For certain $\theta$'s of incidence, it is possible for $n^b_1$ to become 1, even though not at grazing incidence. We observe this behavior when $\varepsilon_1 > \varepsilon_2$ $(\mu_1 = \mu_2)$ in the results of lecture 9.

Odd behavior since: $\sqrt{|n^b_1|} = 1$ and $\times n^b_1 \neq 0, 180^\circ$

$|n^b_1|$ ranges from 2 to 0 as $\theta$ $\rightarrow$ 90$^\circ$ (grazing).

So, the wave is totally reflected, but $|n^b_1| \neq 0$ which seems to imply that fields are not zero in region 2. Strange!

We'll investigate this phenomenon in much detail.

In lecture 9, showed $\theta^c$ vs. $\theta$ for $\varepsilon_1 = 4$, $\varepsilon_2 = 1$ $(\mu_1 = \mu_2 = 1)$.

At approx. 30$^\circ$, $\theta^c$ becomes 90$^\circ$. For larger $\theta$, $\theta^c$ becomes imaginary!! What does that mean? See soon.

Compute $\theta^c$ when $\theta = 90^\circ$. From Snell's law of refraction

$$\theta^c = \sin^{-1}\left(\frac{n_2}{n_1} \sin \theta\right)$$

For $\theta = \frac{\pi}{2}$

$$\sin\left(\frac{\pi}{2}\right) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \sin \theta^c \quad \text{called critical angle of incidence}$$

so

$$\theta^c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

$\theta^c$ is the argument of the $\sin^{-1}$ function, cannot exceed $\pi$ (for real $\theta^c$).

Then this requires for the existence of a critical $\theta^c$ that

$$M_2 \varepsilon_2 \leq M_1 \varepsilon_1$$

Now, if $\mu_1 = \mu_2$, then in order for a critical $\theta^c$ to exist $\varepsilon_1 < \varepsilon_2$. In other words, a critical $\theta^c$ of incidence requires that EM waves traveling from more "dense" EM medium to a less dense one.
Can easily show that

\[ \Gamma_{\perp}^{b} = 1 \quad \Gamma_{\perp}^{b} = 2 \] \hspace{1cm} (3) \hspace{1cm} (4)
\[
\Theta_i \neq \Theta_c \quad \Theta_i \neq \Theta_c
\]

For the example results at the end of this lecture, \( \frac{\epsilon_1}{\epsilon_2} = 4 \) and \( \frac{\mu_1}{\mu_2} = 1 \).

From (2),

\[ \Theta_c = \sin^{-1} \left( \frac{1}{4} \right) = \sin^{-1} \left( \frac{1}{2} \right) = 30^\circ \]

From the plot, \( |\Gamma_{\perp}^{b}| = 1 \) @ \( \Theta_i = 30^\circ \) as predicted. Also, \( |\Gamma_{\perp}^{b}| = 2 \times |\Gamma_{\perp}^{b}| \) where \( \times |\Gamma_{\perp}^{b}| = 0 \)

This is a very interesting behavior. We also see that \( |\Gamma_{\perp}^{b}| = 1 \)

for \( \Theta_i > \Theta_c \). What is happening here? To understand this,

we take a look at the plot of \( \Theta_c \). \( \Theta_c \) increases rapidly from

0 @ \( \Theta_i = 0 \) to 90° @ \( \Theta_i = 30^\circ = \Theta_c \). At the critical \( \chi_c \) of incidence,

\[ \Theta_c = 90^\circ \] as we know. Now, what happens when \( \Theta_i > \Theta_c \)?

The argument of \( \sin \chi_i > 1 \), so there is no longer a real angle of

transmission. Instead, \( \Theta_c \) becomes a complex angle.

What does that mean? No geometrical interpretation. Rather, it is much more meaningful to work directly with the wavenumber

\( \beta^\perp \). As we saw earlier,

\[ \beta_x^\perp = \beta_x \] \hspace{1cm} (4)
From (8) & (13), we can deduce that

\[ \Theta_x > 0, \quad \Theta_y = 0 \]

so that \((\Theta_x \neq 0)\), as that \((\Theta_y = 0)\)

From (8)

\[ E_t = \frac{\Theta x}{\Theta_y} E_0 \exp(-j \beta (x + \frac{x^2}{2} \frac{\Theta y}{\Theta x})) \]

From (7)

\[ \Theta_x = \frac{\Theta x}{\Theta_y} E_0 \exp(-j \beta (x + \frac{x^2}{2} \frac{\Theta y}{\Theta x})) \]

From (8)

\[ \Theta_y = 0 \]

From (7)

\[ E_t = \frac{\Theta x}{\Theta_y} E_0 \exp(-j \beta (x + \frac{x^2}{2} \frac{\Theta y}{\Theta x})) \]

From (7)

\[ \Theta_x = \frac{\Theta x}{\Theta_y} E_0 \exp(-j \beta (x + \frac{x^2}{2} \frac{\Theta y}{\Theta x})) \]

From (8)

\[ \Theta_y = 0 \]

At the critical angle of incidence, \( \Theta_y = 90^\circ \)

\[ \beta_x = \beta_2 \cos \Theta_2 \]

\[ \beta_y = \beta_2 \sin \Theta_2 \]

\[ R_x = 0 \]

\[ R_y = 0 \]

\[ \beta_x = \beta_2 \cos \Theta_2 \]

\[ \beta_y = \beta_2 \sin \Theta_2 \]

\[ R_x = 0 \]

\[ R_y = 0 \]
A wave that propagates along an interface but decays away from it as \( z \), is often called a surface wave. Depending on \( \beta_i \), this attenuation can happen very quickly so that the field is tightly bound to the interface.

Combining all of these terms, we could have a tightly bound slow surface wave, depending on \( \beta_i \).

Referring back to the beginning of this section, we were puzzled by results that showed for certain \( \xi \) of incidence of a UWA incident on a less-dense half-space, \( \Pi^2 = 1 \) and \( \Gamma = 0 \). Seemed to violate our concept of conservation of power.

Let's check this. Your text shows that \( \Theta = \Theta_e \).

\[
S_{av}^t = \frac{1}{2} \text{Re} \left\{ \vec{E}_t \times \vec{H}_t^{+*} \right\} = \frac{1}{8} \frac{2 |E_0|^2}{\beta^2} \left\lbrack \frac{W}{m^2} \right\rbrack \tag{5-40}, (15)
\]

Notice there is time-avg. power flow in \( x \) direction, but none in the \( z \) direction (normal to interface). Interesting.

Your text also shows that

\[
S_{av}^i = \frac{1}{2} \text{Re} \left\{ \vec{E}_i \times \vec{H}_i^{+*} \right\} = \left( \hat{\theta} \sin \Theta_e + \hat{\phi} \cos \Theta_e \right) \frac{|E_0|^2}{2 \eta} \tag{5-41a}
\]

\[
S_{av}^r = \frac{1}{2} \text{Re} \left\{ \vec{E}_r \times \vec{H}_r^{+*} \right\} = \left( \hat{\theta} \sin \Theta_e - \hat{\phi} \sin \Theta_e \right) \frac{|E_0|^2}{2 \eta} \tag{5-41b}
\]

We see here that \( S_{av}^i = -S_{av}^r \) meaning all time-avg. power incident in \( z \) direction is reflected back. Consistent with (5-40).

Do we expect that \( S_{av}^t = S_{av}^i \) \( (= S_{av}^r) \) ? To answer this question, perhaps we can consider their geometry.
Answer is: not necessarily. We do know that for any plane \( \perp \) to \( x \), \( S_{x, \perp} = S_{x, \parallel} \).

Even though fields in region 2 are not zero for \( \theta > \theta_c \), there is no time avg. power flow in \( x \) (normal to interface). There is, however, time-avg. power flow in \( x \) (parallel to interface).

**Critical Angle & Parallel-Pol**

Besides particular field expressions, the results we've derived here concerning the critical \( \theta \) of incidence, power flow = 0 normal to interface in region 2 for \( \theta > \theta_c \), non-uniform plane waves, surface waves, slow waves, etc. are valid for a 0-pol UFW obliquely oriented on a half space.

This makes sense because Snell's law is identical for these UFW polarizations. We derived \( \theta_c \) & other phenomena from these laws.
Example N12.1 (Text example 5-6). Classic problem. A light ray is incident on a dielectric slab as shown. Determine range of $\varepsilon_r$ so that light is contained in slab and propagates down this waveguide.

\[ \theta' \geq \theta_c = \sin^{-1}\left(\frac{1}{\sqrt{\varepsilon_r}}\right) \]  
(197)

which means

\[ \theta_c \leq \frac{\pi}{2} - \theta' = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\sqrt{\varepsilon_r}}\right) \]  
(20)

By phase match at the interface $z=0$, 

\[ \alpha_0 \sin \theta_i = \beta_0 \sqrt{\varepsilon_r} \sin \theta_t \]

\[ \Rightarrow \sin \theta_t = \frac{1}{\sqrt{\varepsilon_r}} \sin \theta_i \]  
(21)

By definition

\[ \cos^2 \theta_t = 1 - \sin^2 \theta_t \]

\[ \cos^2 \theta_t = \pm \sqrt{1 - \sin^2 \theta_t} \]  
(22)

Square (20); sum (22):

\[ \cos^2 \theta_t \geq \frac{1}{\varepsilon_r} \quad \text{or} \quad 1 - \sin^2 \theta_t \geq \frac{1}{\varepsilon_r} \]

and sum (21): \[ \varepsilon_r - \varepsilon_r \left(\frac{1}{\varepsilon_r} \sin^2 \theta_i\right) \geq 1 \]  
\[ \Rightarrow \varepsilon_r \geq 1 + \sin^2 \theta_i \]
To accommodate all $\Theta_i$, then maximum $\epsilon_r$ is

$$\epsilon_r > 1 + \max_i \left[ \sin^2 \Theta_i \right] = 2 \quad \text{when } \Theta_i = 90^\circ$$

When $\Theta = 0$, all wave will prop. in $z$ and be contained in cylindrical wgt. As $\Theta$ increases, $\epsilon_r$ increases, more reflection at front face, however.

Demo with laser pointer

& glass of water