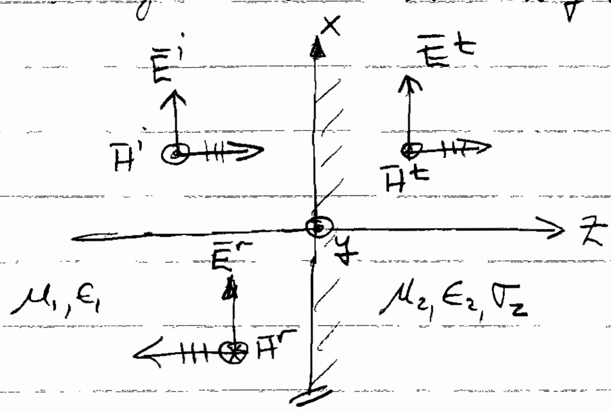
Whites
EE692 AEE

Lecture 13 – UPW Incident on Lossy Media. Skin Depth

To this point, we've assumed in our plane wave studies that both half spaces were lossless. To one extent or another, all materials are lossy so it will be useful for us to consider the effects of loss.

¶ We'll accomplish this task by studying UPW illumination of a lossy half space. In particular, region 1 will be lossless & region 2 will be lossy:



There are multiple sources for energy loss of an EM wave. Here we're considering just electric conduction. This energy loss causes the EM wave amplitude to decrease as it propagates through the lossy material, converting it to heat.

As you saw in Ch. 3 and in your homework, the wave equation for $\bar{E}$ is

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0 \qquad (3\text{-}37),(1)$$

where $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ (3-37d), (2)

s.t. $\gamma = \pm\sqrt{j\omega\mu(\sigma+j\omega\epsilon)} = \pm(\alpha + j\beta)$ (3-37e), (3).

Solutions for α & β given in (4-28)

...This leads to solutions of the form $e^{\mp \gamma_2 z}$.
...Specifically, for the geometry shown above:

$$\bar{E}^i = \hat{x} E_0 e^{-j\beta_1 z}, \quad \bar{E}^r = \hat{x} E_0 \Gamma^b e^{+j\beta_1 z}, \quad \bar{E}^t = \hat{x} E_0 T^b e^{-\alpha_2 z} e^{-j\beta_2 z} \tag{5-47), (4}$$

...and

$$\bar{H}^i = \hat{y} \frac{E_0}{\eta_1} e^{-j\beta_1 z}, \quad \bar{H}^r = -\hat{y} \frac{E_0 \Gamma^b}{\eta_1} e^{+j\beta_1 z}, \quad \bar{H}^t = \hat{y} \frac{E_0 T^b}{\eta_2} e^{-\alpha_2 z} e^{-j\beta_2 z} \tag{5-47), (5}$$

...The coeffs Γ^b & T^b will now be complex numbers as will
... η_2 - α_2 & β_2 are real (positive) numbers
... However,

...

...Enforcing b.c.'s @ the interface, we find Γ^b & T^b are
...the same as for lossless materials:

$$\Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \& \quad T^b = \frac{2\eta_2}{\eta_2 + \eta_1} \tag{5-50), (6}$$

...So, there is not much difference to these solutions than
...lossless case.

Skin Depth

...One consequence of material losses is the EM field decays
...exponentially as we see in $\bar{E}^t$ & $\bar{H}^t$ in (4) & (5).

...Since $\bar{J}_c = \sigma \bar{E}$, then for the half space geometry
...in the previous figure

$$\bar{J}_{c,2}(z) = \sigma_2 \bar{E}^t = \hat{x}\, \sigma_2 E_0 T^b e^{-\alpha z} e^{-j\beta z} \tag{7}$$

Considering the magnitude of this current density

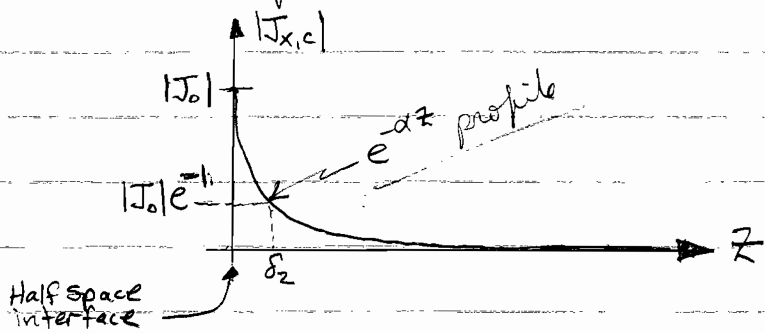
$$\left|J_{x,c,2}(z)\right| = \left|E_0 T^b e^{-\alpha_2 z} e^{-j\beta_2 z}\right| = \left|E_0 T^b e^{-\alpha_2 z}\right| \quad (8)$$

If we call $\quad J_0 = \left|J_{x,c,2}(z=0)\right| \underset{(8)}{=} \left|E_0 T^b\right| \quad (9)$

then we can express (8) as

$$\left|J_{x,c}(z)\right| = \left|J_0\right| e^{-\alpha_2 z} \quad (10)$$

This current density appears as



The distance into the conductor at which the magnitude of $\bar{J}_c$ has been reduced by the factor e^{-1} is called the <u>skin depth</u>, δ, mentioned briefly in lecture 6:

$$\delta_2 = \frac{1}{\alpha_2} = \left\{\omega\sqrt{\mu\epsilon}\left[\frac{1}{2}\left(\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2} - 1\right)\right]^{1/2}\right\}^{-1} \text{[m]} \quad (11),\ (4\text{-}34)$$

If region 2 is a "good" conductor ($\sigma_2 \gg \omega\epsilon_2$)

$$\delta_2 \approx \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{[m]} \quad (4\text{-}43),\ (12)$$

This skin depth can be very small for metals at high frequencies. For example, consider Au w/ $\sigma = 4.1 \times 10^7$ $(\mu = \mu_0)$:

f	$\delta \approx \sqrt{\frac{1}{\pi f \mu_0 \sigma}}$
10 MHz	24.9 μm
100 MHz	7.86 μm
1 GHz	2.49 μm
10 GHz	0.79 μm
30 GHz	0.45 μm

For $z \gtrsim 5\delta_z$, the EM field can be considered essentially zero.

A physical interpretation for skin depth is illustrated in the text on p. 209:

The integral over all current density magnitude is:

$$|J_S| \equiv \int_0^\infty |J_{x,c}|\, dz = \int_0^\infty |J_0|\, e^{-z/\delta_z}\, dz = -\delta_z |J_0|\, e^{-z/\delta_z}\Big|_0^\infty$$

$$\therefore \quad |J_S| \equiv \delta_z |J_0| \qquad (13)$$

In otherwords, the magnitude of the entire current density per unit width is equal to that at the surface considered constant to a distance δ_z. Interesting.