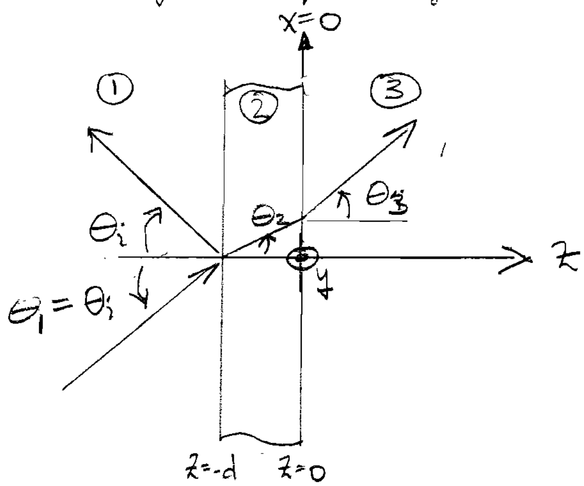
Lecture 14 - UPW Scattering by a Single Slab Layer

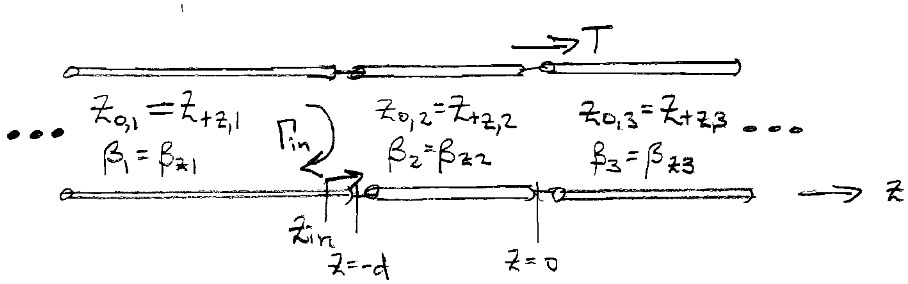
There are many practical applications where adding layers of materials to a surface enhances its performance. For example, low reflection coatings in optics are used to enhance transmission through lenses.

For the next two lectures, we will examine this problem by modeling with UPWs incident on planar, infinite material slabs. We'll begin w/ a single slab in this lecture followed by multiple layers in the next.



By phase match, if materials 1 & 3 are the same, we can see that $\theta_1 = \theta_3$! Neat.

It is very useful to use the TL models for the UPWs propagating in these materials:



$$\text{E pol:} \quad Z_{+z1} = \frac{\eta_1}{\cos\theta_i}, \quad Z_{+z2} = \frac{\eta_2}{\cos\theta_2}, \quad Z_{+3} = \frac{\eta_3}{\cos\theta_3} \qquad (1)$$

$$\text{H pol:} \quad Z_{+z1} = \eta_1 \cos\theta_i, \quad Z_{+z2} = \eta_2 \cos\theta_2, \quad Z_{+3} = \eta_3 \cos\theta_3 \qquad (2)$$

To determine the reflection coeff Γ_{in}, we need to first find the <u>total</u> wave impedance at the front face Z_{in}:

- E pol: $$Z_{in} = -\left.\frac{E_{y1}}{H_{x1}}\right|_{z=-d^-} = -\left.\frac{E_{y2}}{H_{x1}}\right|_{z=-d^+} \quad (3)$$
- H pol: $$Z_{in} = \left.\frac{E_{x1}}{H_{y1}}\right|_{z=-d^-} = \left.\frac{E_{x2}}{H_{y2}}\right|_{z=-d^+} \quad (4)$$

Then $$\Gamma_{in} = \frac{Z_{in} - Z_{+z,1}}{Z_{in} + Z_{+z,1}} \quad (5)$$

We can determine Z_{in} using methods we've learned for TLs. One way is to apply the common Z_{in} expression

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \quad (6)$$

For the single slab problem above,

$$Z_0 \Rightarrow Z_{+z,2}, \quad Z_L = Z_{+z,3}, \quad \beta l = \beta_2 d$$

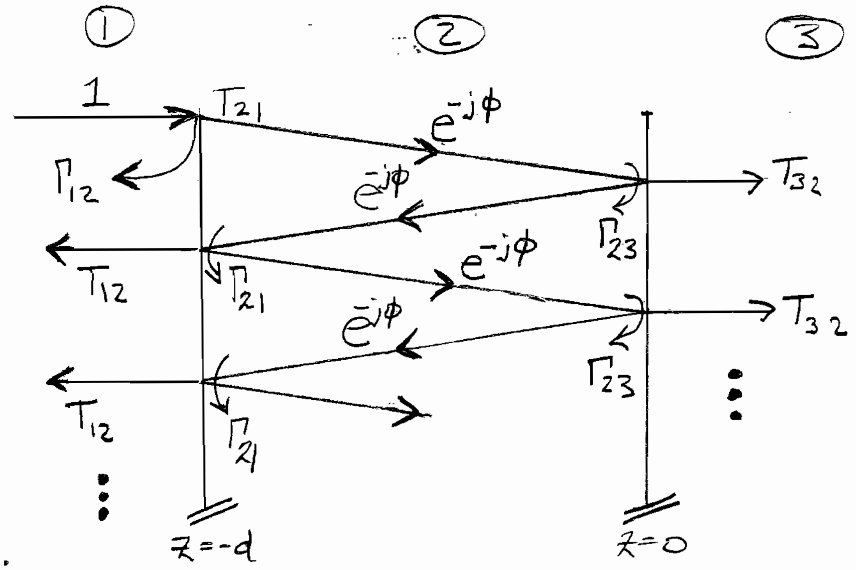
s.t.

$$Z_{in} = Z_{+z,2} \frac{Z_{+z,3} + jZ_{+z,2} \tan(\beta_2 d)}{Z_{+z,2} + jZ_{+z,3} \tan(\beta_2 d)} \quad (7)$$

using (7) in (5), we can compute Γ_{in}.

While this approach produces accurate solutions, there is another way to construct solutions for Γ_{in} & T that is more physically insightful as well as easily expandable to multiple layers.

We'll consult the ladder diagram below to explicitly account for the multiple scattering effects which sum to produce the ~~ov~~overall response. (Pozar, Microwave Engineering, 3rd ed., p. 244 - 246).



where $\phi = \beta_2 d$.

The coeffs Γ_{ij} & T_{ij} are "intrinsic" or "partial" coeffs. In other words, the Fresnel coeffs for that interface. page 3A

The total input reflection coeff at the front face ($z=-d$) is

$$\Gamma_{in}(z=-d^-) = \Gamma_{12} + T_{21} e^{-j\phi} \Gamma_{23} e^{-j\phi} T_{12} +$$

$$+ \left(T_{21} e^{-j\phi} \Gamma_{23} e^{-j\phi}\right) \Gamma_{21} e^{-j\phi} \Gamma_{23} e^{-j\phi} T_{12} + \cdots$$

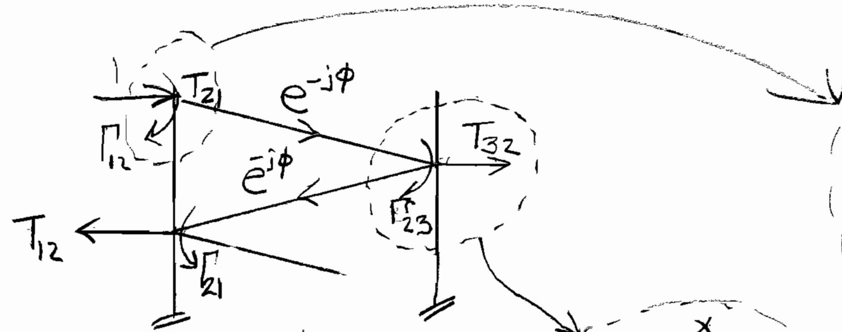
$$= \Gamma_{12} + T_{21} T_{12} \Gamma_{23} e^{-j2\phi} + T_{21} T_{12} \Gamma_{21} \Gamma_{23}^2 e^{-j4\phi} + \cdots$$

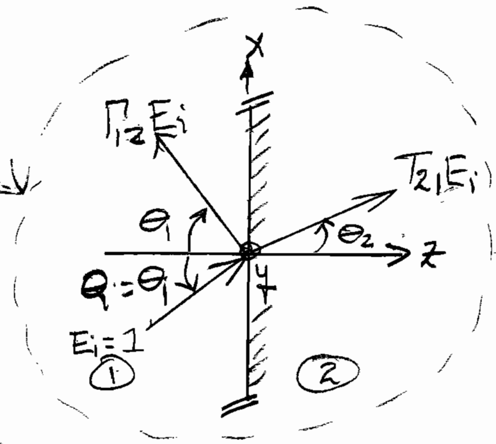
$$= \Gamma_{12} + T_{21} T_{12} \Gamma_{23} e^{-j2\phi} \cdot \sum_{n=0}^{\infty} \Gamma_{21}^n \Gamma_{23}^n e^{-j2n\phi} \qquad (8)$$

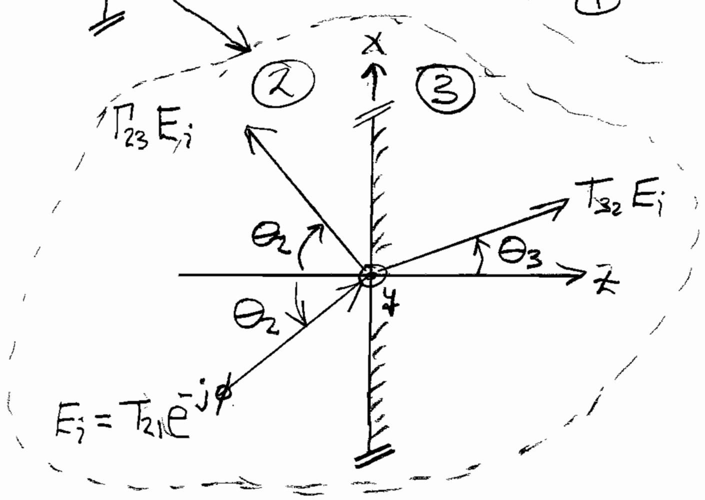
The summation in (8) is a geometric series

In other words, these are ref. & trans coeffs for a half space and depend on the wave polarization for obliquely incident UPWS.

Let's consider these in a bit more detail.







Back to p3

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{for } |x|<1 \qquad (9)$$

Applying (9), then (8) becomes

$$\Gamma_{in} = \Gamma_{12} + \frac{T_{21} T_{12} \Gamma_{23} e^{-j2\phi}}{1 - \Gamma_{21} \Gamma_{23} e^{-j2\phi}} \qquad |\Gamma_{21}\Gamma_{23}| < 1 \qquad (10)$$

[Plots]

Also think of this as multiple scattering by the two interfaces – leads to $\frac{1}{1-(\ )}$ factor.