Whites
EE692 AEE

Lecture 16 – Low Reflection, Multi-Layer Slab Design

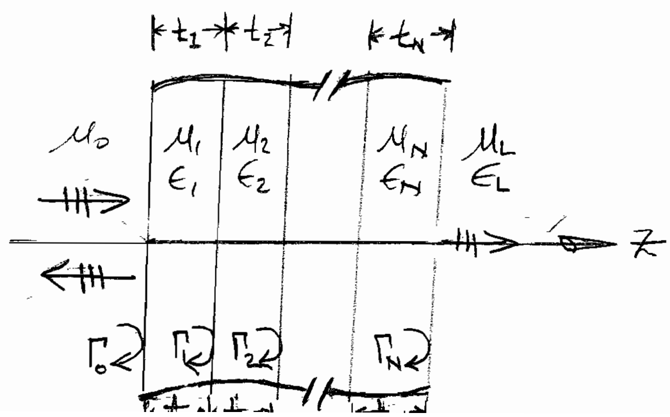
We will now apply some of the principles we've learned in the previous two lectures to the design of multi-layered slab structures with the intention of producing a small reflection over a desired frequency bandwidth.

There are a number of approaches. The most accurate would be to use the exact slab scattering equations, but these would quickly become too complicated for an analytical soln. Another approach would be to use these exact equations with a numerical minimization algorithm such as genetic algorithms (GA), simulated annealing (SA), or another.

Could also use TLs in ADS & the built-in optimization.

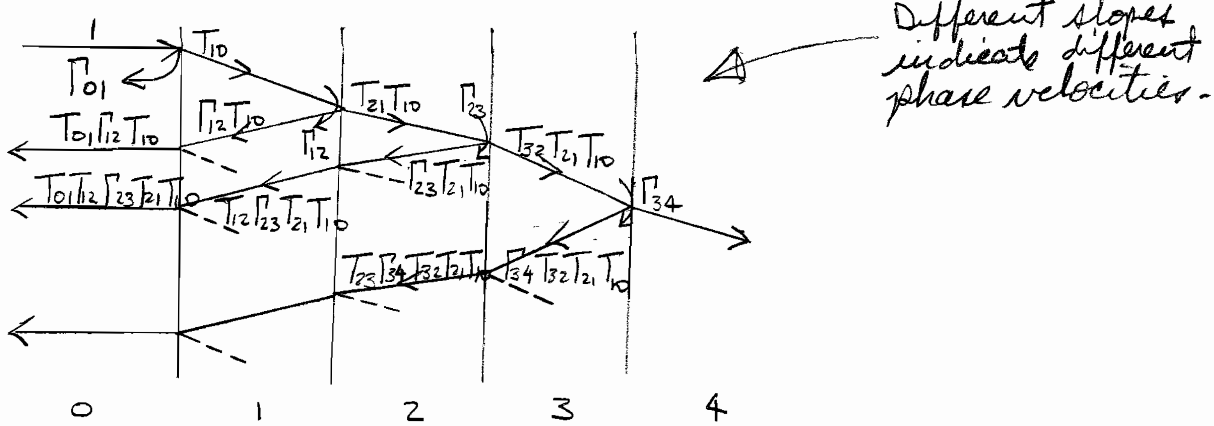
Here, we will use an approximate approach that works quite well when the reflections are small, which is what we want.

The geometry is that shown in Fig 5-20 of the text:



Our task is to choose the # of layers, their material parameters, & their thicknesses to provide a "small" reflection at the front face. That is, we wish to design a multi-section impedance transformer (See Pozar, Sect 5.5).

To a first approximation, the ref. coeff at the front face is due to the primary wave reflections from each of the interfaces. A ladder diagram is helpful:



If the reflections are small, then to a first-order approx:

$$\Gamma_{in} \approx \Gamma_{01} + T_{01}\Gamma_{12}T_{10}\,e^{-j2\phi_1} + T_{01}T_{12}\Gamma_{23}T_{21}T_{10}\,e^{-j2(\phi_1+\phi_2)} + \cdots \tag{1}$$

Since the reflections are small, then these partial transmission coeff will be ≈ 1. Consequently,

$$\Gamma_{in} \approx \Gamma_0 + \Gamma_1 e^{-j2\phi_1} + \Gamma_2 e^{-j2(\phi_1+\phi_2)} + \cdots \tag{2}$$

where will label the partial ref coeffs as Γ_i where

$$\Gamma_i \equiv \Gamma_{i,i+1} = \frac{\eta_{i+1} - \eta_i}{\eta_{i+1} + \eta_i} \tag{3}$$

To make this design process tractable, will assume:

(i) all slabs are one-quarter wavelength long @ the design freq (though these will likely be of different physical length)

(ii) the structure is symmetric s.t.

$$\Gamma_0 = \Gamma_N, \quad \Gamma_1 = \Gamma_{N-1}, \quad \cdots$$

Hence, because of (i):

$$\Gamma_{in} \approx \Gamma_0 + \Gamma_1 e^{-j2\phi} + \Gamma_2 e^{-j4\phi} + \cdots + \Gamma_{N-1} e^{-j2(N-1)\phi} + \Gamma_N e^{-j2N\phi} \qquad (5\text{-}74), (4)$$

where $$\phi = \beta_n t_n = \frac{2\pi}{\lambda_n} \cdot \left(\frac{\lambda_{n,0}}{4}\right) = \frac{\pi}{2}\left(\frac{f}{f_0}\right) \qquad (5\text{-}74b), (5)$$

and f_0 is the design frequency. At the design frequency $f = f_0$, $\phi = \frac{\pi}{2}$.

Because of (ii), we can express (4) as

$$\Gamma_{in} \approx \Gamma_0 \left[1 + e^{-j2N\phi}\right] + \Gamma_1 \left[e^{-j2\phi} + e^{-j2(N-1)\phi}\right] + \cdots \qquad (6)$$

This sum has two forms depending if the number of layers is an even number or odd. We can write (6) as

$$\Gamma_{in} \approx e^{-jN\phi} \left\{ \Gamma_0 \left[e^{+jN\phi} + e^{-jN\phi}\right] + \Gamma_1 \left[e^{j(N-2)\phi} + e^{-j(N-2)\phi}\right] + \text{Term} \right\} \qquad (7)$$

where $$\text{Term} = \begin{cases} \Gamma_{(N-1)/2}\left(e^{j\phi} + e^{-j\phi}\right) & N \text{ odd} \quad (\text{even \# layers}) \\ \Gamma_{N/2} & N \text{ even} \end{cases}$$

Applying the cosine identity

$$\Gamma_{in}(f) \approx 2e^{-jN\phi}\left\{\Gamma_0 \cos[N\phi] + \Gamma_1 \cos[(N-2)\phi] + \cdots + \Gamma_n \cos[(N-2n)\phi] + \cdots + \text{Term}\right\} \quad (5\text{-}75), (8)$$

where

$$\text{Term} = \begin{cases} \Gamma_{(N-1)/2} \cos\phi & N \text{ odd (even\# layers)} \\ \frac{1}{2}\Gamma_{N/2} & N \text{ even} \end{cases} \quad (9)$$

For symmetrical transformer. One way to look at (8) is a Fourier cosine series expansion for the frequency variation of $\Gamma_{in}(f)$. This is true provided Γ_{in} is a real function (lossless media). The fundamental period in ϕ of the periodic variation of Γ_{in} in (8), using (5), is:

$$\cos[(N-2n)\phi] \Longrightarrow T_\phi = \frac{2\pi}{\omega_\phi} = \frac{2\pi}{2} = \pi$$

This corresponds to the frequency range over which the thickness of each slab changes by $\lambda/2$.

So, by properly choosing the Γ_n in (8) we can create a desired frequency variation in Γ_{in}, which was presumed to be small. We will consider two popular choices next.

However, it is instructive to compare this design of multi-layer slab structure with the design of multi-section quarter-wave matching transformers in microwave engineering. They're identical! (See Pozar, 3rd, sec. 5.5 – 5.7.)

Binomial Design

In the binomial, (or maximally flat) design, the variation of $\Gamma_{in}(f)$ wrt f near the design frequency is as small as possible. This occurs when the first N-1 derivatives of $|\Gamma_{in}|$ vanish, where N is the # of slab layers.

in (8)

This type of response can be realized if we choose

$$\Gamma_{in}(f) = A\,(1+e^{-j2\phi})^{N} \tag{10}$$

where
$$A = 2^{-N}\,\frac{\eta_L-\eta_0}{\eta_L+\eta_0} \tag{11}$$

It can be shown that this maximally flat design requires that the Γ_n in (8) be:

$$\underline{\underline{\Gamma_n = A\,C_n^N}} \qquad (5\text{-}77),(12)$$

where C_n^N are the binomial coefficients given by

$$C_n^N = \frac{N!}{(N-n)!\,n!} \qquad n=0,1,2,\cdots,N \qquad (5\text{-}76a),(13)$$

Because this is a symmetrical structure,

$$C_n^N = C_{N-n}^N$$

Special values of the binomial coefficients include (recall that $0! = 1! = 1$):

- $C_0^N = \dfrac{N!}{(N-0)!\,0!} = \dfrac{N!}{N!} = 1$
- $C_1^N = \dfrac{N!}{(N-1)!\,1!} = \dfrac{N!}{(N-1)!} \cdot \dfrac{N}{N} = N \cdot \dfrac{N!}{N!} = N$
- $C_{N-1}^N = C_1^N = N$

With these Γ_n coefficients specified in (12), we're now in a position to determine the necessary intrinsic impedances of every slab layer. This can be accomplished using (3), presuming η_0 &/or η_L are specified. This completes the analysis of the binomial design approximate transformer.

Because the impedances are relat. (to the binomial coeffs through (1)), this is called a binomial transformer design. This theory is approximate: the range of η_L is restricted to approximately

$$\frac{\eta_0}{2} \lesssim \eta_L \lesssim 2\eta_0 \qquad (14)$$

As given in the text, the fractional bandwidth for this binomial design is given as

$$\frac{\Delta f}{f_0} \approx 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{\Gamma_m}{(\eta_L-\eta_0)/(\eta_L+\eta_0)}\right]^{1/N} \qquad (5\text{-}80),\ (15)$$

where Γ_m is the maximum ref. coef. magnitude desired anywhere within the passband. One application of Eqn (15) is it can be used to determine the number of sections (N) needed to achieve a desired maximum ref. coff mag (Γ_m) over the bandwidth Δf centered @ f_0.

[Pozar Fig. 5.15]

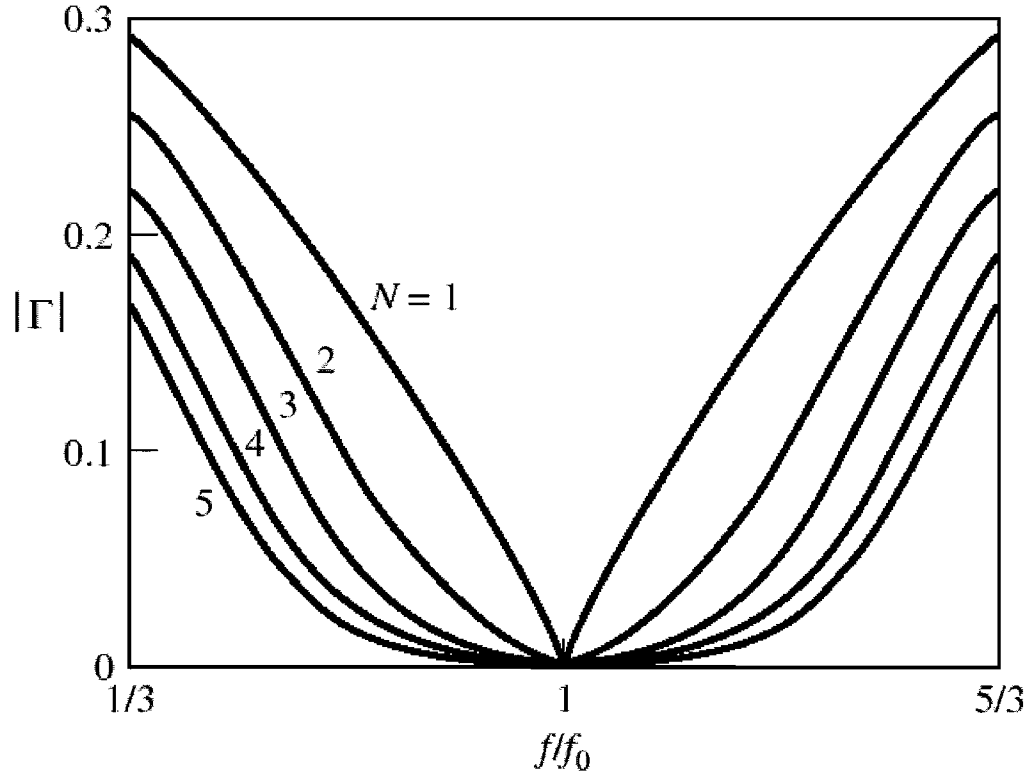


Figure 5.15 (p. 250)

Reflection coefficient magnitude versus frequency for multisection binomial matching transformers of Example 5.6 Z_L = 50Ω and Z_0 = 100Ω.

Tschebyscheff (Chebyshev) Design

This design provides more bandwidth than the binomial transformer (with an equal number of layers) by increasing the amount of "ripple" of Γ_{in} in the passband.

This design is accomplished by equating the coeffs Γ_n in (8) w/ a Tschebyscheff polynomial, $T_n(\cos\phi)$.

$$T_1(x) = x$$
$$T_2(x) = 2x^2 - 1$$
$$T_3(x) = 4x^3 - 3x$$
$$T_4(x) = 8x^4 - 8x^2 + 1$$
$$T_n(x) = 2x T_{n-1} - T_{n-2}$$ ← recurrence relation (numerically stable?)

These polynomials oscillate between ±1 for x in the range $|x|<1$. Outside this range they increase in magnitude w/o bound.

[Pozar Fig 5.16]

It can be shown that the coeffs Γ_n in the approximate $\Gamma_{in}(f)$ in (8) should (somehow) be chosen s.t.

$$\Gamma_{in}(f) \approx e^{-jN\phi} \frac{\eta_L - \eta_0}{\eta_L + \eta_0} \frac{T_N(\sec\phi_m \cos\phi)}{T_N(\sec\phi_m)} \qquad (5\text{-}81), (16)$$

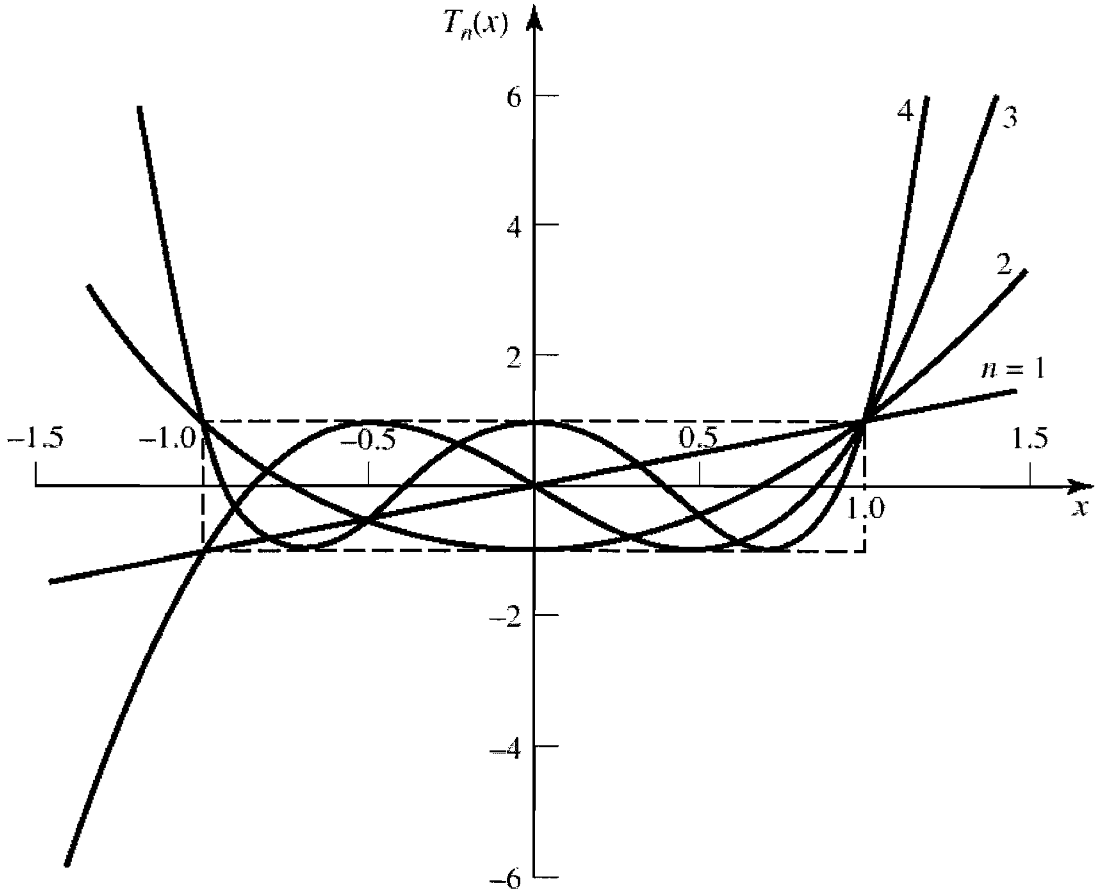


Figure 5.16 (p. 251)
The first four Chebyshev polynomials $T_n(x)$.

where ϕ_m ($\div \pi - \phi_m$) are the edges of the passband when $T_n(x) = \pm 1$. At these frequencies,

$$\Gamma_m = \left| \frac{\eta_L - \eta_0}{\eta_L + \eta_0} \; \frac{1}{T_N(\sec\phi_m)} \right| \qquad (5\text{-}82), (7)$$

There are no simple formulas for Γ_n in this design, though they can be determined by hand. (Assigned in your homework)