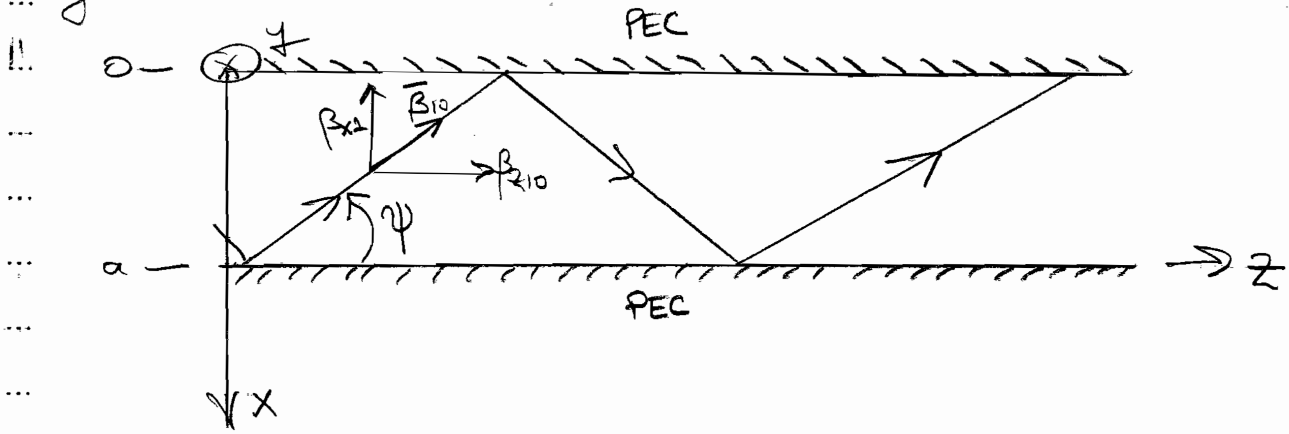
$$\bar{E} = -\hat{y}\,\frac{\omega\mu}{2\beta_{c10}^2}\,\beta_{x1}A_{10}\left[e^{j(\beta_{x1}x - \beta_{z10}z)} - e^{-j(\beta_{x1}x + \beta_{z10}z)}\right] \qquad (8)$$

The two terms in the bracket have the form of UPWs! Both are prop in x & z, with no propagation in y. In particular, the first term represents a UPW prop in $-\hat{x}$ & $+\hat{z}$ directions, while second prop in $+\hat{x}$ & $+\hat{z}$.



The region where fields are non zero is inside guide. Can also think of these UPWs existing everywhere & by interference are zero along planes @ $x = 0$ & a.

We can also look at this as rays bouncing back & forth as the wave propogates down the guide.



As in the text, will define angle ψ as above, where

$$\bar{\beta}_{10} = \hat{x}\beta_{x1} + \hat{z}\beta_{z10} = \hat{x}\beta\sin\psi + \hat{z}\beta\cos\psi \quad (8\text{-}44),\ (9)$$

We can compute ψ by equating $\hat{z}$ comps:

$$\beta_{z10} = \beta\cos\psi \Rightarrow \psi = \cos^{-1}\left(\frac{\beta_{z10}}{\beta}\right) \quad (10)$$

From the dispersion relation

$$\beta_{z10} = \pm\sqrt{\beta^2 - \beta_{c10}^2} = \pm\beta\sqrt{1-\left(\frac{\beta_{c10}}{\beta}\right)^2}$$

or $$\frac{\beta_{z10}}{\beta} = \pm\sqrt{1-\left(\frac{f_{c10}}{f}\right)^2} \qquad (14)$$

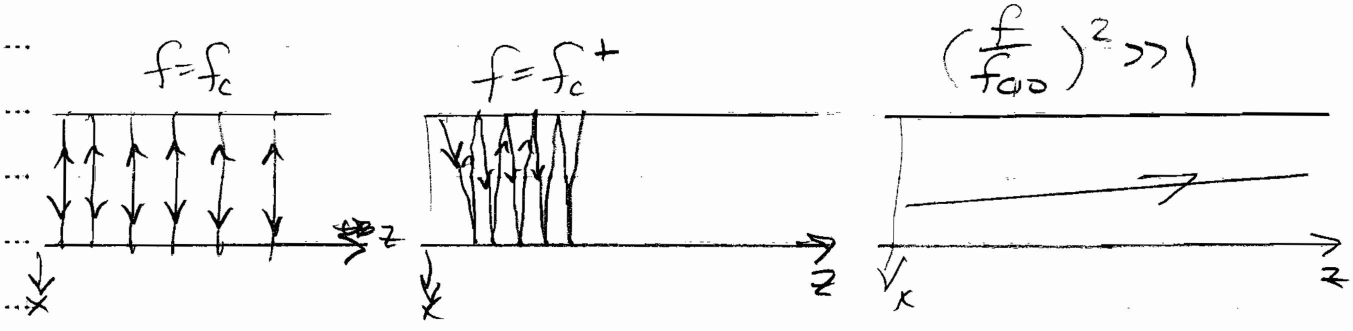
Sub (14) → (10) gives

$$\psi = \cos^{-1}\left[\sqrt{1-\left(\frac{f_{c10}}{f}\right)^2}\right] \qquad (8\text{-}49b), (12)$$

Interesting to investigate variation of ψ as f increases from f_c to ∞. When $f=f_c$, $\psi=90°$. Means wave bouncing back & forth across wvgd, but no propagation down wvgd.

As f increases from f_c, wave bouncing back & forth rapidly per unit length. As f gets very large ($\left(\frac{f_{c10}}{f}\right)^2 \ll 1$), then $\psi \to 0°$. Very little bouncing back & forth per unit length.

Three situations illustrated below



This picture of wave prop in wvgd very helpful in understanding dispersion of signals prop. down guide.

The "guide wavelength" λ_g is defined as

$$\lambda_g = \lambda_{zmn} = \frac{2\pi}{\beta_{zmn}} \qquad (8\text{-}50), (13)$$

As $f \to f_c$, the guide wavelength approaches ∞.

As $\left(\frac{f_{c10}}{f}\right)^2 \ll 1$ in (11), $\beta_{z10} \approx \beta$, the free space wavenumber.

!!

Phase velocity ranging from ∞ to c_0.
This velocity of constant phase fronts.

But how fast is information transmitted down waveguide? At $f = f_c$, wave just bouncing back & forth - no prop. $\Rightarrow v_g = 0$.
As $\left(\frac{f}{f_{c10}}\right)^2 \gg 1$ then information traveling at c_0. $v_g \approx v_p$ for $\left(\frac{f}{f_{c10}}\right)^2 \gg 1$.
Hence

Section 8.3.2

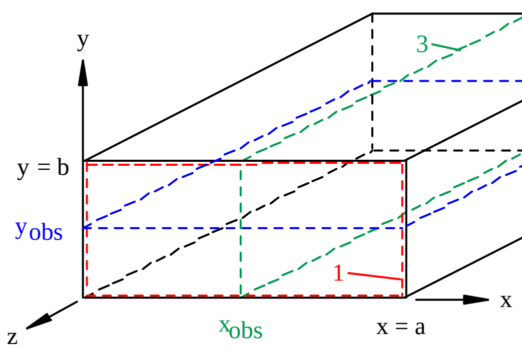
The Fields of TE Modes in a Rectangular Waveguide

Purpose

To compute and visualize the $TE_{m,n}$ modal fields inside a rectangular waveguide. A number of animation clips are generated in this worksheet from which the behavior of the **E** and **H** fields for TE_{mn} modes inside this rectangular waveguide can be observed as functions of both space and time.

Enter parameters

The geometry for this rectangular waveguide is shown in the figure below. The TE modes of this rectangular waveguide are assumed to be propagating in the +z direction.



Note that this geometry is slightly different than that shown in Fig. 8.8 of the text. However, none of the results are changed since the geometry of the waveguide shown here is simply rotated with respect to that shown in the text.

Choose the waveguide dimensions, the TE mode indices, the parameters of the material inside the waveguide and the frequency:

$a := 0.02286$	$b := 0.01016$	x and y dimensions of the waveguide (m).
$m := 1$	$n := 0$	TE mode indices m and n.
$\varepsilon_r := 1$	$\mu_r := 1$	Relative ε and μ of the space inside the waveguide.
$f := 10 \cdot 10^9$		Frequency (Hz).

Compute the radian frequency, material parameters and the phase constant:

$$\omega := 2 \cdot \pi \cdot f$$

$$\varepsilon_0 := 8.854\cdot 10^{-12} \qquad \mu_0 := 4\cdot\pi\cdot 10^{-7}$$

$$\varepsilon := \varepsilon_r\cdot\varepsilon_0 \qquad \mu := \mu_r\cdot\mu_0$$

$$\beta := \omega\cdot\sqrt{\mu\cdot\varepsilon}$$

$$T_p := \frac{1}{f} \qquad T_p = 1\times 10^{-10}$$ Period of one time cycle (s).

Compute wavenumbers β_x, β_y and β_z and the cutoff frequency

The transverse wavenumbers for the TE_{mn} mode (which are identical to those for **TM modes** in the rectangular waveguide) can be computed using Equations (77a) and (77b) in Chap. 8 of the text:

$$\beta_x := \frac{m\cdot\pi}{a} \qquad \beta_y := \frac{n\cdot\pi}{b}$$

The cutoff frequency for this mode is from (78) in Chap. 8 of the text:

$$f_c := \frac{1}{2\cdot\pi\cdot\sqrt{\mu\cdot\varepsilon}}\cdot\sqrt{\beta_x^{\,2}+\beta_y^{\,2}}$$ Cutoff frequency (Hz).

This TE $m = 1$, $n = 0$ mode will propagate inside the waveguide so long as the frequency of operation:

$$f = 1.000\times 10^{10}$$ (Hz)

is greater than the cutoff frequency:

$$f_c = 6.557\times 10^{9}$$ (Hz)

Now compute the longitudinal wavenumber using (79) in Chap. 8 of the text:

$$\beta_z := \text{if}\left(f \ge f_c, \sqrt{\beta^2-\beta_x^{\,2}-\beta_y^{\,2}}, -j\cdot\sqrt{\beta_x^{\,2}+\beta_y^{\,2}-\beta^2}\right)$$

$$\beta_z = 158.235$$ (rad/m)

For f greater than the cutoff frequency, f_c, of the TE $m = 1$, $n = 0$ mode, β_z will be a purely real number whereas for f less than f_c, β_z will be purely imaginary.

The fields of a TE_{mn} mode will generally be nonzero provided *both* m and n are not equal to zero.

Define functions for the E and H fields of the TE modes

We now will define the electric and magnetic fields inside the rectangular waveguide. The TE modal fields are given in Equations (92) in Chap. 8 of the text as:

$C := 1$ Arbitrarily choose the constant C equal to 1.

- The time-domain cartesian components of the electric field –

$$E_x(x,y,z,t) := \mathrm{Re}\left(\frac{j\cdot\omega\cdot\mu\cdot\beta_y}{\beta_x^{\ 2}+\beta_y^{\ 2}}\cdot C\cdot\cos(\beta_x\cdot x)\cdot\sin(\beta_y\cdot y)\cdot\exp(-j\cdot\beta_z\cdot z)\cdot\exp(j\cdot\omega\cdot t)\right)$$

$$E_y(x,y,z,t) := \mathrm{Re}\left(-\frac{j\cdot\omega\cdot\mu\cdot\beta_x}{\beta_x^{\ 2}+\beta_y^{\ 2}}\cdot C\cdot\sin(\beta_x\cdot x)\cdot\cos(\beta_y\cdot y)\cdot\exp(-j\cdot\beta_z\cdot z)\cdot\exp(j\cdot\omega\cdot t)\right)$$

$$E_z(x,y,z,t) := 0$$

- The time-domain cartesian components of the magnetic field –

$$H_x(x,y,z,t) := \mathrm{Re}\left(\frac{j\cdot\beta_z\cdot\beta_x}{\beta_x^{\ 2}+\beta_y^{\ 2}}\cdot C\cdot\sin(\beta_x\cdot x)\cdot\cos(\beta_y\cdot y)\cdot\exp(-j\cdot\beta_z\cdot z)\cdot\exp(j\cdot\omega\cdot t)\right)$$

$$H_y(x,y,z,t) := \mathrm{Re}\left(\frac{j\cdot\beta_z\cdot\beta_y}{\beta_x^{\ 2}+\beta_y^{\ 2}}\cdot C\cdot\cos(\beta_x\cdot x)\cdot\sin(\beta_y\cdot y)\cdot\exp(-j\cdot\beta_z\cdot z)\cdot\exp(j\cdot\omega\cdot t)\right)$$

$$H_z(x,y,z,t) := \mathrm{Re}\left(C\cdot\cos(\beta_x\cdot x)\cdot\cos(\beta_y\cdot y)\cdot\exp(-j\cdot\beta_z\cdot z)\cdot\exp(j\cdot\omega\cdot t)\right)$$

The electric and magnetic fields of this TE $m = 1$, $n = 0$ mode will be plotted in three separate plane cuts as shown in the figure at the beginning of this worksheet. These planes are (1) the xy plane at z = 0 (an end-on view), (2) an xz plane at $y = y_{obs}$ (a top-down view) and (3) a yz plane at $x = x_{obs}$ (a side view). Animation clips will be generated for the electric and magnetic fields in all three of these planes.

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(1) Plot the E and H fields in the xy plane at z = 0

Choose the number of points to plot in the x and y directions:

$npts_x := 20$	$npts_y := 10$	Number of points to plot in x and y.
$x_{start} := 0$	$x_{end} := a$	x starting and ending points (m).
$y_{start} := 0$	$y_{end} := b$	y starting and ending points (m).

Generate a list of x_a and y_a points at which to plot the fields:

$$i := 0 .. npts_x - 1 \qquad xa_i := x_{start} + i \cdot \frac{x_{end} - x_{start}}{npts_x - 1}$$

$$j := 0 .. npts_y - 1 \qquad ya_j := y_{start} + j \cdot \frac{y_{end} - y_{start}}{npts_y - 1}$$

Choose the number of time instances at which to compute the fields in one time period:

$n_{pts\_per\_period} := 20$ Number of points to plot per period.

$t_{start} := 0 \qquad t_{end} := T_p$ Time to start and end plot (s).

Define the variable time in terms of the constant FRAME:

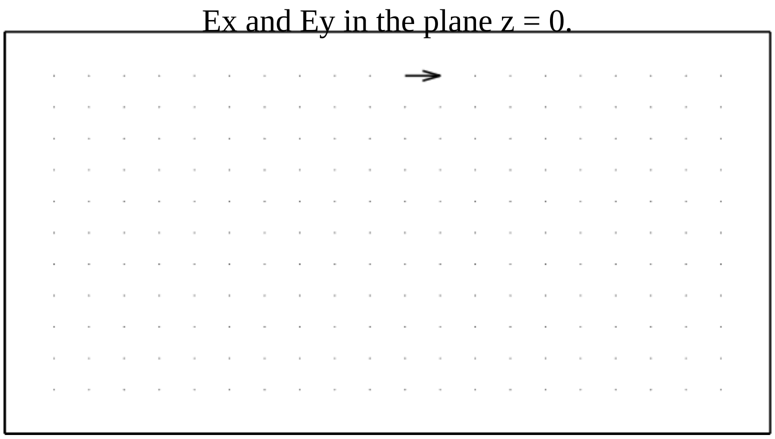
$$t_{inc} := \frac{T_p}{n_{pts\_per\_period}} \qquad time := t_{start} + FRAME \cdot t_{inc}$$

Compute the x and y components of **E** and **H** at the matrix of x_a and y_a points specified at each time instant:

$$exa_{i,j} := E_x(xa_i, ya_j, 0, time) \qquad hxa_{i,j} := H_x(xa_i, ya_j, 0, time)$$

$$eya_{i,j} := E_y(xa_i, ya_j, 0, time) \qquad hya_{i,j} := H_y(xa_i, ya_j, 0, time)$$

Now create an animation clip of these **E** and **H** fields. For best results, in the "Animate" dialog box, choose To = 19 then save the file and replay the animation in a video player that supports continuous loop playback.



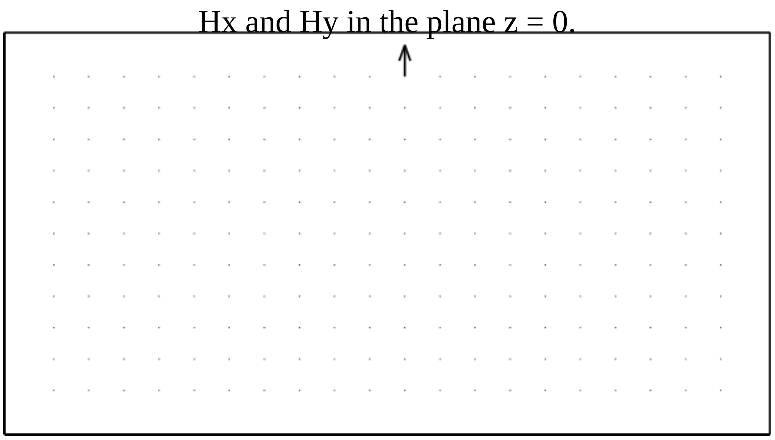
– y = b

– y = 0

For the TE mode
$m = 1$, $n = 0$
at time (in periods, T_p)

$$\frac{\text{time}}{T_p} = 0.000$$

(e_{ya}, e_{xa})



– y = b

– y = 0

For the TE mode
$m = 1$, $n = 0$
at time (in periods, T_p)

$$\frac{\text{time}}{T_p} = 0.000$$

(h_{ya}, h_{xa})

| x = 0 | x = a

Note: The single vector arrow at the top and center of these two plots is not physical. The purpose of this constant vector is simply to prevent Mathcad from distorting this animation through its automatic autoscaling of the vector plot at each time step.

It is perhaps most interesting to view both of these field patterns in the same animation clip. This can be accomplished by dragging the dashed black selection box around both of the plots when generating the animation clip.

After viewing this animation clip, you will see that for propagating modes the electric and magnetic fields are in "time phase" and "space quadrature" with each other as we saw for the **parallel-plate** waveguide and the **TM modes** in the rectangular waveguide.

The TE_{10} mode is the dominant mode in a rectangular waveguide when the guide is wider than it is tall (a > b). Generally waveguides are designed to propagate only a single mode in order to reduce the effects of signal distortion resulting from many propagating modes – called *multimode distortion*. The waveguide is always *dispersive* since β_z is a function of ω, but the amount of signal distortion is lessened considerably if only a single mode propagates the signal. Therefore, it is worthwhile to study the behavior of the dominant TE_{10} mode in the above animations and in the following two sections of this worksheet.

(2) Plot the E and H fields in an xz plane at $y = y_{obs}$

Choose the xz plane in which to observe the fields and the number of points to plot in the z direction:

$$y_{obs} := \frac{b}{2}$$

y observation plane (m). Choose between 0 and b.

$$npts_z := 20$$

Number of points to plot in z.

$$z_{start} := 0 \qquad z_{end} := \frac{2\cdot\pi}{\left|\beta_z\right|}$$

z starting and ending points (m).

Generate a list of z_a points at which to plot the fields:

$$k := 0 .. npts_z - 1 \qquad z_{a_k} := z_{start} + k\cdot\frac{z_{end} - z_{start}}{npts_z - 1}$$

Compute the y component of **E** and the x and z components of **H** at the matrix of x_a and z_a points specified at each time instant:

$$e_{yb_{i,k}} := E_y\left(x_{a_i}, \frac{b}{2}, z_{a_k}, time\right) \qquad h_{xb_{i,k}} := H_x\left(x_{a_i}, \frac{b}{2}, z_{a_k}, time\right)$$

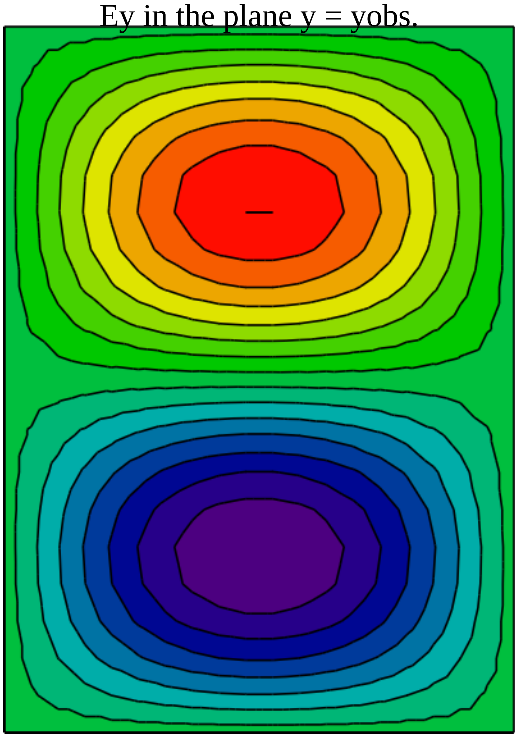
$$h_{zb_{i,k}} := H_z\left(x_{a_i}, \frac{b}{2}, z_{a_k}, time\right)$$

Now create an animation clip of these **E** and **H** fields. For best results, in the "Animate" dialog box, choose To = 19 then save the file and replay the animation in a video player that supports continuous loop playback.

For the TE mode
$m = 1$, $n = 0$

At time (in periods, T_p) $\dfrac{time}{T_p} = 0.000$

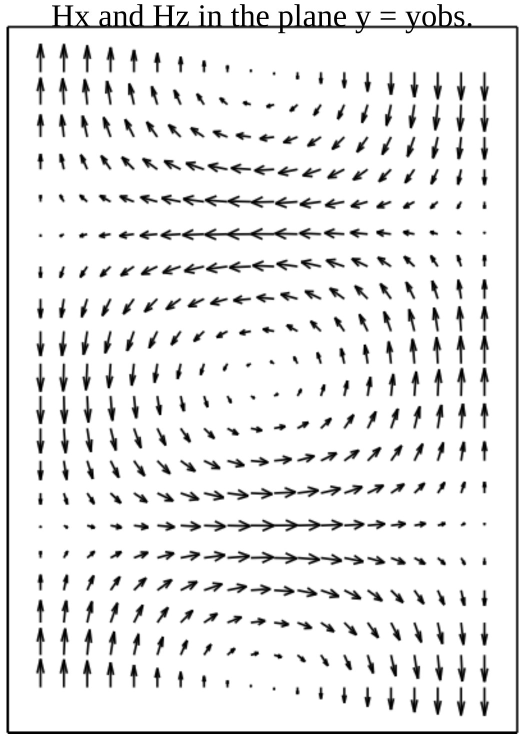
Ey in the plane y = yobs.



$- z = \lambda_z -$

$- z = 0 -$

Hx and Hz in the plane y = yobs.



e_{yb} (h_{xb}, h_{zb})

x = 0 x = a x = 0 x = a

Note: For the proper operation of this animation clip, the TE $m = 1$, $n = 0$ mode should be propagating. In other words, $\beta_z = 158.235$ should be a real number.

It is perhaps most interesting to view both of these field patterns in the same animation clip. This can be accomplished by dragging the dashed black selection box around both of the plots when generating the animation clip.

The z direction is along the vertical axis in these two plots. The fields are shown for one wavelength in this z direction, i.e., from z = 0 to z = λ_z. The propagation of this mode in the +z direction (vertical direction) is quite evident in the animation clip provided $f > f_c$. The E_y field, shown in the contour plot, is directed normal to the plane of the plot – i.e., either into or out of the screen. The warmer colors indicate a positive value (E_y pointing out of the screen) with red showing a maximum positive value, while cooler colors indicate a negative value with deep blue showing the maximum negative value. When the frequency is greater than the cutoff frequency for this mode, this entire field pattern propagates in the +z direction (vertically in this plot) as is apparent when viewing the animation clip.

animation clip.

For the TE_{10} mode (m = 1, n = 0), you can directly compare these fields with the field lines sketched in Fig. 8.11c of the text.

(3) Plot the E and H fields in a yz plane at $x = x_{obs}$

Choose the yz plane in which to observe the fields and the number of points to plot in the y and z directions:

$x_{obs} := \frac{a}{2}$ x observation plane (m). Choose between 0 and a.

$npts_y := 12$ $npts_z := 24$ Number of points to plot in y and z.

Generate a list of y_b and z_b points at which to plot the fields:

$$j := 0 .. npts_y - 1 \qquad yb_j := y_{start} + j \cdot \frac{y_{end} - y_{start}}{npts_y - 1}$$

$$k := 0 .. npts_z - 1 \qquad zb_k := z_{start} + k \cdot \frac{z_{end} - z_{start}}{npts_z - 1}$$

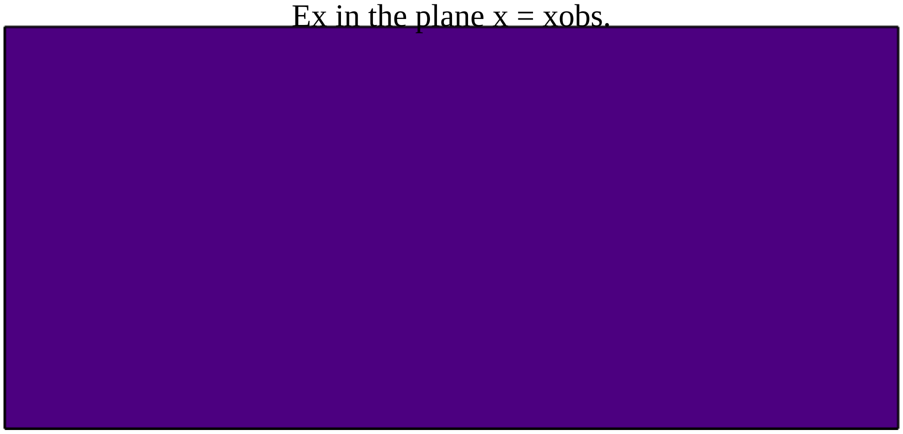
Compute the x component of **E** and the y and z components of **H** at the matrix of y_b and z_b points specified at each time instant:

$$exc_{k,j} := E_x\left(x_{obs}, yb_j, zb_k, time\right) \qquad hyc_{k,j} := H_y\left(x_{obs}, yb_j, zb_k, time\right)$$

$$hzc_{k,j} := H_z\left(x_{obs}, yb_j, zb_k, time\right)$$

Now create an animation clip of these **E** and **H** fields. For best results, in the "Animate" dialog box, choose To = 19 then save the file and replay the animation in a video player that supports continuous loop playback.

Ex in the plane x = xobs.



– y = b

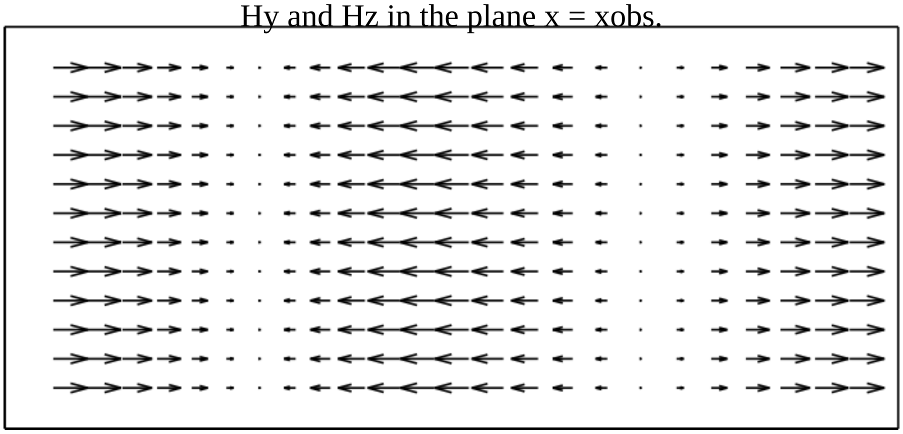
For the TE mode
$m = 1$, $n = 0$
at time (in periods, T_p)

$$\frac{\text{time}}{T_p} = 0.000$$

– y = 0

e_{xc}

Hy and Hz in the plane x = xobs.



– y = b

For the TE mode
$m = 1$, $n = 0$
at time (in periods, T_p)

$$\frac{\text{time}}{T_p} = 0.000$$

– y = 0

(h_{zc}, h_{yc})

| z = 0 | z = λ_z

Note: For the proper operation of this animation clip, the TM $m = 1$, $n = 0$ mode should be propagating. In other words, $\beta_z = 158.235$ should be a real number.

It is perhaps most interesting to view both of these field patterns in the same animation clip. This can be accomplished by dragging the dashed black selection box around both of the plots when generating the animation clip.

In the contour plot for E_x, the warmer colors indicate a positive value (E_x pointing out of the screen) with red showing a maximum positive value, while cooler colors indicate a negative value with deep blue showing the maximum negative value. For some values of n (which includes n = 0), E_x can be zero for all time in the plane x = a/2.

a/2.

For further experimentation, you may wish to change the mode index n at the beginning of this worksheet and observe what happens to the **E** and **H** field patterns shown in these last two plots. When incrementally increasing n, count the number of variations in the field patterns in the y direction. (Remember to increase the operating frequency f so that these higher-order modes propagate.) What is the relationship between the integer n and the number of variations in the field patterns is the y direction? Compare this relationship with that for the integer m and the number of variations in the field patterns in the x direction obtained in the **TM mode** worksheet.

End of worksheet.

