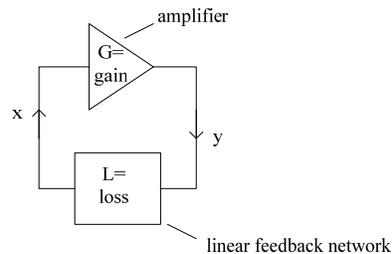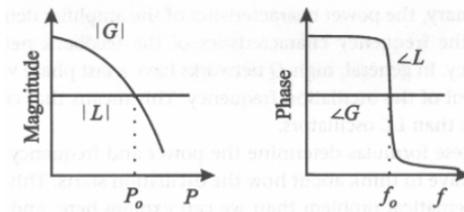# Lecture 34 – Amplifier Stability.

You've seen in EE 322 that a simple model for a feedback oscillator has an amplifier and a feedback network connected as:

Oscillation occurs at the output power P_0 and frequency f_0 where:

Anytime a portion of a circuit has gain, the circuit may oscillate, even though that is not the intended function of the circuit. For example, an amplifier circuit has gain and if it's not properly designed, it may oscillate.

This concept is referred to as circuit instability.

A circuit that is not stable **may** oscillate. This "oscillation" may not be easily detected. That is, you will probably not measure nice sinusoids at the circuit nodes.

Instead, the circuit may just not "work," or you will see DC voltages that "are not possible," or the waveforms are incredibly "noisy", etc. A very frustrating experience, especially if you don't understand circuit stability.

Even a brief period of oscillation could permanently damage a circuit because of large voltages and power levels that might be generated.

A thorough stability analysis requires a large signal analysis of the nonlinear circuit. **Very** difficult.

Here, we will perform a much simpler two-port, *S*-parameter analysis. This is sufficient only for linear, small signal circuit applications. This analysis can be considered accurate as a test for startup instabilities. It can also provide a starting point for large signal analysis.

Negative Resistance

Consider the generic linear, small-signal transistor amplifier circuit we saw in the last lecture:
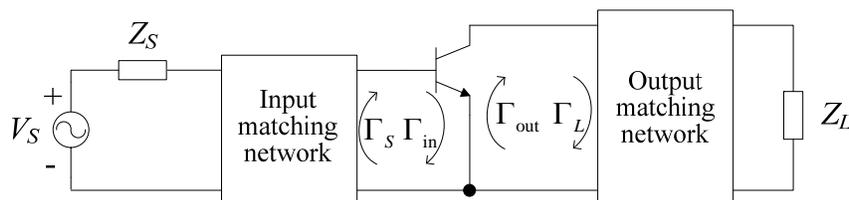
We will imagine the transistor is characterized by the S matrix:

$$[S_t] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{1}$$

Further, we will assume the matching networks are passive so that

$$|\Gamma_S| < 1 \quad \text{and} \quad |\Gamma_L| < 1 \tag{2}$$

Oscillation is possible in this circuit if a signal "incident" on the input or output port of the transistor is "reflected" with a gain > 1. That is, if

$$|\Gamma_{in}| > 1 \quad \text{or} \quad |\Gamma_{out}| > 1 \tag{3}$$

the circuit may become unstable and oscillate.

This could occur when noise in the circuit is incident on either port, is reflected with gain, is reflected again at the corresponding matching network, and so on. It is then possible that at some frequency, such noise could be amplified repeatedly to such a level that the device is forced into nonlinear operation and instability.

In order for $|\Gamma| > 1$, the real part of the impedance seen looking into the port must be negative. That is,

$$R_{in} < 0 \quad \text{or} \quad R_{out} < 0 \tag{4}$$

for instability. This region lies outside the unit circle on a Smith chart, as we'll see shortly.

As we discussed in the previous lecture

$$\Gamma_{\text{in}} = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1 - \Gamma_L S_{22}} \qquad (12.3a),(5)$$

$$\Gamma_{\text{out}} = S_{22} + \frac{\Gamma_S S_{12} S_{21}}{1 - \Gamma_S S_{11}} \qquad (12.3b),(6)$$

Using these in (3), we find that for the circuit to be unconditionally stable:

$$\left|\Gamma_{\text{in}}\right| = \left|S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1 - \Gamma_L S_{22}}\right| < 1 \qquad (12.19a),(7)$$

and

$$\left|\Gamma_{\text{out}}\right| = \left|S_{22} + \frac{\Gamma_S S_{12} S_{21}}{1 - \Gamma_S S_{11}}\right| < 1 \qquad (12.19b),(8)$$

[As an aside, if $\left|\Gamma_S\right| > 1$ or $\left|\Gamma_L\right| > 1$, then (7) and (8) are requirements only for conditional stability.]

Stability Circles

It can be very helpful to generate a graphical depiction of the range of Γ_S and Γ_L values that may lead to instability. Stability circles are one way to do this. They are particularly helpful because of the extra information we gain because they are drawn on the Smith chart.

Stability circles define the <u>boundary</u> between stable and potentially unstable Γ_L or Γ_S.

To determine these boundaries, we will set $\left|\Gamma_{in}\right|=1$ (or $\left|\Gamma_{out}\right|=1$) and draw these curves in the Γ_L (or Γ_S) plane.

For the load stability circle, from (5) we find

$$\left|\Gamma_{in}\right|=\left|S_{11}+\frac{\Gamma_L S_{12} S_{21}}{1-\Gamma_L S_{22}}\right|=1 \qquad (12.20),(9)$$

After some manipulation, as shown in the text, (9) can be rearranged to

$$\left|\Gamma_L - C_L\right|= R_L \qquad (10)$$

which is an equation for a circle in the *complex* Γ_L plane.

In (10)
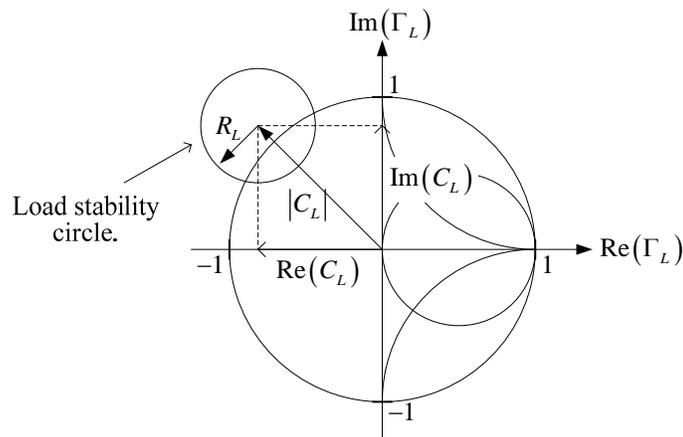$$C_L = \frac{\left(S_{22}-S_{11}^{*}\Delta\right)^{*}}{|S_{22}|^2 - |\Delta|^2} \qquad (12.25a),(11)$$

is the center of the circle in the complex Γ_L plane, and

$$R_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \qquad (12.25b),(12)$$

is the radius, where

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \qquad (12.21),(13)$$

For a conditionally stable circuit, the load stability circle may look something like this on the extended Smith chart:

Again, this circle defines those Γ_L values where $\left|\Gamma_{in}\right| = 1$. Inside the load stability circle (and with $\left|\Gamma_L\right| \leq 1$) are those Γ_L that produce a stable *or* unstable circuit. We don't know which yet. The converse can be said for those Γ_L outside of the circle (and with $\left|\Gamma_L\right| \leq 1$).

So how do we identify the stable region? Very easily, as it turns out. We will choose a special load Z_L that will quickly identify the region of stability. Specifically, we'll choose Z_L such that through the output matching network $\Gamma_L = 0$ so that from (5)

$$\left|\Gamma_{in}\right| = \left|S_{11}\right| \tag{14}$$

Consequently, if $\left|S_{11}\right| < 1$, then the region containing the origin in the Γ_L plane (and $\left|\Gamma_L\right| \leq 1$) is the stable region since in that region $\left|\Gamma_{in}\right| < 1$. Otherwise, if $\left|S_{11}\right| > 1$, then the region inside the stability circle (and $\left|\Gamma_L\right| \leq 1$) is the stable region. These two situations are illustrated below.
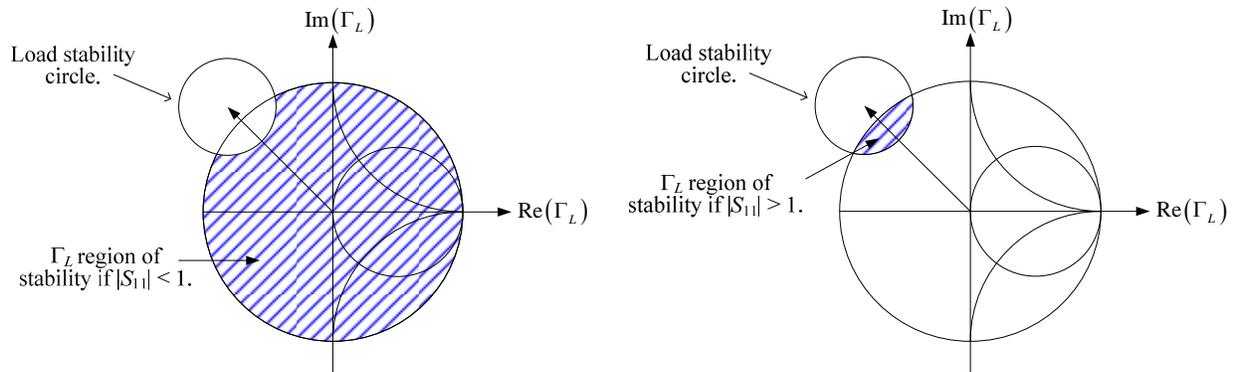
In order for the circuit to be unconditionally stable, the stability circle must lie entirely outside the Smith chart.

Further, if $|S_{11}| > 1$ or $|S_{22}| > 1$ then it is **not** possible for the amplifier circuit to be unconditionally stable because we can always have a load (or source) impedance such that $\Gamma_L = 0$ or $\Gamma_S = 0$. Therefore, again from (5) and (6)

$$|\Gamma_{in}| = |S_{11}| > 1 \quad \text{or} \quad |\Gamma_{out}| = |S_{22}| > 1 \tag{15}$$

Both equations indicate the circuit is always potentially unstable.

It can be shown that the circuit is unconditionally stable if

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta| + |S_{21}S_{12}|} > 1 \qquad (12.30),(16)$$

This is the "mu stability factor." Larger values of μ imply greater stability. There is also a $k - \Delta$ test, but it's not as useful. *ADS* will compute these stability factors, as well as plot single frequency stability circles.

This discussion has shown the stability circles for the load. Beginning with (6), this process can be repeated for the source stability circle. The region of stability for the source stability circle is now defined by S_{22}. If $|S_{22}| < 1$ the region of stability contains the origin in the complex Γ_S plane.

Example N34.1. The S parameters of an Infineon BFP 520 microwave transistor are listed below for a CE amplifier with $V_{CE} = 2$ V and $I_C = 20$ mA. These parameters are referenced to a 50-Ω system impedance at a frequency of 4 GHz. Determine the stability of this amplifier assuming $|\Gamma_S| < 1$ and $|\Gamma_L| < 1$.
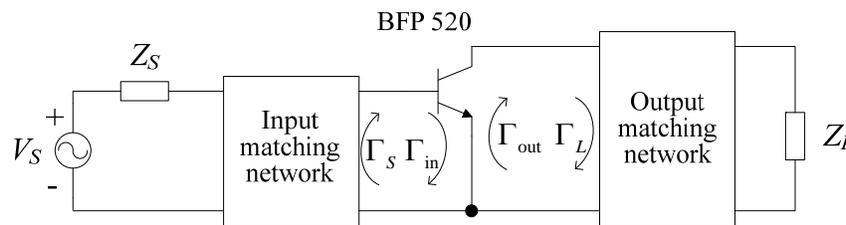
$$S_{11} = 0.2552 \angle 156.2° \qquad S_{12} = 0.0994 \angle 41.5°$$
$$S_{21} = 5.636 \angle 53.1° \qquad S_{22} = 0.1544 \angle -95.3°$$

We will use the μ stability factor to test the stability. From (13)
$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.527 \angle -83.0°$$
so that from (16)
$$\mu = \frac{0.9349}{0.2747 + 0.5602} = 1.12 > 1$$

Since $\mu > 1$ this implies that the amplifier is unconditionally stable, at least at 4 GHz.

However, it is important to test for stability at all frequencies less than f_T of the transistor (where gain > 1). For example, from the BFP 520 data sheet at 0.1 GHz

$$S_{11} = 0.7251\angle -8.4° \qquad S_{12} = 0.0041\angle 92.8°$$

$$S_{21} = 31.637\angle 171.4° \qquad S_{22} = 0.9363\angle -4.4°$$

Using these values in (16) we find that $\mu = 0.798 < 1$. Hence, at 100 MHz the amplifier is only conditionally stable (potentially unstable).

While 100 MHz may not be in the passband of the amplifier circuit, it is possible that nonlinearities at 100 MHz could mix with incoming signals and produce output in the passband. Then the whole circuit would oscillate! Tricky.
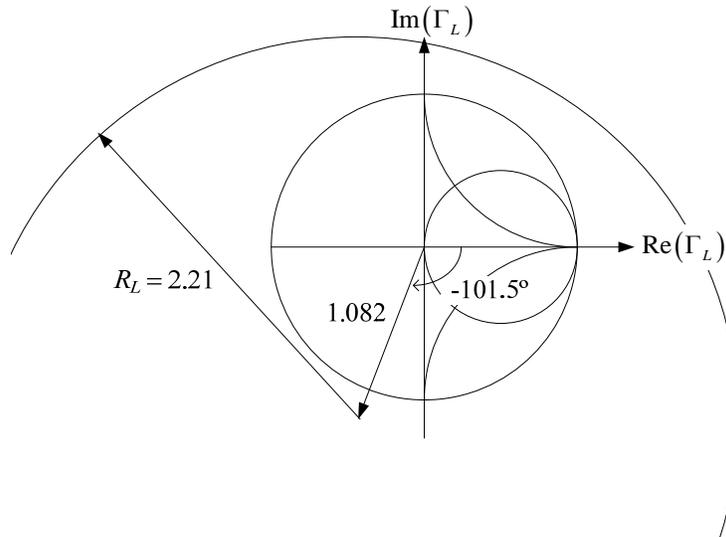
Next, we'll sketch the load stability circles at 4 GHz and at 0.1 GHz. From (11) and (12) at 4 GHz

$$C_L = \frac{(S_{22} - S_{11}^* \Delta)^*}{|S_{22}|^2 - |\Delta|^2}$$

$$= \frac{\left[0.1544\angle -95.3° - (0.2552\angle -156.2°)(0.527\angle -83.0°)\right]^*}{0.1544^2 - 0.527^2}$$

such that $\quad C_L = \dfrac{0.2747\angle 78.5°}{-0.2539} = 1.082\angle -101.5°$

The radius of this load stability circle is from (12)

$$R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{(0.099\angle 41.5°)(5.636\angle 53.1°)}{0.1544^2 - 0.527^2} \right| = 2.21$$

Is the region of stability inside or outside this circle? Since $|S_{11}| < 1$ the region of stability is inside the circle $\Rightarrow$ the circuit is unconditionally stable.

We see from this example that if the circuit is unconditionally stable (as this one is at 4 GHz), there really isn't any need to plot the stability circle.

At 0.1 GHz, we'll use ADS to compute both the load and source stability circles. (The stability circles for 4 GHz are also shown for comparison.)
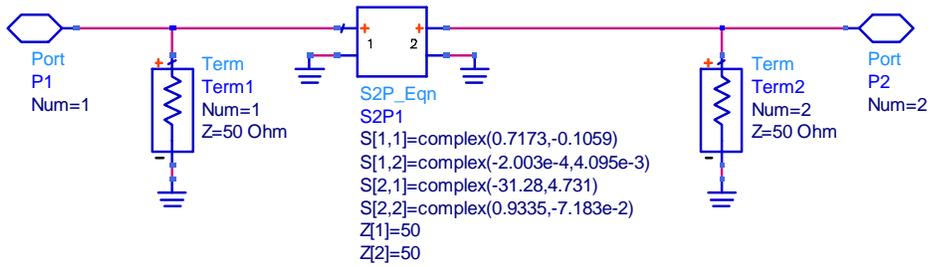
S-PARAMETERS

S_Param
SP1
Start=0.1 GHz
Stop=0.1 GHz
Step=0.1 GHz

Port
P1
Num=1

Term
Term1
Num=1
Z=50 Ohm

S2P_Eqn
S2P1
S[1,1]=complex(0.7173,-0.1059)
S[1,2]=complex(-2.003e-4,4.095e-3)
S[2,1]=complex(-31.28,4.731)
S[2,2]=complex(0.9335,-7.183e-2)
Z[1]=50
Z[2]=50

Term
Term2
Num=2
Z=50 Ohm

Port
P2
Num=2

L_StabCircle
L_StabCircle1
L_StabCircle1=l_stab_circle(S,51)

S_StabCircle
S_StabCircle1
S_StabCircle1=s_stab_circle(S,51)

Mu
Mu1
Mu1=mu(S)

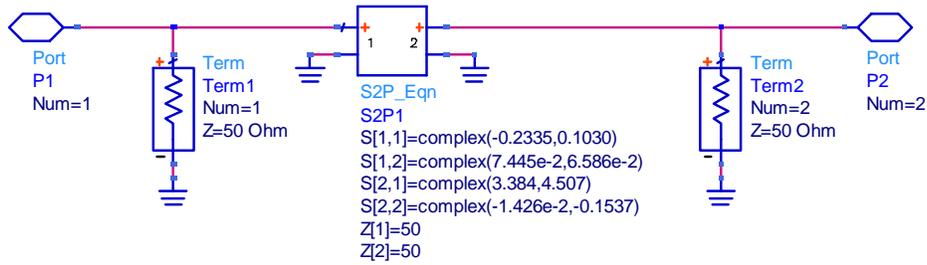| freq	Mu1
100.0MHz	0.798

l_stab_region(S)	s_stab_region(S)
Outside	Outside
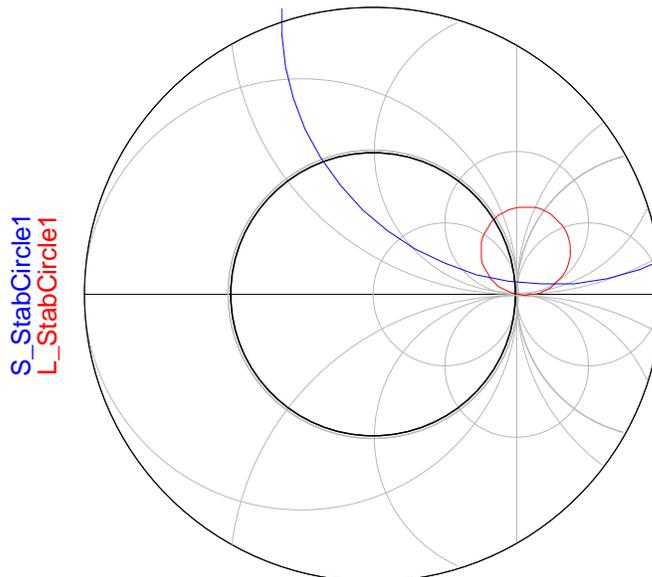
indep(L_StabCircle1) (0.000 to 51.000)
indep(S_StabCircle1) (0.000 to 51.000)

S-PARAMETERS

S_Param
SP1
Start=4 GHz
Stop=4 GHz
Step=0.1 GHz

Port
P1
Num=1

Term
Term1
Num=1
Z=50 Ohm

S2P_Eqn
S2P1
S[1,1]=complex(-0.2335,0.1030)
S[1,2]=complex(7.445e-2,6.586e-2)
S[2,1]=complex(3.384,4.507)
S[2,2]=complex(-1.426e-2,-0.1537)
Z[1]=50
Z[2]=50

Term
Term2
Num=2
Z=50 Ohm

Port
P2
Num=2

L_StabCircle
L_StabCircle1
L_StabCircle1=l_stab_circle(S,51)

S_StabCircle
S_StabCircle1
S_StabCircle1=s_stab_circle(S,51)

Mu
Mu1
Mu1=mu(S)

freq	Mu1
4.000GHz	1.119

l_stab_region(S)	s_stab_region(S)
Inside	Inside

indep(L_StabCircle1) (0.000 to 51.000)
indep(S_StabCircle1) (0.000 to 51.000)