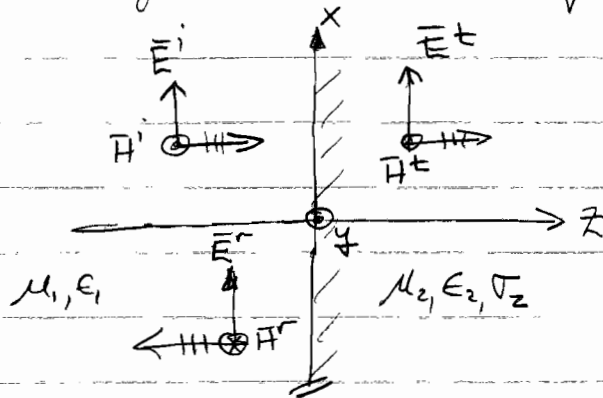


in our plane wave studies that

To this point, we've assumed both half spaces were lossless. To one extent or another, all materials are lossy so it will be useful for us to consider the effects of loss.

9. We'll accomplish this <sup>task</sup> by studying UPW illumination of a lossy half space. In particular, region 1 will be lossless & region 2 will be lossy:



There are multiple sources for energy loss of an EM wave. Here we're considering just electric conduction. This energy loss causes the EM <sup>wave</sup> amplitude to decrease as it propagates through the lossy material, converting it to heat.

As you saw in Ch. 3 and in your homework, the wave equation for  $\bar{E}$  is

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0 \quad (3-37), (1)$$

where  $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$  (3-37d), (2)

s.t.  $\gamma = \pm \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \pm (\alpha + j\beta)$  (3-37e), (3)  
 solution for  $\alpha + j\beta$  given in (4-28)

... This leads to solutions of the form  $e^{\mp \gamma_2 z}$ .

... Specifically, for the geometry shown above:

...  $\vec{E}^i = \hat{x} E_0 e^{-j\beta_1 z}$ ,  $\vec{E}^r = \hat{x} E_0 \Gamma^b e^{+j\beta_1 z}$ ,  $\vec{E}^t = \hat{x} E_0 T^b e^{-\alpha_2 z} e^{-j\beta_2 z}$  (4) (5-47)

... and

...  $\vec{H}^i = \hat{y} \frac{E_0}{\eta_1} e^{-j\beta_1 z}$ ,  $\vec{H}^r = -\hat{y} \frac{E_0 \Gamma^b}{\eta_1} e^{+j\beta_1 z}$ ,  $\vec{H}^t = \hat{y} \frac{E_0 T^b}{\eta_2} e^{-\alpha_2 z} e^{-j\beta_2 z}$  (5) (5-47)

... The coeffs  $\Gamma^b$  &  $T^b$  will now be complex <sup>numbers</sup> as will

...  $\eta_2$ ,  $\alpha_2$  &  $\beta_2$  are real (positive) numbers

... However,

... Enforcing b.c.'s @ the interface, we find  $\Gamma^b$  &  $T^b$  are the same as for lossless materials:

$$\Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad ; \quad T^b = \frac{2\eta_2}{\eta_2 + \eta_1} \quad (5-50), (6)$$

... So, there is not much difference to these solutions than lossless case.

### Skin Depth

... One consequence of <sup>material</sup> loss is the EM field decays exponentially as we see in  $\vec{E}^t$  &  $\vec{H}^t$  in (4) & (5).

... Since  $\vec{J}_c = \sigma \vec{E}$ , then for the half space geometry in the previous figure

...  $\vec{J}_{c2}(z) = \sigma_2 \vec{E}^t = \hat{x} \sigma_2 E_0 T^b e^{-\alpha_2 z} e^{-j\beta_2 z}$  (7)

Considering the magnitude of this current density

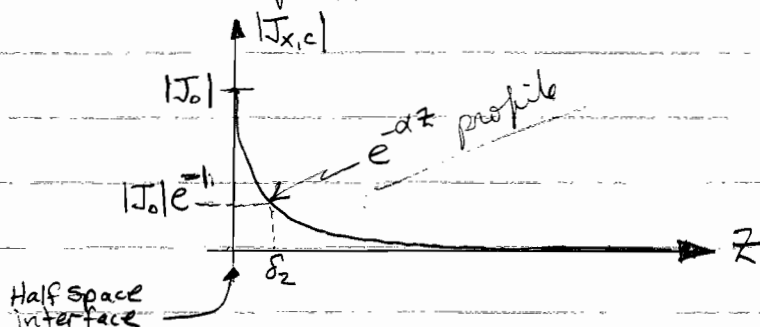
$$|\mathbf{J}_{x,c,2}(z)| = |E_0 T^b e^{-\alpha_2 z} e^{-j\beta_2 z}| = |E_0 T^b e^{-\alpha_2 z}| \quad (8)$$

If we call  $J_0 = |\mathbf{J}_{x,c,2}(z=0)| \stackrel{(8)}{=} |E_0 T^b|$  (9)

then we can express (8) as

$$|\mathbf{J}_{x,c}(z)| = |J_0| e^{-\alpha_2 z} \quad (10)$$

This current density appears as



The distance into the conductor at which the magnitude of  $J_0$  has been reduced by the factor  $e^{-1}$  is called the skin depth,  $\delta$ , mentioned briefly in lecture 6:

$$\delta_2 = \frac{1}{\alpha_2} = \left\{ \omega \sqrt{\mu \epsilon} \left[ \frac{1}{2} \left( \sqrt{1 + \left( \frac{\sigma_2}{\omega \epsilon_2} \right)^2} - 1 \right) \right]^{1/2} \right\}^{-1} \quad [\text{m}] \quad (11), (34)$$

If region 2 is a "good" conductor ( $\sigma_2 \gg \omega \epsilon_2$ )

$$\delta_2 \approx \sqrt{\frac{2}{\omega \mu \sigma}} \quad [\text{m}] \quad (4-43), (12)$$

... This skin depth can be very small for metals  
 ... at high frequencies. For example, consider Au  
 ... w/  $\sigma = 4.1 \times 10^7$  ( $\mu = \mu_0$ ):

$f$	$\delta \approx \sqrt{\frac{1}{\pi f \mu_0 \sigma}}$
10 MHz	24.9 $\mu\text{m}$
100 MHz	7.86 $\mu\text{m}$
1 GHz	2.49 $\mu\text{m}$
10 GHz	0.79 $\mu\text{m}$
30 GHz	0.45 $\mu\text{m}$

... For  $z \gg 5\delta_2$ , the EM field can be considered essentially  
 ... zero.

... A physical interpretation for skin depth is illustrated  
 ... in the text on p. 209:

The integral over  
 all current density  
 magnitude is:

$$|J_S| \equiv \int_0^{\infty} |J_{x,c}| dz = \int_0^{\infty} |J_0| e^{-z/\delta_2} dz = -\delta_2 |J_0| e^{-z/\delta_2} \Big|_0^{\infty}$$

$$\therefore |J_S| \equiv \delta_2 |J_0| \quad (13)$$

... In other words, the magnitude of the <sup>surface</sup> current density  
 ... per unit width is equal to that at the surface  
 ... considered constant to a distance  $\delta_2$ . Interesting.