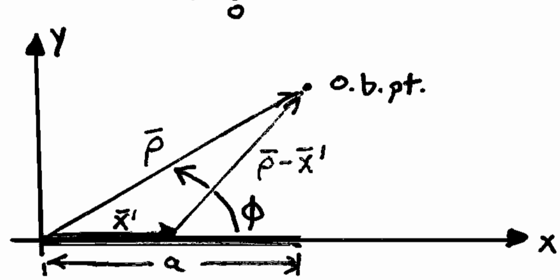
Far-field Calculations & Echo Width for 2-D problems

Once the TM^z and TE^z strip problems have been solved for the currents, we can calculate the total field everywhere in space.

Considering the TM^z scattering by a pec strip first →

$$E_z^s(\bar{\rho}) = -jk\eta A_z(\bar{\rho}) = -\frac{k\eta}{4}\int_0^a J_z(x')\, H_0^{(2)}(k|\bar{\rho}-\bar{x}'|)\, dx' \qquad (1)$$



For observation points far from the strip such that $k|\bar{\rho}-\bar{x}'| \to \infty$, the asymptotic form of the Hankel function

$$H_0^{(2)}(z) \sim \sqrt{\frac{j2}{\pi z}}\, e^{-jz}$$

with the approximations

$$|\bar{\rho}-\bar{x}'| \approx \begin{cases} \rho - x'\cos\phi & : \text{phase} \\ \rho & : \text{amplitude} \end{cases}$$

gives

$$H_0^{(2)}(k|\bar{\rho}-\bar{x}'|) \sim \sqrt{\frac{j2}{\pi k\rho}}\, e^{-jk(\rho - x'\cos\phi)}$$

$$= \sqrt{\frac{j2}{\pi k\rho}}\, e^{-jk\rho}\, e^{jkx'\cos\phi}$$

Substitution in (1)

$$\underline{\underline{E_z^s(\bar{\rho}) \sim -\frac{k\eta}{4}\sqrt{\frac{j2}{\pi k\rho}}\, e^{-jk\rho}\int_0^a J_z(x')\, e^{jkx'\cos\phi}\, dx'}} \qquad (2)$$

Eqn. 2 shows how the far field can be computed by a constant times the Fourier transform of the current. Since we used a pulse expansion for the current, the Fourier xform will be a sum of phase shifted sinc functions. (No numerical integration is required.)

The above analysis can be repeated for all three Cartesian components and generalized to give the far-field

$$\bar{E}^s(\bar{\rho}) \sim -\frac{k\eta}{4}\sqrt{\frac{j2}{\pi k\rho}}\, e^{-jk\rho} \int_{s'} \bar{J}(\bar{\rho}')\, e^{j\bar{k}\cdot\bar{\rho}'}\, ds'$$

$$\text{w/} \quad \bar{k}\cdot\bar{\rho}' = k\cos\phi\, x' + k\sin\phi\, y'$$

which can be used for the TE^z scattering problem, for example.

In the far field, $E_\rho \to 0$ leaving E_z & E_ϕ non-vanishing components only, for the 2-D problem.

In a manner similar to 3-D problems, one can define a Radar Cross Section for 2-D geometries as an RCS per unit length. For 2-D problems this is referred to as an <u>Echo Width</u> or 2-D RCS, $\sigma_{2\text{-}D}$. It is defined as -

> The <u>width</u> intercepting the amount of incident power which, when radiated by cylindrical omni-directional radiation, produces at the receiver the same power density as the original target.

Mathematically -
$$\lim_{\rho \to \infty} \left[\frac{\sigma_{2-D} P_i}{2\pi\rho} \right] = P_s$$

or

$$\sigma_{2-D} = \lim_{\rho \to \infty} \left[2\pi\rho \frac{P_s}{P_i} \right]$$

The definitions of monostatic and bistatic remain the same here for 2-D as for 3-D.

In decibels -

$$\sigma_{2-D} = 10 \log_{10} \left\{ \lim_{\rho \to \infty} 2\pi\rho \frac{P_s}{P_i} \right\}$$

in dbm or dB/λ.