Lecture 32: High Pass and Bandpass Microwave Filters. Resonant Stub Filters.

As has been stated in recent lectures, the low pass prototype can also be used to design high pass, bandpass, and bandstop filters, in addition to low pass filters.

To achieve this, the prototype filter must be "converted," in addition to impedance and frequency scaled. We will discuss the design process for high pass and bandpass filters in this lecture.

High Pass Filter Transformation

The frequency substitution

$$\omega \to \frac{-\omega_c}{\omega} \qquad (8.68),(1)$$

in a transfer function converts a low pass filter response to a high pass one.

Referring to the low pass prototype filters in Fig. 8.25, we will use (1) to convert the impedances of the series inductances and the shunt capacitances.

- Series Inductance. With $Z_L = j\omega L$ and substituting (1):

$$j\omega L_k \to -j\frac{\omega_c}{\omega}L_k = \frac{1}{j\omega C_k'} \tag{2}$$

where
$$C_k' \equiv \frac{1}{\omega_c L_k} \tag{8.69a),(3}$$

We can deduce from this result that the series inductances of the low pass prototype filter are converted to series capacitances. This is indicative of a high pass filter.

The purpose of the negative sign in (1) is also apparent from (3): it yields physically realizable capacitor values.

- Shunt Capacitance. With $Y_C = j\omega C$ and substituting (1):

$$j\omega C_k \to -j\frac{\omega_c}{\omega}C_k = \frac{1}{j\omega L_k'} \tag{4}$$

where
$$L_k' \equiv \frac{1}{\omega C_k} \tag{8.69b),(5}$$

Here we see that the substitution in (1) transforms the shunt capacitances in the low pass prototype to shunt inductances. This is also indicative of a high pass filter.

Including impedance scaling, the complete conversion of a low pass prototype circuit to a high pass filter is accomplished using

$$\boxed{C_k' = \frac{1}{R_0\omega_c L_k}} \tag{8.70a),(6}$$

$$\boxed{L_k' = \frac{R_0}{\omega_c C_k}} \tag{8.70b),(7}$$

Bandpass Filter Transformation

The conversion of the low pass prototype to bandpass or bandstop filters is only slightly more involved than for high pass filters.

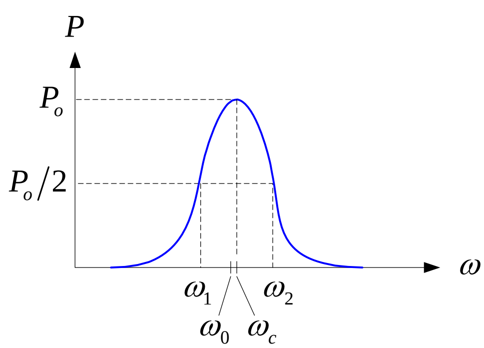
The conversion to a bandpass filter is accomplished with the substitution

$$\omega \to \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \qquad (8.71),(8)$$

where

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} \qquad (8.72),(9)$$

is the fractional bandwidth of the passband:



Notice that ω_0 is not the center frequency ω_c. The text defines

$$\boxed{\omega_0 = \sqrt{\omega_1 \omega_2}} \qquad (8.73)$$

which is the geometric mean of ω_1 and ω_2, rather than the more common arithmetic-mean definition. This was done to make the design equations simpler.

As we did with the high pass filter, we'll apply the transformation (8) to the series inductors and shunt capacitors in the low pass prototype circuit.

- Series Inductance. With $Z_L = j\omega L$ and substituting (8):

$$j\omega L_k \underbrace{\rightarrow}_{(8)} j\frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)L_k = j\omega\underbrace{\frac{L_k}{\omega_0\Delta}}_{L_k'}+\frac{1}{j\omega}\underbrace{\frac{\omega_0 L_k}{\Delta}}_{1/C_k'} \tag{10}$$

From this result we see that the series inductors in the low pass prototype are transformed to a series LC combination with elements

$$\boxed{L_k' = \frac{L_k R_0}{\omega_0\Delta}} \quad \text{(series element 1 of 2)} \qquad \text{(8.74a),(11)}$$

and $$\boxed{C_k' = \frac{\Delta}{\omega_0 L_k R_0}} \quad \text{(series element 2 of 2)} \qquad \text{(8.74b),(12)}$$

where we've also included the impedance scaling.

- Shunt Capacitance. With $Y_C = j\omega C$ and substituting (8):

$$j\omega C_k \underbrace{\rightarrow}_{(8)} j\frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)C_k = j\omega\underbrace{\frac{C_k}{\omega_0\Delta}}_{C_k'}+\frac{1}{j\omega}\underbrace{\frac{\omega_0 C_k}{\Delta}}_{1/L_k'} \tag{13}$$

From this result we see that the shunt capacitors in the low pass prototype are transformed to a parallel LC combination with elements

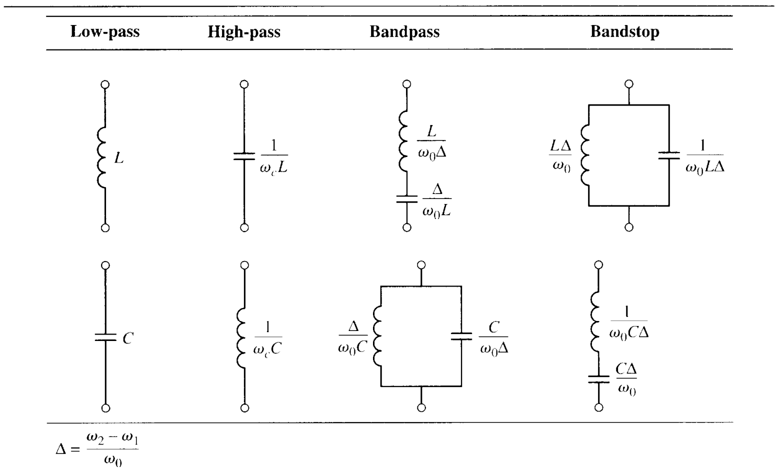
$$\boxed{C_k' = \frac{C_k}{\omega_0 R_0 \Delta}} \quad \text{(shunt element 1 of 2)} \qquad (8.74\text{d}),(14)$$

$$\boxed{L_k' = \frac{R_0 \Delta}{\omega_0 C_k}} \quad \text{(shunt element 2 of 2)} \qquad (8.74\text{c}),(15)$$

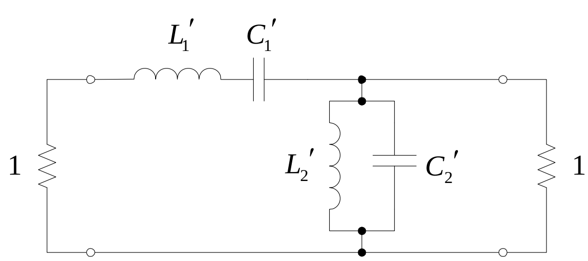
where we've also included impedance scaling.

Table 8.6 in the text succinctly summarizes these transformations:

TABLE 8.6 Summary of Prototype Filter Transformations



How do we implement these filters in microstrip? Consider a second order bandpass filter:



We could use the first Kuroda identity to transform L_1' to a shunt capacitance, but what about the series capacitance C_1'?

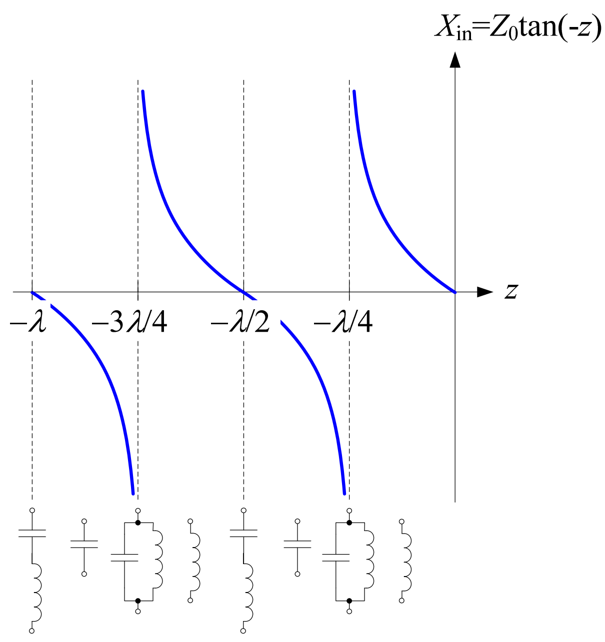
Kuroda's fourth identity transforms a series capacitance to a series capacitance. That's no help here. We're stuck!

Stubs as Resonators

Instead of that approach, one can also use $\lambda/4$-long open or short circuit stubs to act as resonators. As shown in Section 6.2 of the text:

- $\lambda/4$-long open circuit stub = series resonant circuit,
- $\lambda/4$-long short circuit stub = parallel resonant circuit.

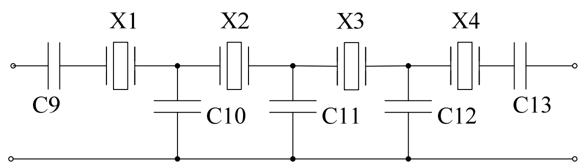
In the latter case with a short circuit termination:



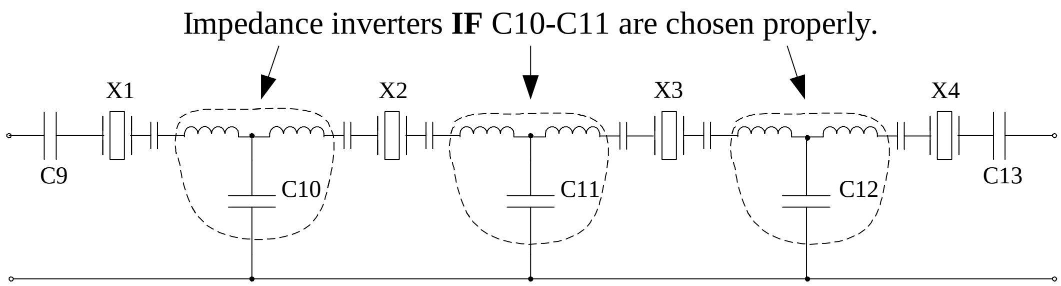
So, one could use a $\lambda/4$-long short circuit stub to act as the shunt parallel resonant circuit for the bandpass filter.

But how would one realize the effective series resonant circuit? In particular, if we are implementing this circuit in microstrip, how can we connect the stub in series? Obviously, not easily.

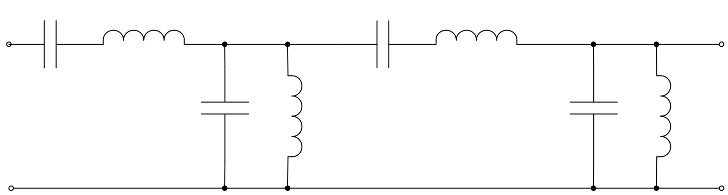
To circumvent this difficulty, we can use a technique directly analogous to what we employed in the analysis of the Cohn IF filter in the NorCal40A:



We understood the operation of this filter as series resonant circuits (the crystals) interconnected by impedance inverters:



which leads to the bandpass-filter interpretation of the Cohn filter:

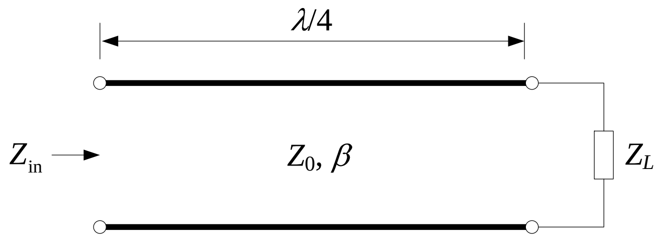


(Refer to Lecture 13 of the *EE 322 Lecture Notes* for more details.)

We can accomplish exactly the same result here by interconnecting shunt short circuit stubs by impedance inverter circuits.

Transmission Line Impedance Inverters

It is very easy to construct an impedance inverter in microstrip. It's nothing more than a $\lambda/4$-long microstrip with the same characteristic impedance as the system:

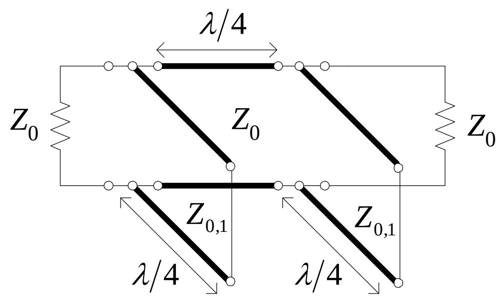


From Lecture 9, the input impedance of this QWT is

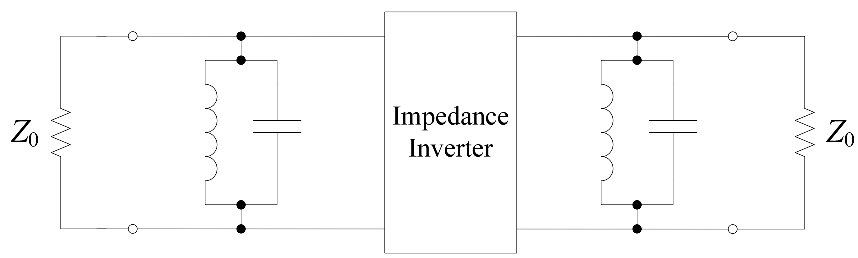
$$Z_{in} = \frac{Z_0^2}{Z_L} \quad \text{or} \quad \frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L} \tag{16}$$

In other words, the normalized input impedance is the inverse of the normalized load impedance. (Think of a one-half rotation around the Smith chart.) Cool!

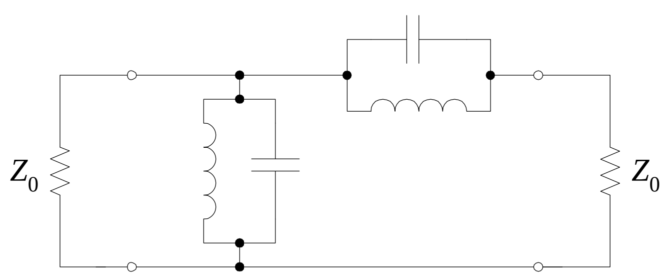
So, to realize a second order bandpass filter, we use two $\lambda/4$-long short circuit stubs (acting as parallel resonant circuits) separated by an impedance inverter:



The effective circuit is



or looking from the left-hand port



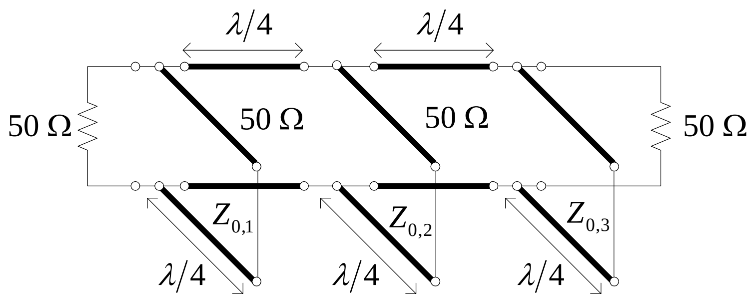
This is the bandpass filter topology are searching for! It has been obtained very easily using stub resonators and TL impedance inverters.

The process for designing these stub filters is quite simple, and is illustrated in the following example.

Example N32.1. Design a third order, maximally flat, bandpass filter using stub resonators that has a 4-GHz center frequency and 50% relative bandwidth in a 50-Ω system.

We first design the low pass prototype circuit. From Table 8.3, $g_1 = 1$, $g_2 = 2$ and $g_3 = 1$.

The filter is to be composed of shorted stub resonators, which we will form using $\lambda/4$-sections of TL:



The required characteristic impedances of these stubs can be expressed in terms of the prototype filter coefficients, g_k, as given by equation (8.131):

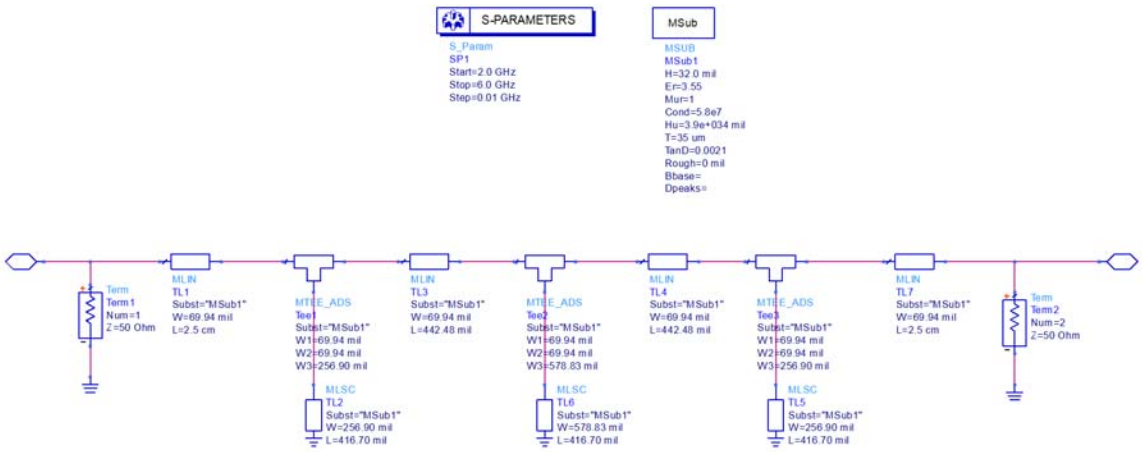
$$\boxed{Z_{0,1} = \frac{\pi Z_0 \Delta}{4g_1}} = \frac{\pi \cdot 50 \cdot 0.5}{4 \cdot 1} = 19.6\ \Omega \tag{17}$$

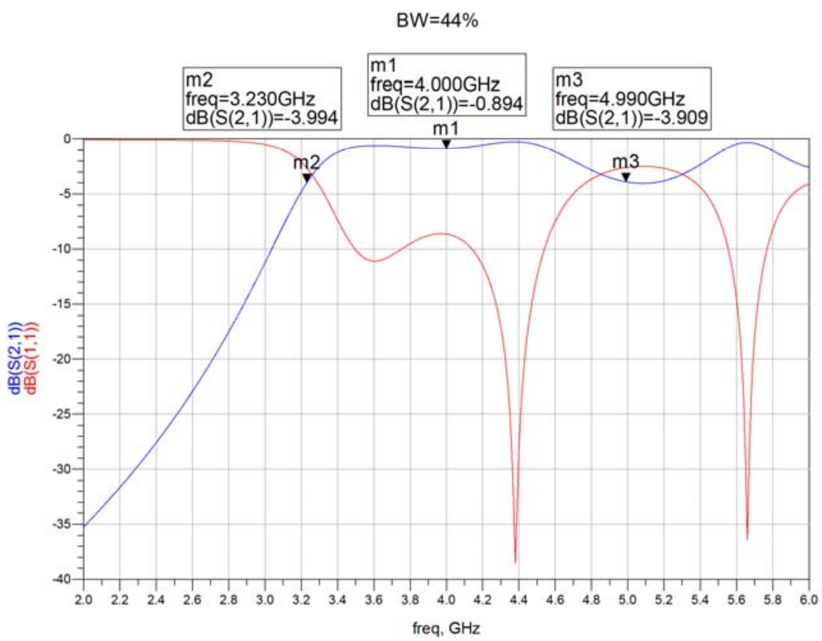
$$\boxed{Z_{0,2} = \frac{\pi Z_0 \Delta}{4g_2}} = \frac{\pi \cdot 50 \cdot 0.5}{4 \cdot 2} = 9.817\ \Omega \tag{18}$$

and $Z_{0,3} = Z_{0,1}$.

Other than computing the physical lengths corresponding to the center frequency and laminate, that's all there is to this design.

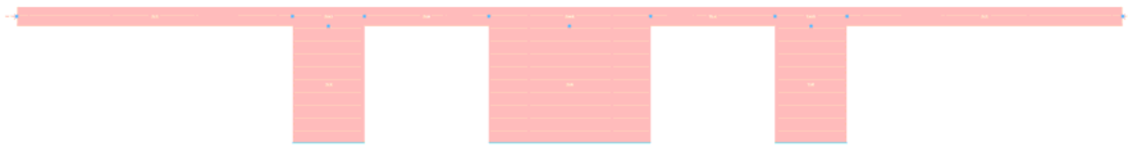
ADS simulation results for this bandpass filter using Rogers 4003C laminate are shown below.





Among other issues, the upper stop band response is not good for this filter.

This initial filter design needs to be tweaked because of the very wide stub widths:



Bandwidths smaller than 50% (which is itself often quite large for a bandpass filter) require even wider strips! Not practical.

For bandstop filters, the strip widths become very narrow. This is what limits the bandwidth of these filters.

For higher *Q* bandpass or bandstop filters, better choices in these situations are coupled line, coupled resonator, or dielectric resonator filters.