It is possible to transport energy from one position to another using EM fields. No surprise: we experience this everyday through cell phones, radios, sat TV.

Poynting's theorem is the mathematical definition for the conservation of power in EM fields. With reference to a volume \( V \) surrounded by \( S \), Poynting's theorem is

\[
P_S = P_e + P_d + \frac{d}{dt}(W_e + W_m) \tag{1-692}, \tag{1}
\]

where

\( P_S \) - EM power supplied to \( V \) \( = -\int_V (\mathbf{E} \cdot \mathbf{J} + \mathbf{H} \cdot \mathbf{M}) \, dV \tag{1-59a}, \tag{2} \)

\( P_e \) - power leaving \( V \) through \( S \) \( = \oint_S (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} \tag{W}, \tag{1-59b}, \tag{3} \)

\( P_d \) - power dissipated in \( V \) \( = \int_V (\mathbf{E} \cdot \mathbf{J}) \, dV \tag{W} \tag{1-59c}, \tag{4} \)

\( W_e \) - Electric energy stored in \( V \) \( = \int_V \frac{1}{2} \varepsilon |\mathbf{E}|^2 \, dV \tag{W}, \tag{1-59d}, \tag{5} \)

\( W_m \) - Magnetic energy stored in \( V \) \( = \int_V \frac{1}{2} \mu |\mathbf{H}|^2 \, dV \tag{W} \tag{1-59b}, \tag{6} \)

In words, Poynting's theorem (1) states that the EM power supplied to a volume equals the sum of (1) the EM power leaving the volume, (2) the power dissipated in \( V \) (through an irreversible process), and (3) the time rate of change in the EM energy stored in \( V \).

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The quantity \( \mathbf{S} = \mathbf{E} \times \mathbf{H} \tag{Wm} \) is the Poynting vector, widely interpreted as the EM power density vector. This vector (often) indicates power flow, though there may be ambiguity in rare circumstances.

The correctness of (5) is not in doubt, as the total power leaving \( S \). It is the interpretation of \( \mathbf{S} \) on the intensity of energy flow at a point that may fail.
Indeed, we can add any vector field \( \overrightarrow{\mathbf{F}}' \) to \( \overrightarrow{\mathbf{F}} \) so that \( \oint_{C} \overrightarrow{\mathbf{F}}' \cdot d\mathbf{r} = 0 \) w/o changing (3). For example, if a vector field \( \overrightarrow{\mathbf{F}}' \) is solenoidal throughout \( V \), \( \nabla \cdot \overrightarrow{\mathbf{F}}' = 0 \) \( \forall \mathbf{F} \in \{V\} \) \( \Rightarrow \) \( \text{(7)} \)

Then, applying the divergence theorem

\[
\int_{V} [\nabla \cdot \overrightarrow{\mathbf{F}}(\mathbf{F})] dV = \oint_{S} \overrightarrow{\mathbf{F}}(\mathbf{F}) \cdot d\mathbf{S}
\]

\( \text{(8)} \)

Using (7) in (8) gives \( \oint_{S} \overrightarrow{\mathbf{F}}' \cdot d\mathbf{S} = 0 \) \( \text{(9)} \)

Therefore, the Divergence theorem at every point in \( V \)

is \( \overrightarrow{\mathbf{F}}(\mathbf{F}) + \overrightarrow{\mathbf{F}}'(\mathbf{F}) \), but the total power leaving \( S \)

is \( \oint_{S} \overrightarrow{\mathbf{F}}(\mathbf{F}) \cdot d\mathbf{S} \).
The vast majority of our work this semester will be with sinusoidally steady state solutions to ME's. We will assume an $e^{j\omega t}$ time dependence, and work with vector phasor fields that have only spatial dependence.

For example, we have
\[
E(t, r) = \Re \{ E(r) e^{j\omega t} \} \quad (1.6\text{a}), (10)
\]
and
\[
B(t, r) = \Re \{ B(r) e^{j\omega t} \} \quad (1.6\text{d}), (11)
\]

Then the point form of Faraday's law becomes
\[
\nabla \times \Re \{ E(r) e^{j\omega t} \} = -\frac{1}{c} \frac{d}{dt} \Re \{ B(r) e^{j\omega t} \}
\]

The operators $\Re \{ \} \times$ differentiation commute. This gives
\[
\Re \left\{ \nabla \times \left[ E(r) e^{j\omega t} \right] \right\} = -\Re \left\{ \frac{d}{dt} \left[ B(r) e^{j\omega t} \right] \right\}
\]

or
\[
\Re \left\{ \nabla E(r) e^{j\omega t} \right\} = -\Re \left\{ B(r) \frac{d}{dt} (e^{j\omega t}) \right\}
\]

Therefore,
\[
\Re \left\{ \nabla E(r) e^{j\omega t} \right\} = \Re \left\{ -j\omega B(r) e^{j\omega t} \right\} \quad (12)
\]

It can be shown that if $A \neq B$ are complex quantities and
\[
\Re \{ A e^{j\omega t} \} = \Re \{ B e^{j\omega t} \}
\]
for all $t$, then $A = B$.

Applying this lemma to (12) gives the phase form of Faraday's law.

\[
\nabla \times E(r) = -j\omega B(r) \quad (13)
\]

Table 1-4 in the text lists the phase form of ME's in both point & integral form.
<table>
<thead>
<tr>
<th>TABLE 1-4</th>
<th>Instantaneous and time-harmonic forms of Maxwell’s equations and continuity equation in differential and integral forms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential form</td>
<td></td>
</tr>
<tr>
<td>( \nabla \times \mathbf{\varepsilon} = -\mathbf{M}_t - \frac{\partial \mathbf{B}}{\partial t} )</td>
<td>( \nabla \times \mathbf{E} = -\mathbf{M}_t - j\omega \mathbf{B} )</td>
</tr>
<tr>
<td>( \nabla \times \mathbf{\mu} = \mathbf{J}_t + \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} )</td>
<td>( \nabla \times \mathbf{H} = \mathbf{J}_t + \mathbf{J}_c + j\omega \mathbf{D} )</td>
</tr>
<tr>
<td>( \nabla \cdot \mathbf{D} = q_{tv} )</td>
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</tr>
<tr>
<td>( \nabla \cdot \mathbf{B} = q_{mv} )</td>
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</tr>
<tr>
<td>( \nabla \cdot \mathbf{J}<em>c = -\frac{\partial q</em>{cv}}{\partial t} )</td>
<td>( \nabla \cdot \mathbf{J}<em>c = -j\omega q</em>{cv} )</td>
</tr>
<tr>
<td>Integral form</td>
<td></td>
</tr>
<tr>
<td>( \oint_{C} \mathbf{\varepsilon} \cdot d\mathbf{L} = -\iint_{S} \mathbf{M}<em>t \cdot ds - \frac{\partial}{\partial t} \iint</em>{S} \mathbf{B} \cdot ds )</td>
<td>( \oint_{C} \mathbf{E} \cdot d\mathbf{L} = -\iint_{S} \mathbf{M}<em>t \cdot ds - j\omega \iint</em>{S} \mathbf{B} \cdot ds )</td>
</tr>
<tr>
<td>( \oint_{C} \mathbf{\mu} \cdot d\mathbf{L} = \iint_{S} \mathbf{J}<em>t \cdot ds + \iint</em>{S} \mathbf{J}<em>c \cdot ds + \frac{\partial}{\partial t} \iint</em>{S} \mathbf{D} \cdot ds )</td>
<td>( \oint_{C} \mathbf{H} \cdot d\mathbf{L} = \iint_{S} \mathbf{J}<em>t \cdot ds + \iint</em>{S} \mathbf{J}<em>c \cdot ds + j\omega \iint</em>{S} \mathbf{D} \cdot ds )</td>
</tr>
<tr>
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</tr>
<tr>
<td>( \sum_{S} \mathbf{J}_c \cdot ds = -\frac{\partial \mathcal{Q}_c}{\partial t} )</td>
<td>( \sum_{S} \mathbf{J}_c \cdot ds = -j\omega \mathcal{Q}_c )</td>
</tr>
</tbody>
</table>
In electrical engineering, the time dependence $e^{-j\omega t}$ is very commonly used. However, in physics it is common to use the dependence $e^{j\omega t}$. Faraday's law can then be written as

$$\nabla \times \vec{E}(r) = j\omega \vec{B}(r)$$

To convert results in a physics text or paper with this convention, make the substitution $i \leftrightarrow -j$.

Occasionally, you will come across a paper using $e^{-j\omega t}$ or $e^{j\omega t}$ convention. Need to be careful when using results from unfamiliar authors.