Lecture 33 – Active Microwave Circuits: Two-Port Power Gains.

We are going to focus on active microwave circuits for the remainder of the semester. There are many types of active circuits such as amplifiers, oscillators, and mixers. We will concentrate only on amplifiers.

It is often a much more involved process to design and construct active circuits that operate correctly than passive ones. Reasons for this include:

- A bias network is required,
- The devices are nonlinear,
- Unintended oscillations produced by circuit instability.

More care, patience, and experience are often required in the design of active RF and microwave circuits than purely passive ones.

The analysis of such circuits is usually very difficult given the nonlinear behavior of the devices. For linear amplifiers, though, a linear analysis is applicable, which helps simplify matters.

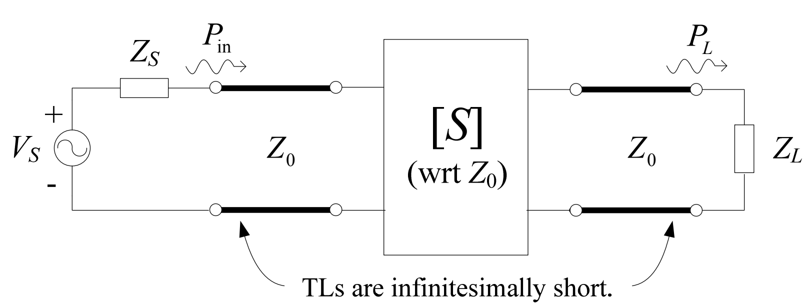
For this reason, we will focus on linear, small signal amplifiers. Furthermore, we will use measured (or given) *S* parameters for the devices (transistors) rather than detailed device parameters (β, C_π, r_π, etc.). Consequently, we can treat the transistor as a two port, but possibly one with gain. This approach works well for the steady state analysis of linear, small-signal amplifiers.

For other types of active circuits, such as oscillators, mixers, or power amplifiers, the nonlinear behavior of the circuit devices must be explicitly accounted for, which precludes the use of S parameters. Much more difficult.

One big difference with active devices is that the magnitude of the S parameters may be greater than one. Often it is only S_{21} that has this characteristic, with port 1 serving as the input and port 2 the output. With passive devices, S parameters with magnitudes greater than unity are physically impossible.

Types of Power Gains

Referring to a generic two-port network circuit such as



there are three commonly used definitions for power gain.

1. Operating Power Gain: $$G = \frac{P_L}{P_{in}} \quad (1)$$

This is the ratio of the time-average power dissipated in a load to the time-average power delivered to the network.

2. Available Gain:
$$G_A = \frac{P_{av,n}}{P_{av,S}} \quad (2)$$

This is the ratio of the maximally available time-average power from the network to the maximally available time-average power from the source.

3. Transducer Gain:
$$G_T = \frac{P_L}{P_{av,S}} \quad (3)$$

This is the ratio of the time-average power dissipated in the load to the maximally available time-average power from the source.

It is this latter transducer gain that you used in EE 322 *Electronics II – Wireless Communication Electronics* to characterize the performance (i.e. gain) of the active devices in circuits.

Among other applications, these three definitions of power gain are used to design different types of amplifiers:

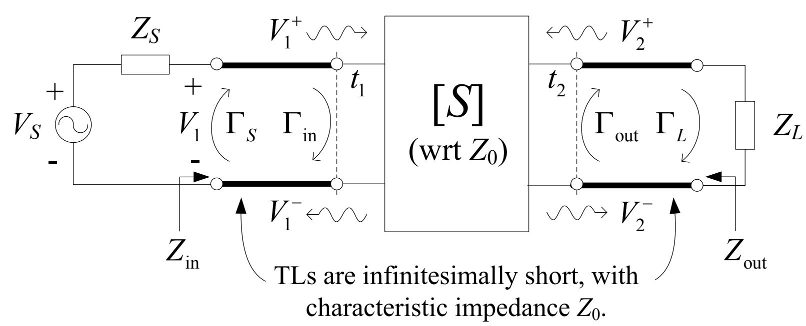
1. Operating Power Gain, *G*. Maximum linear output power amplifiers.
2. Available Power Gain, G_A. Low Noise Amplifiers (LNAs).

3. Transducer Power Gain, G_T. Simultaneously conjugate matched input and output ports (leads to maximum linear gain).

Power Gain Expressions

We will now derive analytical expressions for these power gains in terms of the S parameters of the network, as well as the source and load impedances. These will prove central to the design of linear microwave amplifiers.

Referring to this generic two-port circuit (Fig. 12.1):



then by the definition of the S parameters we can write

$$V_1^- = S_{11}V_1^+ + S_{12}\Gamma_L V_2^- \qquad (12.2a),(4)$$

and

$$V_2^- = S_{21}V_1^+ + S_{22}\Gamma_L V_2^- \qquad (12.2b),(5)$$

In these equations we have used the relationship $V_2^+ = \Gamma_L V_2^-$.

As we showed in Lecture 21 using signal flow graphs

$$\Gamma_{\text{in}} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1 - \Gamma_L S_{22}} \qquad (12.3a),(6)$$

Similarly, it can be show that

$$\Gamma_{\text{out}} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{\Gamma_S S_{12} S_{21}}{1 - \Gamma_S S_{11}} \qquad (12.3b),(7)$$

Next, by voltage division at the source and for an infinitesimally short TL

$$V_1 = \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_S} V_S = V_1^+ + V_1^- = V_1^+ \left(1 + \Gamma_{\text{in}}\right) \qquad (8)$$

so that

$$V_1^+ = \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_S} \frac{V_S}{1 + \Gamma_{\text{in}}} \qquad (9)$$

Now, using $\Gamma_{\text{in}} = \left(Z_{\text{in}} - Z_0\right) / \left(Z_{\text{in}} + Z_0\right)$ and after some algebra, (9) can be reduced to

$$V_1^+ = \frac{1 - \Gamma_S}{1 - \Gamma_S \Gamma_{\text{in}}} \cdot \frac{V_S}{2} \qquad (12.4),(10)$$

There are four different time-average power quantities we need to determine in order to compute (1)-(3):

1. ***P_{in}*:** Time-average power provided by the source

$$P_{\text{in}} = \frac{|V_1^+|^2}{2Z_0} \left(1 - |\Gamma_{\text{in}}|^2\right) \qquad (12.5),(11)$$

Substituting for V_1^+ from (10) gives

$$P_{\text{in}} = \frac{|V_S|^2}{8Z_0} \frac{|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_{\text{in}}|^2}\left(1-|\Gamma_{\text{in}}|^2\right) \qquad (12.5),(12)$$

2. ***P***P_L: Time-average power delivered to the load. This quantity is similar to (11):

$$P_L = \frac{|V_2^-|^2}{2Z_0}\left(1-|\Gamma_L|^2\right) \qquad (12.6),(13)$$

Using (5) and (10) in (13), as shown in the text,

$$P_L = \frac{|V_S|^2}{8Z_0}|S_{21}|^2 \frac{\left(1-|\Gamma_L|^2\right)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{\text{in}}|^2} \qquad (12.7),(14)$$

3. $P_{av,s}$: Maximum available power from the source (and supplied to the circuit). This occurs when $Z_{\text{in}} = Z_S^* \Rightarrow \Gamma_{\text{in}} = \Gamma_S^*$ (i.e., conjugate match). So, from P_{in} in (12) and with $\Gamma_{\text{in}} = \Gamma_S^*$:

$$P_{av,S} \equiv P_{\text{in}}\big|_{\Gamma_{\text{in}}=\Gamma_S^*} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2\left(1-|\Gamma_S|^2\right)}{|1-|\Gamma_S|^2|^2}$$

But with $|1-|\Gamma_S|^2|^2 = \left(1-|\Gamma_S|^2\right)^2$ then

$$P_{av,S} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{1-|\Gamma_S|^2} \qquad (12.9),(15)$$

4. $P_{av,n}$: Maximum available power from the network (and supplied to the load). This occurs when $Z_L = Z_{\text{out}}^* \Rightarrow \Gamma_L = \Gamma_{\text{out}}^*$ (i.e., conjugate match). From (14) and with $\Gamma_L = \Gamma_{\text{out}}^*$:

$$P_{av,n} \equiv P_L\big|_{\Gamma_L=\Gamma^*_{\text{out}}} = \frac{|V_S|^2}{8Z_0}|S_{21}|^2 \frac{\left(1-|\Gamma_{\text{out}}|^2\right)|1-\Gamma_S|^2}{|1-S_{22}\Gamma^*_{\text{out}}|^2|1-\Gamma_S\Gamma_{\text{in}}|^2}$$

Using (6) and after considerable algebra, it can be shown that

$$P_{av,n} = \frac{|V_S|^2}{8Z_0}|S_{21}|^2 \frac{|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{\text{out}}|^2\right)} \qquad (12.11),(16)$$

With these four time-average power quantities in (12) and (14)-(16), we are now in a position to compute the three power gain expressions.

- Operating Power Gain, G. From (1) and substituting (12) and (14):

$$G = \frac{P_L}{P_{\text{in}}} = |S_{21}|^2 \frac{\left(1-|\Gamma_L|^2\right)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2\,|1-\Gamma_S\Gamma_{\text{in}}|^2} \frac{|1-\Gamma_S\Gamma_{\text{in}}|^2}{|1-\Gamma_S|^2\left(1-|\Gamma_{\text{in}}|^2\right)}$$

or

$$G = \underbrace{\frac{1}{1-|\Gamma_{\text{in}}|^2}}_{\text{Source end}}|S_{21}|^2 \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{\text{Load end}} \qquad (12.8),(17)$$

- Available Gain, G_A. From (2) and substituting (15) and (16):

$$G_A = \frac{P_{av,n}}{P_{av,S}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2 \frac{1}{1-|\Gamma_{\text{out}}|^2} \qquad (12.12),(18)$$

- Transducer Gain, G_T. From (3) and substituting (14) and (15):

$$G_T = \frac{P_L}{P_{av,S}} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{\text{in}}|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \qquad (12.13),(19)$$

It can also be shown that G_T can be expressed as

$$G_T = \frac{P_L}{P_{av,S}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{\text{out}}\Gamma_L|^2} \qquad (20)$$

Discussion

(i) All of these gain expressions (17)-(20) are formed by the product of three factors. The first and third describe how the power gain is reduced (or accentuated) by the source and load circuits, respectively.

(ii) G and G_A contain portions of G_T. More specifically, the last two terms in G are the same as those in (19), while the first two terms in G_A are the same as those in (20).

(iii) It is apparent from (17) that G is not dependent on Γ_S (or Z_S). From (18) we deduce that G_A is not dependent on Γ_L (or Z_L). However, G_T is dependent on both Γ_S and Γ_L.

(iv) If the source and load are both conjugate matched, (i.e., $\Gamma_{\text{in}} = \Gamma_S^*$ and $\Gamma_{\text{out}} = \Gamma_L^*$) then $G = G_T$ in (19) and $G_A = G_T$ in (20) such that

$$G = G_T = G_A \quad \left(\neq |S_{21}|^2\right) \qquad (21)$$

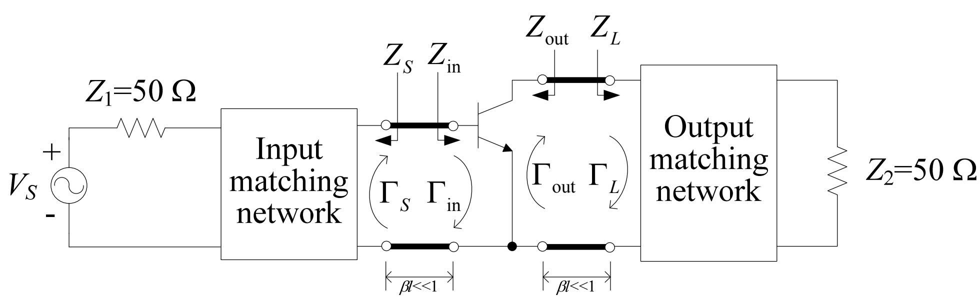
(v) If $\Gamma_S = \Gamma_L = 0$ (i.e., the source and load are matched for zero reflection rather than conjugate matched) then from (19)

$$G_T = \left|S_{21}\right|^2 \qquad (22)$$

while $G = \dfrac{|S_{21}|^2}{1-|\Gamma_{\text{in}}|^2}$ and $G_A = \dfrac{|S_{21}|^2}{1-|\Gamma_{\text{out}}|^2}$.

Example N33.1. (Similar to text example 12.1.) The input and output matching networks shown below are designed to produce $\Gamma_S = 0.5\measuredangle 120^\circ$ and $\Gamma_L = 0.4\measuredangle 90^\circ$. Calculate G, G_A, and G_T given the following S parameters for the transistor.

$$S_{11} = 0.6\measuredangle -160^\circ \ , \quad S_{12} = 0.045\measuredangle 16^\circ$$
$$S_{21} = 2.5\measuredangle 30^\circ \ , \quad S_{22} = 0.5\measuredangle -90^\circ$$



- From (6),

$$\Gamma_{\text{in}} = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1-\Gamma_L S_{22}}$$

$$= 0.6\measuredangle -160^\circ + \frac{0.4\measuredangle 90^\circ \cdot 0.045\measuredangle 16^\circ \cdot 2.5\measuredangle 30^\circ}{1-0.4\measuredangle 90^\circ \cdot 0.5\measuredangle -90^\circ}$$

$$\Gamma_{\text{in}} = 0.627\measuredangle -164.6^\circ$$

- From (7),

$$\Gamma_{\text{out}} = S_{22} + \frac{\Gamma_S S_{12} S_{21}}{1 - \Gamma_S S_{11}}$$

$$= 0.5\measuredangle -90^\circ + \frac{0.5\measuredangle 120^\circ \cdot 0.045\measuredangle 16^\circ \cdot 2.5\measuredangle 30^\circ}{1 - 0.5\measuredangle 120^\circ \cdot 0.6\measuredangle -160^\circ}$$

$$\Gamma_{\text{out}} = 0.471\measuredangle -97.6^\circ$$

With these reflection coefficients and the given S parameters, we can now compute the requested gain quantities.

- From (17),

$$G = \frac{1}{1 - |\Gamma_{\text{in}}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$= \frac{1}{1 - 0.627^2} 2.5^2 \frac{1 - 0.4^2}{|1 - 0.5\measuredangle -90^\circ \cdot 0.4\measuredangle 90^\circ|^2}$$

$$G = 13.52 \quad (11.31\,\text{dB})$$

- From (18),

$$G_A = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |\Gamma_{\text{out}}|^2}$$

$$= \frac{1 - 0.5^2}{|1 - 0.6\measuredangle -160^\circ \cdot 0.5\measuredangle 120^\circ|^2} \cdot 2.5^2 \cdot \frac{1}{1 - 0.471^2}$$

$$G_A = 9.56 \quad (9.80\,\text{dB})$$

- From (19),

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{\text{in}}|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$= \frac{1-0.5^2}{|1-0.5\measuredangle 120^\circ \cdot 0.627\measuredangle -164.6^\circ|^2}\cdot 2.5^2\cdot$$

$$\frac{1-0.4^2}{|1-0.5\measuredangle -90^\circ \cdot 0.4\measuredangle 90^\circ|^2} = 9.44 \quad (9.75\text{ dB})$$

Observe that $G = 13.52 = \dfrac{P_L}{P_{\text{in}}} \Rightarrow P_{\text{in}} = \dfrac{P_L}{13.52}$

while $G_T = 9.44 = \dfrac{P_L}{P_{av,S}} \Rightarrow P_{av,S} = \dfrac{P_L}{9.44}$

We see from these two equations that $P_{\text{in}} < P_{av,S}$. Hence, we can deduce that because $G > G_T$, then the input power, P_{in}, is less than the maximum power available from the source, $P_{av,S}$.

Additionally, with $G_A = \dfrac{P_{av,n}}{P_{av,S}} = 9.56$

and $G_T = \dfrac{P_L}{P_{av,S}} = 9.44$

we can deduce that nearly all of the power available from the network is delivered to the load.