
Impedance matching (or simply “matching”) one portion of a circuit to another is an immensely important part of microwave engineering.

Additional circuitry between the two parts of the original circuit may be needed to achieve this matching.

Why is impedance matching so important? Because:

1. Maximum power is delivered to a load when the TL is matched at both the load and source ends. This configuration satisfies the conjugate match condition.
2. With a properly matched TL, more signal power is transferred to the load, which increases the sensitivity of the device.
3. Some equipment (such as certain amplifiers) can be damaged when too much power is reflected back to the source.

Factors that influence the choice of a matching network include:

1. The desire for a simple design, if possible.
2. Providing an impedance match at a single frequency is often not difficult. Conversely, achieving wide bandwidth matching is usually difficult.
3. Even though the load may change, the matching network may need to perform satisfactorily in spite of this, or be adjustable.

We will discuss three methods for impedance matching in this course:

1. $L$ networks,
2. Single stub tuners (using shunt stubs),
3. Quarter wave transformers.

You’ve most likely seen all three of these before in other courses, or in engineering practice.

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**Matching Using $L$ Networks**

Consider the case of an arbitrary load that terminates a TL:

![Diagram of TL with load](image)

To match the load to the TL, we require $\Gamma_L = 0$. However, if $Z_L \neq Z_0$ additional circuitry must be placed between $Z_L$ and $Z_0$ to bring the VSWR = 1, or least approximately so:
For $\Gamma_L = 0$, this implies $Z_{\text{in}} = Z_0$. In other words, $R_{\text{in}} = \Re[Z_0]$ and $X_{\text{in}} = 0$, if the TL is lossless.

So, we need at least two degrees of freedom in the matching network in order to transform $Z_L$ at the load to $Z_0$ seen at the input to the matching network.

This describes impedance matching in general. For an $L$ network specifically, the matching network is either (Fig. 5.2):

$$R_L > Z_0: \quad \begin{array}{c}
\cdot \cdot \cdot \quad Z_0, \beta \quad jX \\
\cdot \cdot \cdot \quad (jB)^{-1} \quad Z_L
\end{array}$$

$$R_L < Z_0: \quad \begin{array}{c}
\cdot \cdot \cdot \quad Z_0, \beta \quad jX \\
\cdot \cdot \cdot \quad (jB)^{-1} \quad Z_L
\end{array}$$

where $Z_L = R_L + jX_L$.

This network topology gets its name from the fact that the series and shunt elements of the matching network form an “L” shape.

There are eight possible combinations of inductors and capacitors in the $L$ network:
Notice that this type of matching network is **lossless**; or at least the loss can potentially be made extremely small with proper component choices.

As in the text, we’ll solve this problem two ways: first analytically, then using the Smith chart.

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**Analytical Solution for \( L \)-Network Matching**

- Assume \( R_L > Z_0 \). From Fig. 5.2(a):

  \[
  Z_{in} = jX + \left( jB + \frac{1}{R_L + jX_L} \right)^{-1}
  \]  

Through the proper choice of \( X \) and \( B \) we wish to force \( Z_{in} = Z_0 \). Solving (1) for the \( B \) and \( X \) that produce this outcome (by equating real and imaginary parts, as shown in the text) we find that
\[ B = \frac{X_L \pm \sqrt{R_L/Z_0} \sqrt{R_L (R_L - Z_0) + X_L^2}}{R_L^2 + X_L^2} \]  
\[ (5.3a),(2) \]

and

\[ X = \frac{1}{B} + \frac{X_L Z_0 - Z_0}{R_L - BR_L} \]  
\[ (5.3b),(3) \]

Comments:

1. Since \( R_L > Z_0 \), the argument is positive in the second square root of (2). (B must be a real number.)
2. Note that there are two possible solutions for \( B \) in (2).
3. \( X \) in (3) also has two possible solutions, depending on which \( B \) from (2) is used.

- Assume \( R_L < Z_0 \). Using Fig. 5.2(b) with \( Z_{in} = Z_0 \), we obtain

\[ Z_{in} = Z_0 = \left( jB + \frac{1}{Z_L + jX} \right)^{-1} \]  
\[ (5.4),(4) \]

Solving this equation by equating real and imaginary parts as shown in the text gives

\[ X = \pm \sqrt{R_L (Z_0 - R_L) - X_L} \]  
\[ (5.6a),(5) \]

and

\[ B = \pm \frac{1}{Z_0 \sqrt{\frac{Z_0 - R_L}{R_L}}} \]  
\[ (5.6b),(6) \]

Comments:

1. Since \( R_L < Z_0 \), the argument is positive for the square root in (5).
2. There are two solutions for both \( X \) and \( B \). Use the top signs in both (5) and (6) for one solution and the bottom signs for the other.
Smith Chart Solution for \( L \)-Network Matching

\( L \)-network matching can also be computed graphically using the Smith chart. This approach is less accurate than the analytical approach.

However, more insight into the matching process is often obtained using the Smith chart. For example, the contribution each element makes to the matching is quite clear.

The process of using the Smith chart to design the matching network is probably best illustrated by example. Example 5.1 in the text illustrates the design of an \( L \) network when \( R_L > Z_0 \) (Fig. 5.2a). Here, we’ll give an example when \( R_L < Z_0 \).

**Example N7.1** Design an \( L \) network to match the load \( 25 + j30 \) \( \Omega \) to a TL with \( Z_0 = 50 \) \( \Omega \) at the frequency \( f = 1 \) GHz.

Since \( R_L < Z_0 \), we’ll use the circuit topology in Fig. 5.2b:
We’ll solve this problem using the Smith chart and then confirm it analytically.

**Steps for a Smith chart solution:**

1. \( z_L = \frac{Z_L}{Z_0} = \frac{1}{2} + j \frac{3}{5} \) p.u.Ω. Mark this point on the chart.

2. The overall concept behind this type of \( L \)-network matching is to add a reactance \( x \) to \( z_L \) such that the sum of admittances \( b \) and \( (z_L + jx)^{-1} \) yield \( y_{in} = 1 + j0 = z_{in} \) (the center of the Smith chart). In such a case, the TL sees a matched load.

So in this particular example, and referring to the Smith chart below, we need to add a normalized impedance \( jx = -0.1 \) p.u.Ω in order to move to the \( y = 1 + jb \) circle as it appears on the Smith **impedance** chart. As we learned in Lecture 6, \( 1 + jb \) describes a circle on the Smith **admittance** chart. On the Smith **impedance** chart, this \( 1 + jb \) circle appears rotated by 180°, as shown by the dashed circle in the figure below.

3. Convert this impedance to an **admittance** value by reflecting through the origin to the diametrically opposed point on the constant VSWR circle. (We’re now working on a Smith **admittance** circle.)
4. **Add the normalized susceptance** $b = 1.0$ p.u. S to reach the center of the Smith (admittance) chart. Here $\Gamma = 0$ and $y_{in} = 1 + j0$, which means the TL now sees a matched load.

Un-normalizing, we find that

$$jX = jx \cdot Z_0 = -j0.1 \cdot 50 = -j5.0 \ \Omega$$
\[ jB = jb \cdot Y_0 = j1.0 \cdot \frac{1}{50} = j0.02 \ \text{S} \]

What are the \( L \) and \( C \) values of these elements? We can identify the type of element by the sign of the reactance or susceptance:

<table>
<thead>
<tr>
<th></th>
<th>Inductor</th>
<th>Capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance</td>
<td>( Z_L = j\omega L )</td>
<td>( Z_C = \frac{1}{j\omega C} = -j\frac{\omega}{\omega C} )</td>
</tr>
<tr>
<td>Admittance</td>
<td>( Y_L = \frac{1}{j\omega L} = -j\frac{\omega}{\omega L} )</td>
<td>( Y_C = j\omega C )</td>
</tr>
</tbody>
</table>

- Since \( X < 0 \), we identify this as a capacitor. Therefore,
  \[ jX = \frac{-j}{\omega C_2} = -j5.0 \ \Omega \]
  For operation at 1 GHz, we need
  \[ C_2 = \frac{1}{5 \cdot 2\pi f} = 31.8 \ \text{pF} \]

- Since \( B > 0 \), we also identify this as a capacitor. Therefore,
  \[ jB = j\omega C_1 = j0.02 \ \text{S} \]
  For operation at 1 GHz, we need
  \[ C_1 = \frac{0.02}{2\pi f} = 3.18 \ \text{pF} \]

The final circuit is:
Let’s check to see if we have really achieved a match at 1 GHz:

- \( Z_{C2} + Z_L = \frac{1}{j\omega C_2} + Z_L = -j5 + 25 + j30 = 25 + j25 \ \Omega \).

- \( Z_{in} = Z_{C1} \| (Z_{C2} + Z_L) = \left[ j0.02 + (25 + j25)^{-1} \right]^{-1} = 50 + j0 \ \Omega \).

So, we have indeed achieved a match since \( Z_{in} = Z_0 \).

Next, for the analytical solution we apply (5)

\[
X = \pm \sqrt{R_L (Z_0 - R_L)} - X_L = \pm 25 - 30 = \begin{cases} -5 \ \Omega \\ -55 \ \Omega \end{cases}
\]

while from (6)

\[
B = \pm \frac{1}{Z_0} \sqrt{\frac{Z_0 - R_L}{R_L}} = \begin{cases} 0.02 \ \text{S} \\ -0.02 \ \text{S} \end{cases}
\]

This analytical solution agrees exactly with our Smith chart solution.

It is readily apparent that there is a second analytical solution for this matching network. It is equally valid.
On the Smith chart, this second solution is found by adding $jx$ to $z_L$ until the second intersection with the $1 + jx$ admittance circle is reached.
The Yin-Yang of Matching:  
**Part 1—Basic Matching Concepts**

By Randy Rhea  
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**When an electrical signal propagates through media, a portion of the signal is reflected at the interface between sections with differing impedances. This is analogous to light reflection in optical systems. The reflected signal may pose problems, and the power in the reflected signal reduces the transmitted power.**

Figure 1 shows the impedance chart developed by Phillip Smith [1]. Any impedance with a positive real part may be displayed on the standard, unity radius Smith chart. The horizontal line is pure resistance. Circles with a center on this line are constant resistance. Arcs converging at center right are constant reactance. The top half of the chart is inductive and the bottom half is capacitive. The chart is sometimes normalized to the desired reference impedance of the system such as 50 ohms. For example, the inductive impedance $50 + j50$ ohms becomes $1 + j1$ when normalized and is plotted at the intersection of the circle labeled 1 and the top arc labeled 1. An admittance version of the chart is a left-right mirror of the impedance chart. A detailed description of the chart is given in Smith’s book and a CD-ROM tutorial by Glenn Parker [2].

The center of the chart is the reference impedance of the system. An impedance at this point represents an ideal match. The length of a vector from the center of the chart to any impedance point is the magnitude of the reflection coefficient, $|\rho|$. The angle of the vector is from the center of the chart with respect to the real axis on the right. An ideal match has $|\rho| = 0$ and total reflection from a pure reactive impedance at the circumference of the chart has $|\rho| = 1$. The reflection coefficient may be computed from

$$\rho = \frac{Z - Z_0}{Z + Z_0}$$  \hspace{1cm} (1)

where $Z$ is the impedance and $Z_0$ is the reference impedance of the system.

The magnitude of the reflection coefficient is a radial parameter. In other words, it is a function of radial distance from the center of the chart. Other useful radial formats are related to $\rho$.

$$RL_{dB} = -20 \log |\rho|$$  \hspace{1cm} (2)
For example, in a 50 ohm system, an impedance $50 + j50$ has a reflection coefficient of 0.4472 at 63.43°, a return loss of 7.0 dB, a VSWR of 2.618 and an insertion loss of 0.9691 dB. The insertion loss, $L_A$, is due to reflection. It does not account for dissipation loss that is present with lossy components or resistance in the circuit. Table 1 lists radical parameters. This table is useful for disciplines other than matching. For example, a filter designed for 0.177 dB passband ripple has a passband return loss of 15 dB.

### Complex-Conjugate Match

Consider a load impedance of $Z_L = R_L - jX_L$ at 100 MHz where $R_L = 50$ ohms and $X_L = -33.86$ ohms as shown in Figure 2. This load is matched to the 50 ohm source using a series 53.89 nH inductor with a reactance +33.86 ohms at 100 MHz. At this frequency the inductor and load reactance series resonate effectively connecting the 50 ohm source directly to the 50 ohm load resistance. Given in Figure 3 are the transmission and reflection responses of the original load and the matched load. The match is achieved only at the series resonant frequency of the load capacitance and matching inductance. In fact, the match is worse above 140 MHz than without the matching network. This is a characteristic of matching networks; the presence of reactive elements often provides selectivity.

The impedance seen looking toward the source through the matching inductor is $50 + j33.86$ ohms. This is the complex conjugate of the load impedance. In fact, at any node in a lossless matching network, the impedance to the left is the complex conjugate of the impedance to the right. This is another characteristic of matching networks.

If the load reactance is in parallel with the resistance, then a shunt element can cancel the load susceptance and provide a parallel resonant match. A single inductor or capacitor can provide a match only if the load and source resistance are equal, or near enough so to provide an adequate match. In general, we are not so fortunate and two matching elements are required.

### L-Network Matching

This section introduces L-networks for matching both real and complex impedances. At a single frequency, any positive-real complex impedance can be matched to any other positive-real complex impedance using no more than two reactive elements. Consider the common need to match a complex impedance to a real impedance such as 50 ohms. Given in Figure 4 are the eight unique topologies comprised of two L-C elements. Each topology is capable of matching certain complex load terminations on the network’s right to a real source resistance on the network’s left. The charts are normalized to the desired source resistance. The matchable space is enclosed by the green curves in Figure 4. Notice these curves are the familiar shape of the Chinese Yin-Yang for the four topologies that include both an inductor and a capacitor. I first became aware of these concepts when reading Smith’s book Electronic Applications of the Smith Chart [1]. Smith’s unique ability to graphically express important concepts encompasses yin-yang!
Certain impedances are matched by only two topologies. For example, impedances with real components greater than the reference impedance are matched by only type 1 and 2 networks. Other impedances are matched by four topologies. For example, \( Z = 0.5 + j1 \) is matched by types 1, 4, 5 or 7.

Phil’s book describes how to find the element values for these L-networks by using graphical techniques. Here are formulas for finding the element values analytically. For a reference impedance \( Z_0 = R_0 - j\theta \), and a load termination impedance \( Z_L = R_L - jX_L \), the load admittance is (eq. 5)

\[
Y_L = G_L + jB_L = \frac{R_L}{R_L^2 + X_L^2} - j\frac{X_L}{R_L^2 + X_L^2}
\]

then setting the following:

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Figure 4 - Diagrams of matchable impedance space for L-networks of type 1 through 8.
we then have

\[
A = \sqrt{\frac{G_L}{R_0} - G_L^2}
\]  \hspace{1cm} (6)

\[
B = \sqrt{R_L(R_0 - R_L) - X_L}
\]  \hspace{1cm} (7)

\[
C = -\sqrt{R_L(R_0 - R_L) - X_L}
\]  \hspace{1cm} (8)

Consider a load termination impedance of 25 + j43.33 ohms at 100 MHz. The return loss is 4.8 dB and the transmission loss is 1.76 dB.

Referring to Figure 2 we see that type 1, 4, 5 or 7 is used to match this load. Eqs. 5 through 10 are used to find the element values for type 1: L1 is 79.66 nH and C1 is 43.47 pF.

Which of these topologies is best? The amplitude transmission response are given in Figure 5 as computed and displayed by the Agilent GENESYS program suite [3]. Notice that all four provide full transmission at the design frequency, 100 MHz. Notice that the type 1 network is somewhat lowpass in character and that the others are somewhat highpass. The response characteristics are a function of the load impedance. In this case, type 7 provides the widest bandwidth. In general,
responses become more bandpass in character and narrower for larger load reflection coefficients. In some applications wide bandwidth is desired while for others the rejection of interfering signals by a narrow response is desired. The use of a simulator is a convenient way to simultaneously determine element values and compute network responses. Given in Figure 6 is a screen capture of an GENESYS workspace that computes element values and displays the responses for all eight L-networks. The user enters the reference impedance, load resistance, load reactance and center frequency as seen at the upper left. The computed element values are shown on the right. Notice that, for this load impedance, four of the L-networks have negative element values. These are depicted in the schematics as magenta colored inductors. If negative-valued inductors could be realized then all eight L-networks would function as matching networks. This GENESYS workspace may be downloaded from the online post “Matching Tutorial Published in High Frequency Electronics Magazine” at the Founder’s Forum at www.eagleware.com.

**Distributed Networks**

Component values in all matching networks become extreme as the reflection coefficient of the load approaches 1.0. Realizability is then at risk. These issues are even more critical with distributed circuits because of the difficulty in realizing transmission lines with high and low characteristic impedance. The property of reentrance, where transmission line behavior repeats at multiples of 90° or 180°, restricts the use of distributed circuits when good stopbands are desired. Nevertheless, distributed networks are useful for matching.

A microstrip transmission line involves inductance associated with the flow of current in the conductor and capacitance associated with the strip separated from ground by the dielectric substrate. This distributed inductance and capacitance is the basis of the classic L-C model for a transmission line and it accounts for the term “distributed.” If the line is narrow, the inductance is increased but the capacitance is small. A narrow, high-impedance line behaves like an inductor if it is less than 90 degrees in electrical length. A wide, low-impedance line looks capacitive.

Let’s examine this analytically. The impedance, \( Z_S \), at the input of a transmission line of characteristic impedance \( Z_0 \) and length \( \theta \) terminated in a load, \( Z_L \), is given by

\[
Z_S = \frac{Z_0 \left( Z_L - jZ_0 \tan \theta \right)}{Z_0 + jZ_L \tan \theta}
\] (25)
Consider the case where \( Z_L \) is a short, \( 0 + j0 \). Then

\[
Z_s = jZ_0 \tan \theta
\]  

(26)

Since the input impedance of a shorted inductor is \( jX_L \),

\[
X_L = Z_0 \tan \theta
\]  

(27)

This means that the reactance of an inductor in a network may be replaced with a transmission line of characteristic impedance \( Z_0 \) and length \( \theta \). Figure 7 presents the equivalent relationships between lumped and distributed circuits. These equivalents are exact only at the design frequency. The reactance of an inductor increases linearly with increasing frequency while the reactance of a shorted line increases as \( \tan \theta \). If the line is short, that is \( \theta \ll 90^\circ \), then \( \tan \theta \approx \theta \) and the input reactance of a shorted line increases linearly with frequency. Therefore, a shorted line behaves like an inductor over a range of frequencies where the line is much less than 90° long, preferably less than 30°.

The relationships in Figure 7 may be used to replace lumped elements with distributed elements. These equivalents are useful in circuits other than matching networks such as filters, oscillators, and couplers. Generally, the equivalence is better with higher impedance and shorter length lines for inductors, and with lower impedance and shorter length lines for capacitors. For example, a 47 nH inductor has a reactance of 29.53 ohms at 100 MHz. A shorted 30.57°, 50 ohm line provides the same reactance. However, a shorter (16.45°) 100 ohm line would behave like this inductor over a wider frequency range. For the 180° resonator, the line impedance should be either higher or lower impedance than the impedance of the system it is inserted within. The 90° resonators have unique impedance values.

**Distributed L-Networks**

Conspicuously absent in Figure 7 is a transmission line equivalent for a series capacitor. Series capacitors are difficult to realize in distributed form. A microstrip gap requires intolerably close spacing to achieve significant capacitance and the line ends have parasitic capacitance to ground as well as the series capacitance. Edge coupled lines have even larger capacitance to ground. Because of this, the most popular form of distributed highpass filters use transmission lines for the shunt inductors and lumped-element series capacitors.

However, from Figure 4, we can see that L-network types 1, 3, 6 and 8 do not require series capacitors. Using the equivalents in Figure 7, these types may be realized in distributed form.

In summary, once the lumped values are computed,
Single-Stub Tuner

Given in Figure 8 are single-stub tuners with shorted and open stubs and their matchable space plotted on the Smith chart. The shorted-stub tuner is similar to L-network type 8, while the open-stub tuner is similar to type 3. However, by using transmission line theory embodied in Eq. 25, a more general solution is achieved that does not require short line lengths. Also, the more general solution matches a larger space than a lumped Type 1 network. The formulas given here are for the case where the characteristic impedance of the lines in the network equal the reference impedance of the system. This is convenient because, if a match to 50 or 75 ohms is desired, lines of that impedance are typically available. Allowing other transmission line characteristic impedances may provide wider bandwidth.

The following equations are more concise when the load impedance is normalized to $Z_0$.

$$Z_o'(\text{SeriesLines}) = \frac{\alpha L_{\text{Lumped}}}{\sin \theta}$$

(28)

$$Z_o'(\text{Shunt Shorted Stub}) = \frac{\alpha L_{\text{Lumped}}}{\tan \theta}$$

(29)

$$Z_o'(\text{Shunt Open Stub}) = \frac{\tan \theta}{\alpha C_{\text{Lumped}}}$$

(30)

Next the required susceptance of the stub is found by

$$B_S = \frac{(R'_L + X'_L \tan \theta_{\text{series}} + X'^2_L - 1)\tan \theta_{\text{series}} - X'_L}{R'^2_L + (X'_L + \tan \theta_{\text{series}})^2}$$

(38)

Then

$$\theta_{sa} = \tan^{-1} \frac{1}{B_S}$$

(39)

$$\theta_{oa} = -\tan^{-1} B_S$$

(40)

Suppose we desire to match an antenna with an input impedance of 35.5 $-$ j107 ohms to 50 ohms using the single-stub tuner. Then the series 50 ohm line is 47.8° long and the shorted 50 ohm stub is 21.3° long.

Next Issue

In the next issue, this article concludes with Part 2, covering the topics of impedance transformation through transmission line sections, multiple-section transformers, absorbing reactance using filters, and analysis of load characteristics for selection of the preferred matching method.

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References

3. Agilent Technologies, EEsof EDA Division, Santa Rosa, CA. www.agilent.com/find/eesof
4. G. Matthaei, L. Young, and E.M.T. Jones, *Microwave Filters, Impedance Matching Networks and Coupling*
Figure 8 · On the left is a single-stub transmission-line tuner with a shorted stub and on the right is a tuner with an open stub. The matchable space of the shorted stub includes much of the chart but excludes the smaller unity conductance circle on the left.