Lecture 35 – Single Stage Amplifier: Design for Maximum Gain.

Amplifiers must be designed for different performance requirements that depend on the application. Examples of these different requirements are maximum gain, maximum output power, specific gain, circuit stability with varying load impedance, wide bandwidth, and low noise.

Often, the first amplifier in a microwave-frequency receiver will be a low noise amplifier (LNA). After the signal level is raised well above the noise level, gain often becomes more important than noise in amplifier design.

However, we will first consider the design of amplifiers that are needed for large gain rather than for their noise or bandwidth characteristics.

To realize maximum gain, the input and output matching networks are simultaneously conjugate matched to the transistor. We also need the entire amplifier system (the transistor and matching networks) to be matched to the system impedance.

We will use the transducer gain approach to design the amplifier of Figure 12.2 for maximum gain:

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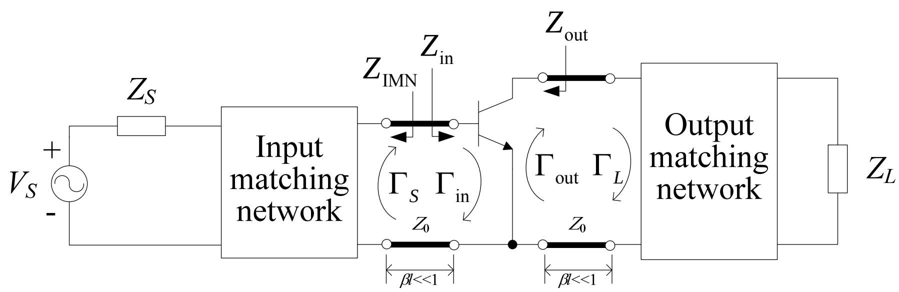


In this approach, we begin with the transducer gain we developed in Lecture 33:

$$G_T = \frac{P_L}{P_{av,S}} = G_S G_0 G_L \tag{1}$$

where
$$G_S = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{\text{in}}\Gamma_S|^2} \qquad \text{(12.16a),(2)}$$

$$G_0 = |S_{21}|^2 \qquad \text{(12.16b),(3)}$$

and
$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \qquad \text{(12.16c),(4)}$$

Maximum power transfer from the input matching network (IMN) to the transistor occurs when the two are conjugate matched so that $Z_{\text{in}} = Z_{\text{IMN}}^*$. For a real-valued system impedance, this leads to the requirement (can you verify this?)

$$\Gamma_{\text{in}} = \Gamma_S^* \qquad \text{(12.36a),(5)}$$

Similarly, maximum power is transferred from the transistor to the output matching network (OMN) when

$$\Gamma_{\text{out}} = \Gamma_L^* \qquad \text{(12.36b),(6)}$$

When (5) and (6) are realized in the amplifier circuit, maximum transducer gain will be obtained (for lossless matching networks). The value of this maximum transducer gain, $G_{T_{\max}}$, is found by substituting (5) into (1)

$$G_{T_{\max}} = \frac{1}{1-|\Gamma_S|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \qquad (12.37),(7)$$

In order to achieve this gain, the IMN and OMN must be simultaneously designed so that (5) and (6) are satisfied. To develop these design equations, we substitute the expressions for Γ_{in} and Γ_{out} stated in Lecture 34 into (5) and (6) giving

$$\Gamma_{MS}^* \equiv \Gamma_S^*\Big|_{\text{C.M.}} = \Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_{ML}}{1-S_{22}\Gamma_{ML}} \qquad (12.38a),(8)$$

$$\Gamma_{ML}^* \equiv \Gamma_L^*\Big|_{\text{C.M.}} = \Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_{MS}}{1-S_{11}\Gamma_{MS}} \qquad (12.38b),(9)$$

where the ‘M’ subscript indicates values obtained with conjugate matching (C.M.). [So, with these definitions, it is Γ_{MS} and Γ_{ML} that are used in (7) for Γ_S and Γ_L, respectively.]

We can use these two equations (8) and (9) to solve for the two unknowns Γ_{MS} and Γ_{ML}. As shown in the text, these solutions are

$$\boxed{\Gamma_{MS} = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}} \qquad (12.40a),(10)$$

and

$$\boxed{\Gamma_{ML} = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}} \qquad (12.40b),(11)$$

(not '$\pm$' as in the text – see Gilmore and Besser, vol. II, p. 79). In (10) and (11):

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \qquad (12.41a),(12)$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \qquad (12.41b),(13)$$

$$C_1 = S_{11} - S_{22}^*\Delta \qquad (12.41c),(14)$$

$$C_2 = S_{22} - S_{11}^*\Delta \qquad (12.41d),(15)$$

and

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \qquad (12.21),(16)$$

These design equations will produce physically realizable, passive matching networks provided the transistor is unconditionally stable (or has been "stabilized").

The final step is to construct these lossless passive matching networks. This will be illustrated in the following example.

Example N35.1 (Text example 12.3). For the amplifier circuit shown earlier in this lecture, design input and output matching networks to achieve maximum transducer gain at 4 GHz. The *S* parameters for the transistor, referenced to 50 Ω, are:

f(GHz)	S_{11}	S_{21}	S_{12}	S_{22}
3.0	$0.80\measuredangle -89^\circ$	$2.86\measuredangle 99^\circ$	$0.03\measuredangle 56^\circ$	$0.76\measuredangle -41^\circ$
4.0	$0.72\measuredangle -116^\circ$	$2.60\measuredangle 76^\circ$	$0.03\measuredangle 57^\circ$	$0.73\measuredangle -54^\circ$
5.0	$0.66\measuredangle -142^\circ$	$2.39\measuredangle 54^\circ$	$0.03\measuredangle 62^\circ$	$0.72\measuredangle -68^\circ$

The first step in this design process is to ensure that the device is unconditionally stable at 4.0 GHz.

Applying the μ test from (12.30), we find

$$\mu = 1.04 > 1 \tag{17}$$

Hence, the transistor is unconditionally stable (but just barely). Consequently a passive, lossless matching network can be designed to produce maximum transducer gain from this transistor.

From (10) and (11) we find for this amplifier at 4.0 GHz that

$$\Gamma_{MS} = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} = 0.872\measuredangle 123^\circ \tag{18}$$

$$\Gamma_{ML} = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} = 0.876\measuredangle 61^\circ \tag{19}$$

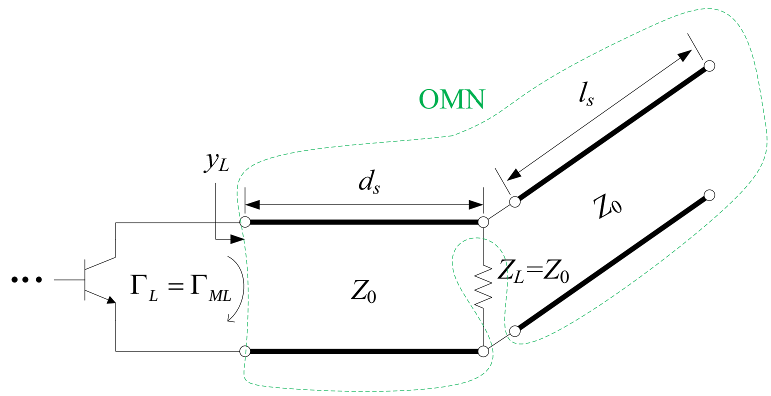
These values of Γ_{MS} and Γ_{ML} will produce the maximum transducer gain in (7) as

$$G_{T_{\max}} = \frac{1}{1-|\Gamma_{MS}|^2}|S_{21}|^2 \frac{1-|\Gamma_{ML}|^2}{|1-S_{22}\Gamma_{ML}|^2} = 16.7 \text{ dB}$$

To achieve this maximum gain, we need to design passive, lossless matching networks that produce (18) and (19).

As in the text, we will use single stub matching networks with open circuit stubs for these matching networks. The text illustrates this procedure for the source (the IMN). Here we will determine the output matching network that produces (19).

The stub matching network connected to the transistor output (in this case the collector and emitter leads) will look like:

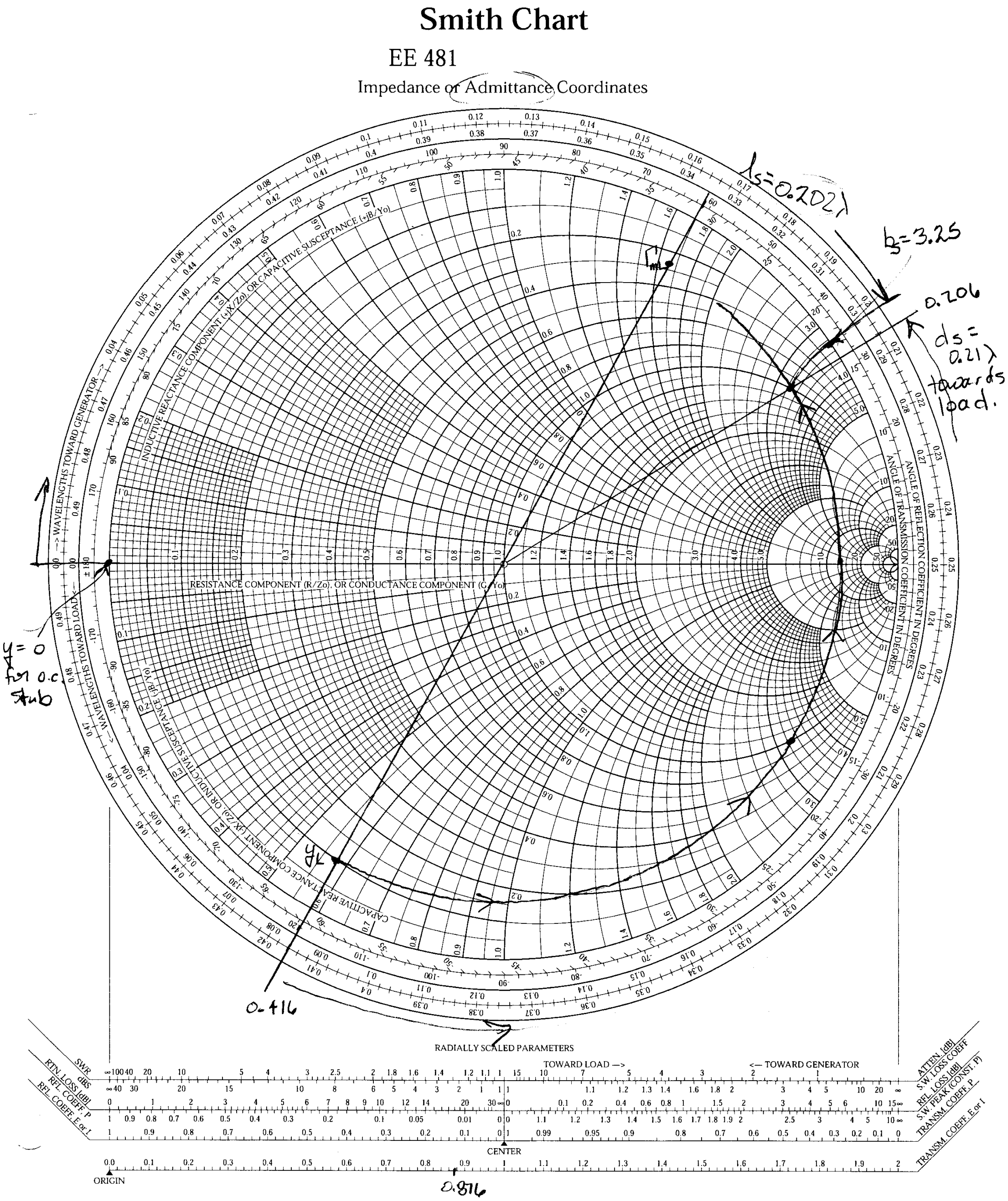


We are given Γ_{ML} so we need to work "backwards" compared to the other matching problems we have done. The procedure is:

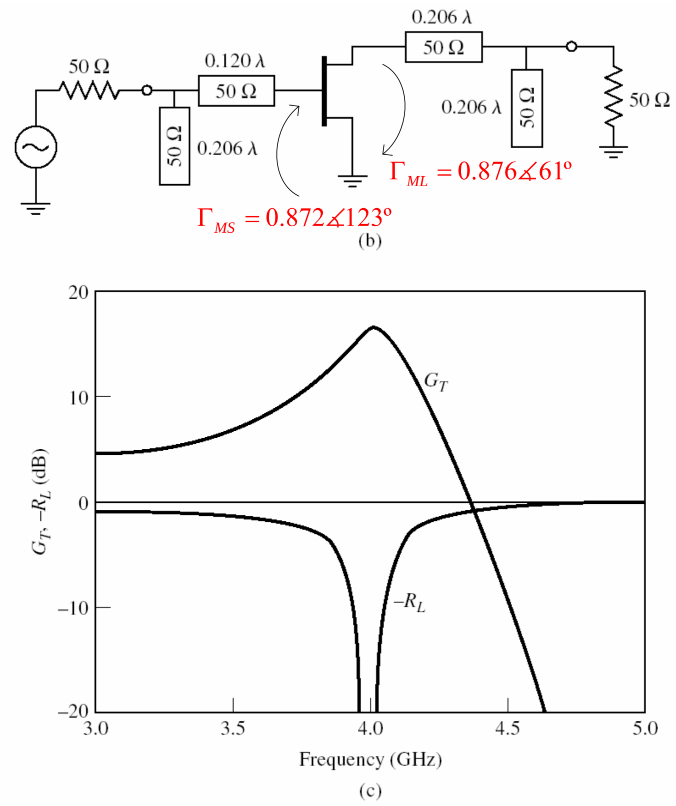
1. Start by marking Γ_{ML} on the Smith impedance chart ($=0.876\measuredangle 61^\circ$). In these "dual use" Smith charts, the phase angle of Γ is only correctly shown for the impedance chart. (See EE 382 *Applied Electromagnetics* Lecture 24 notes on the derivation of the Smith admittance chart.)
2. Compute y_L on the Smith admittance chart.
3. Move distance d_s **towards** the **load** to the unit admittance circle.
4. Compute the stub length in the regular fashion. (Notice, however, that we are not trying to negate a shunt susceptance here as we've done previously with shunt stub tuners. Here we're creating a desired admittance at the load

position by adding $1/Z_L = 1/Z_0$ to the shunt stub input admittance.)

These steps are illustrated on the Smith chart shown below. From this graph we find that $d_s \approx 0.210\lambda$ and $l_s \approx 0.202\lambda$.



The resulting small signal circuit is shown below (Fig. 12.7) along with a plot of G_T and –RL (return loss, not $-R_L$). These were computed using a CAD package and, most likely, interpolating the transistor S parameters between those given at the discrete frequencies 3, 4, and 5 GHz.



The desired gain of 16.7 dB was attained at 4 GHz with a good match at port 1. The response is narrowband because the transistor is not well matched to 50 Ω since $|S_{11}| \approx 0.6 - 0.8$ and $|S_{22}| \approx 0.7 - 0.8$. However, bandwidth was not a consideration in this design. We designed for maximum gain at a specified frequency.

Note that a dc bias network is not shown, nor has its effects been accounted for in these simulated results. The *S* parameters were simply used in the simulation, but the bias network needed to achieve the operating point was neglected. Not good.

Computing Transducer Gain

In the previous example, we verified that maximum gain was achieved by plotting G_T over the frequency range of interest (3-5 GHz). This quantity and RL are easily computed using ADS.

If both the source and load impedances are matched loads ($\Gamma_S = \Gamma_L = 0$), then from (1) it can be shown that

$$\boxed{G_T = |S_{21}|^2} \quad \text{and} \quad \boxed{\text{RL} = -20\log_{10}|S_{11}|}. \tag{20}$$

This wasn't the case, though, in the previous example. However, for a comparison, if $\Gamma_S = \Gamma_L = 0$ then at 4 GHz

$$G_T = 10\log_{10}\left(2.6^2\right) = 8.30 \text{ dB}$$

This is 8.4 dB (i.e., a factor of 6.9) less power gain than if a selected mismatch was designed at the input and output ports of the transistor, as performed in the previous example. Very interesting!

Actually, this topic was discussed earlier in this course in Lecture 5 when we covered maximum power transfer on transmission lines. In the previous amplifier design we see that creating a specific mismatch at the input and output ports of the transistor has allowed us to transfer maximum power because the ports are then conjugate matched. This is precisely what we would expect from the maximum power transfer theorem.