Lecture 11 – Brewster Angle of Incidence

We saw in the results of the last lecture that it is possible for an obliquely incident UPW on a half space to be completely transmitted – no reflection. This occurred for a parallel-pol UPW incident from a more "dense" EM material to a less dense one, and vice versa.

We did not observe this behavior for perpendicularly polarized UPWs regardless of which material was more "dense."

Let's investigate this behavior in more depth. We'll consider ⊥ pol first, then ∥ pol.

Perpendicular Polarization

From Lecture 9 we found that

$$\Gamma_\perp^b = \left.\frac{E_\perp^r}{E_\perp^i}\right|_{z=0} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \qquad (5\text{-}27),\ (1)$$

For $\Gamma_\perp^b = 0 \implies \eta_2\cos\theta_i = \eta_1\cos\theta_t$

or

$$\cos\theta_i = \frac{\eta_1}{\eta_2}\cos\theta_t \qquad (5\text{-}27a),\ (2)$$

Squaring (2) & using Snell's law of refraction $\left(\sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i\right)$ gives

$$\cos^2\theta_i = \left(\frac{\eta_1}{\eta_2}\right)^2\cos^2\theta_t$$

or

$$1-\sin^2\theta_i = \left(\frac{\eta_1}{\eta_2}\right)^2\left[1-\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)^2\right]$$

Solving for $\sin\theta_i$ gives

$$\boxed{\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1}-\frac{\mu_2}{\mu_1}}{\frac{\mu_1}{\mu_2}-\frac{\mu_2}{\mu_1}}}} \qquad (5\text{-}28a),\ (3)$$

For a real angle of incidence θ_i, the sine fct cannot exceed one. This implies from (3) that

$$\frac{\epsilon_2}{\epsilon_1} - \cancel{\frac{\mu_2}{\mu_1}} \leq \frac{\mu_1}{\mu_2} - \cancel{\frac{\mu_2}{\mu_1}}$$

or

$$\boxed{\frac{\epsilon_2}{\epsilon_1} \leq \frac{\mu_1}{\mu_2}} \qquad (5\text{-}29a), (4)$$

A

This is the requirement for total transmission of $\perp$-pol UPW incident on a half space (from region 1 to region 2). If (4) is satisfied, then θ_i found from (3). This incidence angle is called the Brewster angle, θ_B.

Notice that if $\mu_1 = \mu_2$ (nonmagnetic), then (3) becomes

$$\sin\theta_i = \infty \qquad (6)$$

There is no real angle of incidence θ_i that satisfies this equation. Hence, there is no Brewster angle of incidence for a $\perp$-pol UPW incident from one dielectric material to another.

This is consistent w/ our results in lecture 9.

Parallel Polarization

From Lecture 10, we found that

$$\Gamma_{\parallel}^{b} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \qquad (5\text{-}30), (7)$$

For $\Gamma_{\parallel}^{b} = 0 \Rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_i$

or

$$\cos\theta_i = \frac{\eta_2}{\eta_1}\cos\theta_t \qquad (5\text{-}30a), (8)$$

Squaring both sides of this equation & using Snell's law of refraction as before, leads to

Also, the argument must be positive: A

$$\frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1}} \geq 0 \qquad (5)$$

Both criteria (4) & (5) must be satisfied simultaneously for there to exist an χ of incidence where $\Gamma_{\perp}^{b} = 0$.

$$0 \leq \frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1}} \leq 1$$

- If $\frac{\mu_1}{\mu_2} \geq 1$: then LH: $\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1} \geq 0 \Rightarrow \frac{\epsilon_2}{\epsilon_1} \geq \frac{\mu_2}{\mu_1}$

  & RH: $\frac{\epsilon_2}{\epsilon_1} - \cancel{\frac{\mu_2}{\mu_1}} \leq \frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1} \Rightarrow \frac{\epsilon_2}{\epsilon_1} \leq \frac{\mu_1}{\mu_2}$

  then $1 \overset{?}{\geq} \frac{\mu_2}{\mu_1}$ (yes) & $1 \leq \frac{\mu_1}{\mu_2}$ (yes) ← assumed

  For variable μ's, only satisfied if $\epsilon_1 = \epsilon_2$

- If $\frac{\mu_2}{\mu_1} \geq 1$: then LH: $0 \geq \frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1} \Rightarrow \frac{\epsilon_2}{\epsilon_1} \leq \frac{\mu_2}{\mu_1}$

  & RH: $\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1} \geq \frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1} \Rightarrow \frac{\epsilon_2}{\epsilon_1} \geq \frac{\mu_1}{\mu_2}$

  $1 - 2 \qquad 0.5 - 2 = -1.5$

  Choosing μ's, satisfied only for $\epsilon_1 \overset{?}{=} \epsilon_2$

$$\frac{1-2}{0.5-2} = \frac{-1}{-1.5}$$

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2}}} \qquad (5\text{-}31a), (9)$$

For a real $\theta_i \Rightarrow$

[Note that text eqns (5-32) & (5-32a) are incorrect] $\longrightarrow$

$$0 \le \frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2}} \le 1 \qquad (10)$$

If $\frac{\mu_2}{\mu_1} = 1$ (only non-magnetic half spaces)

$$0 \le \frac{\frac{\epsilon_2}{\epsilon_1} - 1}{\frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2}} \le 1 \qquad (11)$$

<u>Two cases:</u>

(1) $\underline{\frac{\epsilon_2}{\epsilon_1} \ge 1}$. LH of (11): $0 \overset{?}{\le} \frac{\epsilon_2}{\epsilon_1} - 1 \Rightarrow \frac{\epsilon_2}{\epsilon_1} \overset{?}{\ge} 1$ yes, assumed.

RH of (11): $\cancel{\frac{\epsilon_2}{\epsilon_1}} - 1 \overset{?}{\le} \cancel{\frac{\epsilon_2}{\epsilon_1}} - \frac{\epsilon_1}{\epsilon_2} \Rightarrow 1 \overset{?}{\ge} \frac{\epsilon_1}{\epsilon_2}$ yes, assumed.

$\therefore$, for $\parallel$ pol, and $\frac{\epsilon_2}{\epsilon_1} \ge 1$, a Brewster angle will be present at some oblique angle of incidence.

(2) $\underline{\frac{\epsilon_2}{\epsilon_1} \le 1}$. LH of (11): $0 \overset{?}{\ge} \frac{\epsilon_2}{\epsilon_1} - 1 \Rightarrow \frac{\epsilon_2}{\epsilon_1} \overset{?}{\le} 1$ yes, assumed.

RH of (11): $\cancel{\frac{\epsilon_2}{\epsilon_1}} - 1 \overset{?}{\ge} \cancel{\frac{\epsilon_2}{\epsilon_1}} - \frac{\epsilon_1}{\epsilon_2} \Rightarrow \frac{\epsilon_1}{\epsilon_2} \overset{?}{\ge} 1$ yes, assumed.

$\therefore$ for all $\parallel$ pol & $\frac{\epsilon_1}{\epsilon_2} \ge 1$, a Brewster $\measuredangle$ will be present at some oblique $\measuredangle$ of incidence.

Combining these two results we see that for $\parallel$-pol & $\mu_1 = \mu_2$, a Brewster angle will be present at some oblique $\measuredangle$ of incidence for any positive, real values for ϵ_1 & ϵ_2.