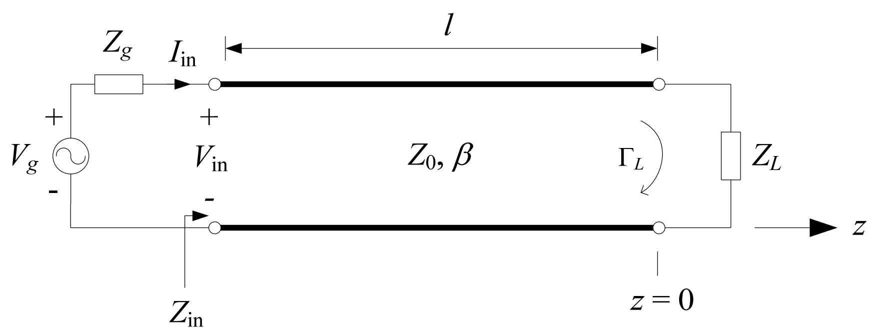
Lecture 5: Generator and Load Mismatches on TLs.

Up to this point, we have focused primarily on terminated transmission lines that lacked a specific excitation. That is, the TL was semi-infinite and terminated by a load impedance.

In this lecture, we'll complete our review of TLs by adding a voltage source together with an arbitrary load (Fig. 2.19):



This TL model is very useful and applicable to a wide range of practical engineering situations.

Quantities of interest in such problems include the input impedance (for matching purposes) and signal power delivered to the load.

We will first consider the computation of the latter quantity assuming the TL is lossless.

Proceeding, the voltage on the TL is expressed by

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

or

$$V(z) = V_o^+ \left(e^{-j\beta z} + \Gamma_L e^{j\beta z} \right) \qquad (2.69),(1)$$

where

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad (2.68),(2)$$

We'll assume that the physical properties of the TL, the source and the load are known. This leaves the complex constant V_o^+ as the only unknown quantity in (1).

Generally speaking, we compute V_o^+ by applying the boundary condition at the TL input. (Recall that we have already applied boundary conditions at the load.) This is accomplished by applying voltage division at the input:

$$V_{\text{in}} = \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g} V_g \qquad (3)$$

Observe that V_{in} is an electrical circuit quantity.

However, at the input to the TL, voltage must be continuous from the generator to the TL. This implies that V_{in} must also equal $V(z=-l)$ on the TL.

Proceeding, then from (1) at the input

$$V(z=-l) = V_o^+ \left(e^{+j\beta l} + \Gamma_L e^{-j\beta l} \right) \qquad (4)$$

Equating (3) and (4) to enforce the boundary condition at the TL input we find

$$V_o^+\left(e^{+j\beta l}+\Gamma_L e^{-j\beta l}\right)=\frac{Z_{\text{in}}}{Z_{\text{in}}+Z_g}V_g$$

or

$$V_o^+ = V_g \frac{Z_{\text{in}}}{Z_{\text{in}}+Z_g}\left(e^{j\beta l}+\Gamma_L e^{-j\beta l}\right)^{-1} \text{ [V]} \qquad (2.70),(5)$$

Maximum Power

Because the TL is lossless, the time average power P_{av} delivered to the input of the TL must equal the time average power delivered to the load. Therefore,

$$P_{\text{av}}=\frac{1}{2}\Re e[V_{\text{in}}I_{\text{in}}^*]=\frac{1}{2}\Re e\left[V_{\text{in}}\frac{V_{\text{in}}^*}{Z_{\text{in}}^*}\right]$$

or

$$P_{\text{av}}=\frac{\left|V_{\text{in}}\right|^2}{2}\Re e\left[\frac{1}{Z_{\text{in}}^*}\right] \text{ [W]} \qquad (2.74),(6)$$

(Note that there is an error in (2.74) in your text.)

Now, substituting (3) into (6) gives

$$P_{\text{av}}=\frac{\left|V_g\right|^2}{2}\left|\frac{Z_{\text{in}}}{Z_{\text{in}}+Z_g}\right|^2\Re e\left[\frac{1}{Z_{\text{in}}^*}\right] \qquad (2.74),(7)$$

If we define $Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$ and $Z_g = R_g + jX_g$, (7) becomes

$$P_{\text{av}}=\frac{\left|V_g\right|^2}{2}\frac{R_{\text{in}}}{\left(R_{\text{in}}+R_g\right)^2+\left(X_{\text{in}}+X_g\right)^2} \qquad (2.75),(8)$$

Employing this last result, we'll consider three special cases for P_{av} in an effort to maximize this quantity. We will assume that Z_g is both nonzero and fixed:

(1.) Load is matched to the TL: $Z_L = Z_0$.

From (2), $\Gamma_L = 0$ in this situation, which also implies that $Z_{in} = Z_0$. [This should be intuitive. If not, see (2.43).] Consequently, from (8) with $R_{in} = Z_0$ and $X_{in} = 0$:

$$P_{av,1} = \frac{|V_g|^2}{2} \frac{Z_0}{(Z_0 + R_g)^2 + (X_g)^2} \qquad (2.76),(9)$$

(2.) Generator is matched to an arbitrarily loaded TL: $Z_{in} = Z_g$ and $\Gamma_L \neq 0$.

Specific values for βl, Z_0, and Z_L would need to be chosen so that $Z_{in} = Z_g$. Then from (8) and with $R_{in} = R_g$ and $X_{in} = X_g$:

$$P_{av,2} = \frac{|V_g|^2}{2} \frac{R_g}{(R_g + R_g)^2 + (X_g + X_g)^2}$$

or

$$P_{av,2} = \frac{|V_g|^2}{8} \frac{R_g}{R_g^2 + X_g^2} \qquad (2.78),(10)$$

(3.) Maximum power transfer theorem applied at the TL input: $Z_{\text{in}} = Z_g^*$. (2.80),(11)

In this situation, $R_{\text{in}} = R_g$, $X_{\text{in}} = -X_g$ (conjugate match), and $\Gamma_L \neq 0$, so that from (8)

$$P_{\text{av},3} = \frac{|V_g|^2}{2} \frac{R_g}{(R_g + R_g)^2 + (-X_g + X_g)^2}$$

or

$$P_{\text{av},3} = \frac{|V_g|^2}{8R_g} \quad (2.81),(12)$$

Previous EE 322 students should recognize this as the maximum available source power.

Now, which of these three situations (9), (10), or (12) provides the most time average power delivered to the load?

- Clearly, $P_{\text{av},3} \geq P_{\text{av},2}$ (equal when $X_g = 0$).
- It can be shown that $P_{\text{av},3} > P_{\text{av},1}$.

Therefore, $P_{\text{av},3}$ is the largest.

In conclusion, to transfer maximum time average power to a load, we generally need to conjugate match the generator impedance to the TL input impedance.

Note that maintaining a low VSWR ($\Gamma_L \approx 0$) doesn't necessarily guarantee maximum P_{av}, though it could. (When?)

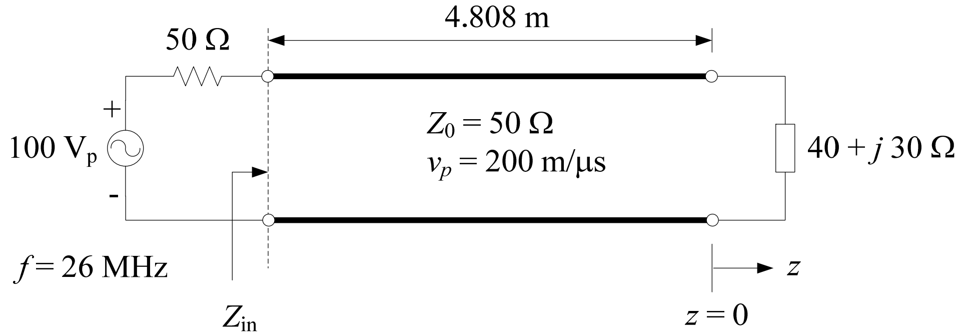
Efficiency

We haven't said anything about efficiency yet. That is, what percentage of the source power is delivered to the load?

With $Z_g = Z_L = Z_0$, the load and the source are both matched to the TL. However, only one half of the source power is delivered to the load so the efficiency is 50%. For a matched line, that's as good as it gets.

One way to increase efficiency is to decrease R_g (from Z_0) and conjugate match the source to the TL input. The line may no longer be matched. Nevertheless, the power from multiple reflections can add in phase to increase the time average signal power delivered to the load.

Example N5.1: For the TL shown, determine the VSWR on the TL and the time-averaged power delivered to the load.



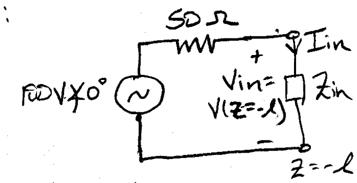
For this TL:

- $\beta = \dfrac{\omega}{v_p} = \dfrac{2\pi \cdot 26\times 10^6}{200\times 10^6} = 0.8168$ rad/m
- $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{40 + j30 - 50}{40 + j30 + 50} = \dfrac{j}{3}$

$\therefore \ \underline{\underline{\text{VSWR}}} = \dfrac{1+|\Gamma_L|}{1-|\Gamma_L|} = \dfrac{1+1/3}{1-1/3} = \underline{\underline{2}}$

There are a few different approaches to determine the time-averaged power delivered to the load in the case of a lossless TL, as we have here.

One approach is to determine the time-averaged power delivered to the input of the TL. That will equal the time-averaged power delivered to the load for a lossless TL. We can construct an equivalent circuit at the input position of the TL as:



From (5): $Z_{in} = Z(-\ell) = Z_0 \dfrac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}$

$\beta\ell = \dfrac{2\pi \cdot 26\times 10^6}{200\times 10^6} \cdot 4.808 = 3.927 \text{ rad} \Rightarrow \tan(\beta\ell) = 1.001$

$\therefore \ \underline{Z_{in}} \approx 50 \cdot \dfrac{40 + j30 + j50}{50 + j40 - 30} = 50 \cdot (2 + j0) = \underline{100\ \Omega}$ ← curious! Z_L is complex, but Z_{in} is purely real. TL's act as <u>impedance transformers</u>.

From the equivalent circuit using voltage division

$\underline{V_{in}} = \dfrac{Z_{in}}{Z_{in} + 50} \cdot V_S = \dfrac{100}{100 + 50} \cdot 100 = \underline{66.67 \text{ V}}$

As stated earlier, since this TL is lossless, all time-averaged power at the input of the TL will be delivered to the load.

$$\therefore \; P_{AV} = \frac{1}{2}\,\text{Re}\left[V(-\ell)\cdot I(-\ell)^*\right] = \frac{1}{2}\,\text{Re}\left[V(-\ell)\cdot\frac{V(-\ell)^*}{Z_{in}^*}\right]$$

$$= \frac{|V(-\ell)|^2}{2}\,\text{Re}\left[\frac{1}{Z_{in}^*}\right]$$

In this case, $\underline{\underline{P_{AV}}} = \frac{66.67^2}{2}\,\text{Re}\left[\frac{1}{100}\right] = \underline{\underline{22.22\text{ W}}}$

← Time-averaged power delivered to the load.

A much longer way to calculate the time-averaged power delivered to the load is to calculate the voltage at the load. From (1) at $z=0$:

$$V(0) = V_L = V_0^+\,(1+\Gamma_L)$$

Then

$$P_{AV} = \frac{1}{2}\,\text{Re}\left\{V(z=0)\cdot I^*(z=0)\right\} = \frac{1}{2}\,|V(0)|^2\cdot\text{Re}\left[\frac{1}{Z_L^*}\right]$$

or

$$= \frac{1}{2}\left|V_0^+(1+\Gamma_L)\right|^2\cdot\text{Re}\left[\frac{1}{Z_L^*}\right] \qquad (13)$$

We need to determine the complex constant V_0^+ to complete the calculation of P_{AV} in (13). Let's rewrite (1) in the form

$$V(z) = V_0^+ e^{-j\beta z}\left(1+\underbrace{\frac{V_0^-}{V_0^+}}_{=\Gamma_L} e^{+j2\beta z}\right)$$

or

$$\boxed{V(z) = V_0^+ e^{-j\beta z}\left(1+\Gamma_L e^{+j2\beta z}\right)} \quad \longleftarrow \text{useful!}$$

At the input to the TL at $z=-\ell$:

$$V(z=-\ell) = \underset{\underset{66.67}{\shortparallel}}{V_{in}} = V_0^+ \underbrace{e^{j\beta\ell}}_{=1\angle 3.937\text{ rad}}\left(1+\underbrace{\Gamma_L e^{-j2\beta\ell}}_{=1/3}\right)$$

Solving for V_0^+:

$$\underline{V_0^+} = \frac{66.67\,(1\angle -3.927\text{ rad})}{1+1/3} = \underline{50\text{ V}\angle 2.356\text{ rad}}$$

Substituting into (13) gives:

$$\underline{\underline{P_{AV}}} = \frac{1}{2}50^2\left|1+\frac{1}{3}\right|^2\cdot\text{Re}\left[\frac{1}{40-j30}\right] = \frac{2500}{2}\cdot 1.054^2\cdot 0.016 = \underline{\underline{22.22\text{ W}}}$$

Agrees with previous calculation at the input.

That's the long way to solve this problem. There's even an easier way to calculate the time-averaged power delivered to the load. Since for this problem the source is matched to the TL characteristic impedance, then from Homework #3

$$P_{\text{av}} = \frac{\left|V_g\right|^2}{8Z_0}\left(1-\left|\Gamma\right|^2\right) \tag{14}$$

such that

$$P_{\text{av}} = \frac{100^2}{8\cdot 50}\left[1-\left(\frac{1}{3}\right)^2\right] = 22.2 \text{ W}$$

as computed above.