

Lecture 36 – Single Stage Amplifier: Design for Specific Gain.

Maximum gain amplifiers discussed in the previous lecture are designed to provide the maximum gain possible from the circuit for a given set of S parameters for a small-signal amplifier. If a **specific gain** value less than the maximum is required, then another design approach must be used, which is the topic of this lecture.

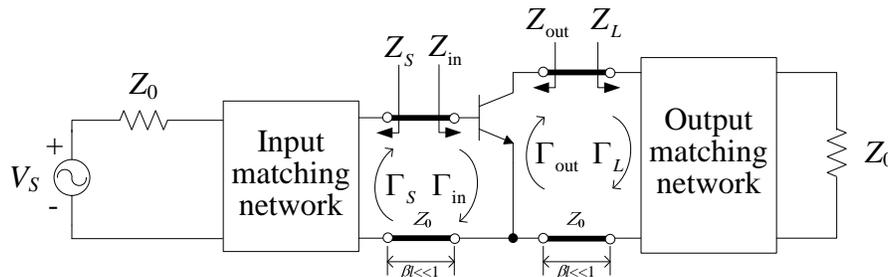
Additional motivation for amplifiers with less than maximum gain is related to the gain-bandwidth product concept. Increased gain usually occurs at the expense of bandwidth and vice versa. Generally speaking, **more bandwidth** from an amplifier may be obtained if the **gain is reduced**.

Unilateral Transistor

Before introducing the method of design for specific gain, we will first discuss the concept of a **unilateral transistor**. This is an approximation in which $S_{12} = 0$ is assumed for a transistor. This is in contrast to a so-called bilateral transistor where $S_{12} \neq 0$.

The unilateral approximation can substantially simplify the design of an amplifier circuit while, in some cases, introducing acceptable error in the gain calculation.

The assumption that $S_{12} = 0$ has some important consequences for the input and output reflection coefficients in a two-port network:



In particular, from (12.3a) for the input reflection coefficient and $S_{12} = 0$:

$$\Gamma_{\text{in}} = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1 - \Gamma_L S_{22}} = S_{11} \quad (12.3a),(1)$$

while from (12.3b) for the output reflection coefficient and $S_{12} = 0$:

$$\Gamma_{\text{out}} = S_{22} + \frac{\Gamma_S S_{12} S_{21}}{1 - \Gamma_S S_{11}} = S_{22} \quad (12.3b),(2)$$

Under the unilateral transistor assumption, we see from these two equations that Γ_{in} is no longer dependent on Γ_L and that Γ_{out} is no longer dependent on Γ_S . Consequently, the **IMN and OMN can be designed independently of one another**. This is perhaps the **chief benefit of the unilateral assumption**.

[This is in contrast to the maximum transducer gain amplifier design of a **bilateral** transistor in the previous lecture where the IMN and the OMN needed to be **designed simultaneously**, which lead to the design equations (12.40a) and (12.40b).]

Using (1) and (2) in the expression for G_T [from (12.13) in the text] provides what is called the **unilateral transducer gain**, G_{TU} , given as

$$G_{TU} = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S S_{11}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \Gamma_L|^2} \quad (12.15), (3)$$

The error in gain calculated by the unilateral approximation is sometimes quantified through the ratio of G_{TU} to the regular (or “bilateral”) transducer gain G_T . As given in the text, this ratio is bounded as

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \quad (12.45), (4)$$

where U is the **unilateral figure of merit** defined as

$$U = \frac{|S_{11}| |S_{21}| |S_{12}| |S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} \quad (12.46), (5)$$

The unilateral assumption is often found acceptable when the error is less than approximately 0.5 dB or so.

We have found that this expression in (4) is actually **not correct**. You will be investigating this in your homework.

Constant Gain Circles

The unilateral transducer gain in (3) is the product of three terms

$$G_{TU} = G_{SU} G_0 G_{LU} \quad (6)$$

where

$$G_{SU} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}, \quad G_{LU} = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (7), (8)$$

and

$$G_0 = |S_{21}|^2 \quad (9)$$

The S parameters of the transistor are fixed by the device itself as well as the particular DC biasing.

To achieve a specific gain, which is less than the maximum, from the transistor amplifier circuit, mismatch will be purposefully designed into the input and/or output “matching” networks to yield the gain we seek.

In other words, the **source and load gain factors** in (7) and (8) will be adjusted downward from their maximum values $G_{SU_{\max}}$ and $G_{LU_{\max}}$ in order to achieve the specified gain. As we discussed in the previous lecture, the maximum G_S occurs when there is a conjugate match at the transistor input ($Z_S = Z_{in}^*$), so that for a unilateral device

$$\Gamma_S = \Gamma_{in}^* \stackrel{(1)}{=} S_{11}^* \quad (10)$$

Likewise, the maximum G_L occurs when there is a conjugate match at the transistor output ($Z_L = Z_{out}^*$) so that

$$\Gamma_L = \Gamma_{\text{out}}^* \stackrel{(2)}{\equiv} S_{22}^* \quad (11)$$

Applying (10) in (7) and (11) in (8) leads to an expression for the **maximum unilateral transducer gain**, $G_{TU_{\max}}$, given as

$$G_{TU_{\max}} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2} \equiv G_{SU_{\max}} G_0 G_{LU_{\max}} \quad (12)$$

where the maximum source and load gain factors for the unilateral device are

$$G_{SU_{\max}} = \frac{1}{1 - |S_{11}|^2} \quad (12.47a),(13)$$

and

$$G_{LU_{\max}} = \frac{1}{1 - |S_{22}|^2} \quad (12.47b),(14)$$

We will define **normalized gain factors** g_S and g_L to quantify the amount of reduction in the source and load gain factors needed to achieve the desired gain. These factors are given as

$$g_S \equiv \frac{G_{SU}}{G_{SU_{\max}}} \stackrel{(7),(13)}{\equiv} \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} (1 - |S_{11}|^2) \quad (12.48a),(15)$$

and

$$g_L \equiv \frac{G_{LU}}{G_{LU_{\max}}} \stackrel{(8),(14)}{\equiv} \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} (1 - |S_{22}|^2) \quad (12.48b),(16)$$

so that $0 \leq g_S \leq 1$ and $0 \leq g_L \leq 1$.

As shown in the text, (15) and (16) can be cast in the forms

$$|\Gamma_S - C_S| = R_S \quad \text{and} \quad |\Gamma_L - C_L| = R_L, \quad (17),(18)$$

respectively, where

$$C_S = \frac{g_S S_{11}^*}{1 - (1 - g_S) |S_{11}|^2} \quad (12.51a), (19)$$

$$R_S = \frac{\sqrt{1 - g_S} (1 - |S_{11}|^2)}{1 - (1 - g_S) |S_{11}|^2} \quad (12.51b), (20)$$

and

$$C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L) |S_{22}|^2} \quad (12.52a), (21)$$

$$R_L = \frac{\sqrt{1 - g_L} (1 - |S_{22}|^2)}{1 - (1 - g_L) |S_{22}|^2} \quad (12.52b), (22)$$

As we've seen many times now, (17) and (18) are equations for circles in the complex Γ_S and Γ_L planes, respectively. Any value of Γ_S or Γ_L on these circles will produce the desired normalized gain factor.

These so-called **constant gain circles** can be drawn on a Smith chart to facilitate the design of an amplifier circuit for a specified gain, which is illustrated in the following example.

Example N36.1 (Text example 12.4). Design an amplifier for 11 dB gain at 4 GHz. Plot the constant gain circles for $G_S = 2$ dB and 3 dB, and $G_L = 0$ dB and 1 dB. Calculate and plot the input return loss and the overall amplifier gain from 3 to 5 GHz. The FET S parameters, referenced to 50- Ω system impedance, are:

$f(\text{GHz})$	S_{11}	S_{21}	S_{12}	S_{22}
3.0	$0.80 \angle -90^\circ$	$2.8 \angle 100^\circ$	0	$0.66 \angle -50^\circ$
4.0	$0.75 \angle -120^\circ$	$2.5 \angle 80^\circ$	0	$0.60 \angle -70^\circ$
5.0	$0.71 \angle -140^\circ$	$2.3 \angle 60^\circ$	0	$0.58 \angle -85^\circ$

Since $S_{12} = 0$, we have a unilateral transistor. (This is just an approximation for a real device, of course, that has a small $|S_{12}|$.)

The first step in this design process is to ensure that the device is unconditionally stable at 4.0 GHz. With $S_{12} = 0$, the mu stability factor is

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta| + |S_{21} S_{12}|} = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* S_{11} S_{22}|}$$

If $|S_{11}| < 1$ and $|S_{22}| < 1$ then $\mu > 1$, so this FET is **unconditionally stable**.

From (13) and (14), the maximum gain factors at 4 GHz are

$$G_{SU_{\max}} = \frac{1}{1 - 0.75^2} = 2.29 \quad (= 3.59 \text{ dB})$$

and

$$G_{LU_{\max}} = \frac{1}{1 - 0.60^2} = 1.56 \quad (= 1.94 \text{ dB})$$

Additionally, from (9)

$$G_0 = |S_{21}|^2 = 2.5^2 = 6.25 \quad (= 7.96 \text{ dB})$$

Consequently, from (12) the maximum unilateral transducer gain from this FET at 4 GHz is the sum of these dB factors:

$$G_{TU_{\max}} = 3.59 \text{ dB} + 1.94 \text{ dB} + 7.96 \text{ dB} = 13.50 \text{ dB}$$

Only 11-dB transducer gain is requested, so there is an **extra 2.5 dB** of gain.

We'll arbitrarily **choose** $G_{SU} = 2$ dB (reducing 1.59 dB) **and** $G_{LU} = 1$ dB (reducing by 0.94 dB) to reach 11 dB overall gain, as were chosen in the text.

From (15) and (16):

$$g_S \equiv \frac{G_{SU}}{G_{SU_{\max}}} = \frac{10^{2/10}}{2.29} = 0.69 \quad \text{and} \quad g_L \equiv \frac{G_{LU}}{G_{LU_{\max}}} = \frac{10^{1/10}}{1.56} = 0.81$$

Using these values in (19)-(22) gives the parameters for the constant gain circles to be

$$C_S = 0.63 \angle 120^\circ \quad \text{and} \quad R_S = 0.17$$

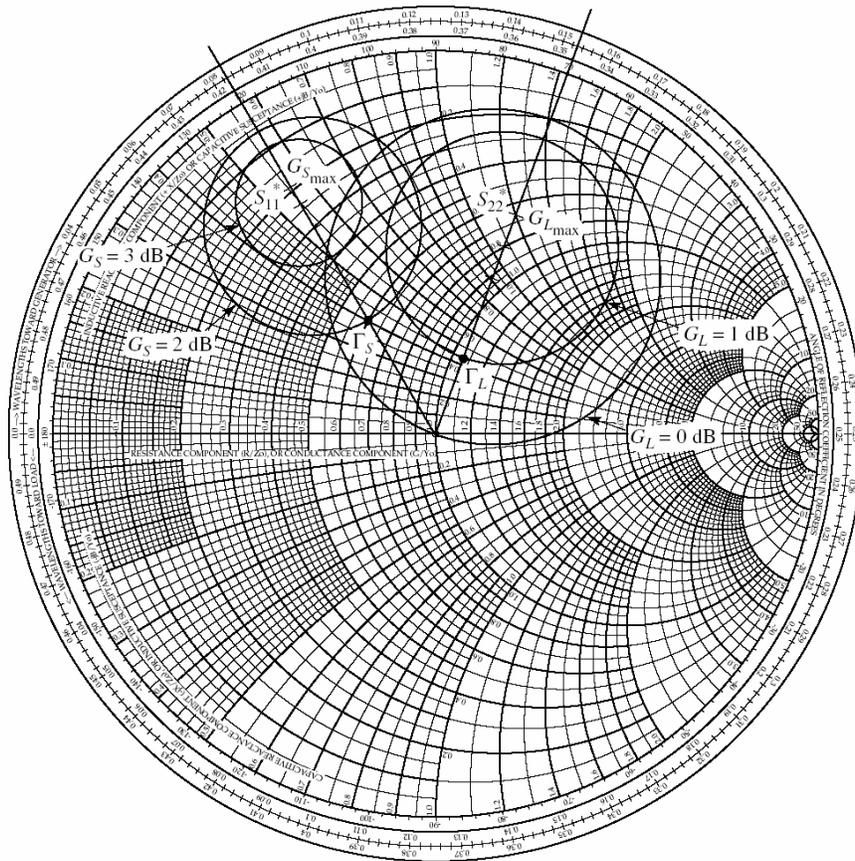
while $C_L = 0.52 \angle 70^\circ \quad \text{and} \quad R_L = 0.30$

These circles are drawn on the Smith chart below (Fig. 12.8), along with the $G_S = 3$ dB and $G_L = 0$ dB circles.

The next step is to choose Γ_S and Γ_L values on these circles. There are an **infinite number of possibilities**, but the points marked on the Smith chart

$$\Gamma_S = 0.33 \angle 120^\circ \quad \text{and} \quad \Gamma_L = 0.22 \angle 70^\circ \quad (23),(24)$$

are those **closest to the origin** and provide the best match at 4 GHz. Presumably, these will also lead to a broader bandwidth.



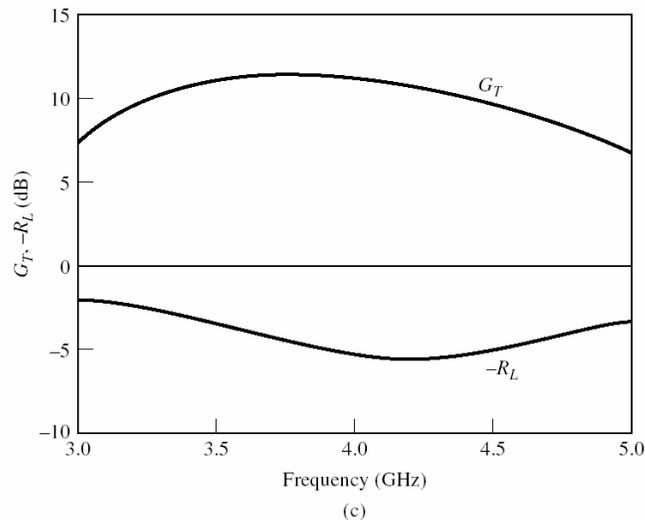
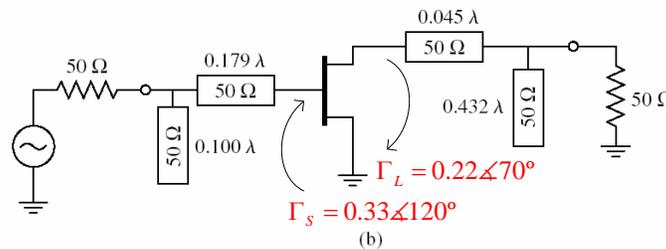
(a)

Notice the points $G_{S_{\max}}$ and $G_{L_{\max}}$. These are actually the points $C_S|_{\max}$ and $C_L|_{\max}$ from (19) and (21), respectively, with $g_S = 1$ and $g_L = 1$, respectively. Think of these as constant gain circles with radius equal to zero, which is what they actually are.

The final step is to design the input and output “matching” networks to provide the necessary amount of mismatch in the IMN and OMN dictated by (23) and (24).

Most any type of lossless matching network could be used for this function. Your text chose shunt open circuit stubs, as shown in the figure below (Fig. 12.8). (The design process for this stub design is identical to that discussed in Example N35.1)

The resulting unilateral transducer gain G_{TU} and return loss (-RL) are shown as well, as computed from a CAD package.



A gain of approximately 11 dB was achieved at 4 GHz, with a return loss of approximately 5 dB (which is not too good, but a necessary byproduct to reduce the gain by 2.5 dB). The relative bandwidth over which G_{TU} varies by less than ± 1 dB is approximately 25%. This is significantly larger than that for the

maximum gain amplifier we designed in Lecture 35, though a different transistor was used in that example so a direct comparison of the results from these two examples is not accurate.