

Lecture 31: Stub Synthesis. Kuroda's Identities. Stub Low Pass Filters.

Another method for synthesizing microwave filters without lumped elements is to use shorted and opened stubs to realize the inductances and capacitances of the filter.

However, the stubs we've seen have only provided a shunt reactance. We also **need a series reactance** for ladder filters. Furthermore, the **separations** between the stubs are generally **not electrically small** and thus will degrade the filter performance if they are neglected.

Kuroda's identities are transformations that prove useful with this type of situation.

Stub Synthesis

The text calls this stub synthesis process “Richard’s transformation.” Here we’ll show a simpler approach.

In Section 2.3 of text, we derived the stub input impedances:

- Short circuit TL: $Z_{\text{in}} = jZ_0' \tan \beta l$ (2.45c),(1)

- Open circuit TL: $Y_{\text{in}} = jY_0' \tan \beta l$ (2.46c),(2)

To be consistent with Section 8.4, the prime indicates an impedance-scaled (i.e., unnormalized) value.

From (1) with $l < \lambda/4$ (or $\beta l < \pi/2$), the input impedance for a **short circuit stub** is a positive reactance (i.e., an **effective inductance**) while from (2), the **open circuit stub** presents a negative reactance (i.e., an **effective capacitance**). We'll use these two properties to construct effective inductances and capacitances for filters.

From (1), it is apparent that we **cannot** express the input impedance in the form ωL since ω appears in the tangent function. We can conclude that this effective inductance **varies with frequency**.

If this is the case, then which f would one choose? One choice is $l = \lambda/8$ ($\beta l = \pi/4$), which is halfway between 0 and $\lambda/4$ (beyond this, the reactance changes sign).

So, with $l = \lambda/8$ then $\tan(\beta l) = 1$ and (1) becomes

$$Z_{\text{in}} = jZ_0' \quad (3)$$

Now, for an inductor at $\omega = \omega_c$

$$Z_L = j\omega_c L \quad (4)$$

Equating (3) and (4) we see that by choosing the characteristic impedance to be

$$Z_0' = \omega_c L \quad (5)$$

then this $\lambda/8$ -long short-circuited TL has the same input impedance as an inductor with inductance L .

We will employ (5) in the design of stub filters. In such an application, the filter coefficients g_k will be associated with unscaled component values (i.e., the *unprimed* L_k and C_k values in Section 8.4). So, from (5)

$$Z_0 \equiv \frac{Z_0'}{\omega_c \underset{(5)}{\omega_c}} = \frac{1}{\omega_c} \omega_c L = L \quad (6)$$

where the unprimed quantity indicates an *unscaled* coefficient.

This relationship in (6) is very useful. It shows us that we can realize an effective inductance with filter coefficient L_k by using a short circuited TL with an unscaled characteristic impedance

$$Z_0 = L_k \quad (7)$$

that is $\lambda/8$ -long at $\omega = \omega_c$, which is the design frequency.

Similarly, one can show that a filter coefficient C_k can be effectively realized by an open circuit stub with unscaled characteristic impedance

$$Z_0 = \frac{1}{C_k} \quad (8)$$

that is $\lambda/8$ -long at $\omega = \omega_c$.

These relationships are shown in Fig. 8.34:

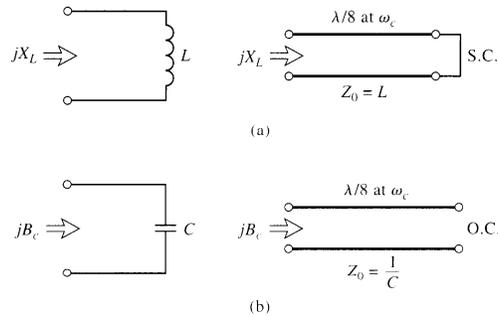


FIGURE 8.34 Richard's transformation. (a) For an inductor to a short-circuited stub. (b) For a capacitor to an open-circuited stub.

These effective L and C values for the stubs change with frequency. This affects, and generally degrades, the filter performance for $f \neq f_c$.

Kuroda's Identities

Now that we can construct stubs to perform as effective inductors and capacitors in a filter [but with $L(f)$ and $C(f)$], we must next address the **creation of series effective reactances** as well as account for the effects that occur when the **stubs are separated** from each other.

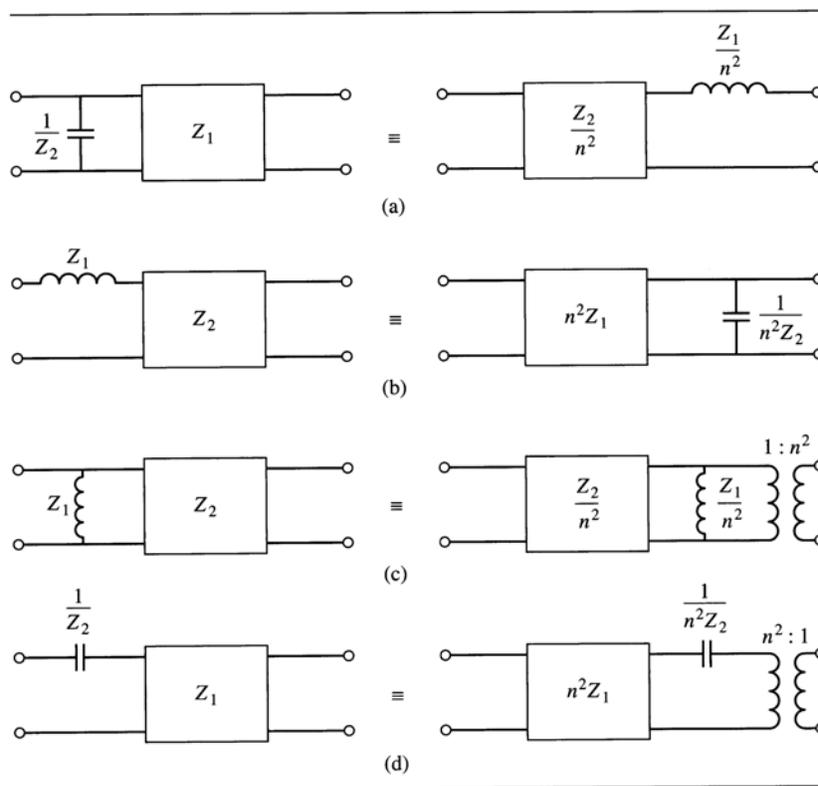
This is an effect we ignored in the low pass prototype filter. We assumed it contained lumped elements that were interconnected without any time delay between them. In other words, all the elements existed at a point in space.

In microwave circuits, this restriction may be difficult to realize in the physical construction. Hence, the distances between stubs may not be electrically small.

The four Kuroda identities allow us to add so-called **redundant TLs** to the microwave filter circuit and transform it into a more practical form.

The **four Kuroda identities** are shown in Table 8.7:

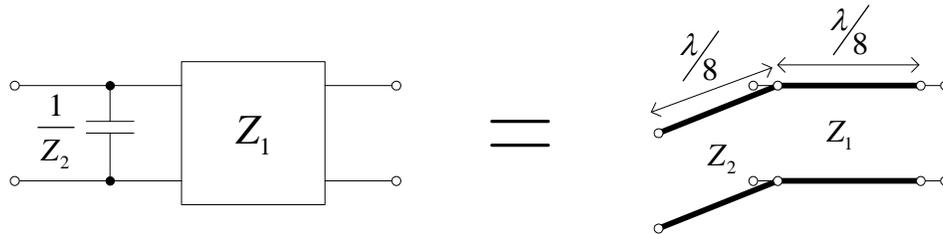
TABLE 8.7 The Four Kuroda Identities ($n^2 = 1 + Z_2/Z_1$)



Each **box** represents a so-called **unit element**, which is simply a TL with the indicated characteristic impedance and a length

$\lambda/8$. The **lumped elements** represent short- or open-circuit stubs acting as *normalized* (i.e., unscaled) series or shunt TLs.

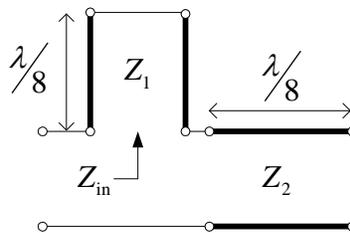
For example, the first figure in entry (a) represents:



The text shows a proof of the first Kuroda identity entry in Table 8.7. We'll prove the second one in the following example.

Example N31.1. Prove the second Kuroda identity in Table 8.7.

The left hand circuit is



From this circuit

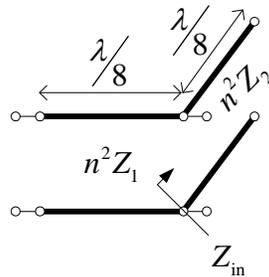
$$Z_{\text{in}} = jZ_1 \tan \beta l = jZ_1 \Omega$$

where $\Omega \equiv \tan \beta l$. Z_{in} is an **un-normalized** value because of Z_1 .

Cascading *ABCD* matrices for this circuit:

$$\begin{aligned}
 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{LHS}} &= \begin{bmatrix} 1 & j\Omega Z_1 \\ 0 & 1 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 1 & j\Omega Z_2 \\ \frac{j\Omega}{Z_2} & 1 \end{bmatrix}}_{(8.79)} \frac{1}{\sqrt{1+\Omega^2}} \\
 &= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 - \Omega^2 \frac{Z_1}{Z_2} & j\Omega Z_2 + j\Omega Z_1 \\ \frac{j\Omega}{Z_2} & 1 \end{bmatrix} \quad (9)
 \end{aligned}$$

The right hand circuit in row 2 of Table 8.7 is



from which we deduce that

$$Z_{in} = -jZ_0 \cot \beta l = \frac{Z_0}{j\Omega}$$

or

$$Z_{in} = \frac{n^2 Z_2}{j\Omega}$$

Cascading $ABCD$ matrices for this circuit:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{RHS}} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega n^2 Z_1 \\ \frac{j\Omega}{n^2 Z_1} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{j\Omega}{n^2 Z_2} & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 - \Omega^2 \frac{Z_1}{Z_2} & j\Omega n^2 Z_1 \\ \frac{j\Omega}{n^2 Z_1} + \frac{j\Omega}{n^2 Z_2} & 1 \end{bmatrix} \quad (10)$$

Comparing (9) and (10), we see that they are equal **provided** $n^2 = 1 + Z_2/Z_1$. That is:

$$j\Omega(Z_1 + Z_2) \stackrel{?}{=} j\Omega n^2 Z_1 = j\Omega \left(1 + \frac{Z_2}{Z_1}\right) Z_1 = j\Omega Z_1 + j\Omega Z_2 \quad \checkmark \text{Yes}$$

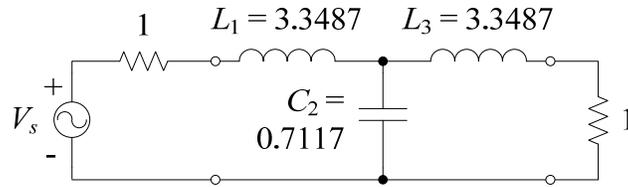
$$\frac{j\Omega}{Z_2} \stackrel{?}{=} \frac{j\Omega}{n^2} \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) = \frac{j\Omega}{1 + Z_2/Z_1} \cdot \frac{1}{Z_2} \left(\frac{Z_2}{Z_1} + 1\right) = \frac{j\Omega}{Z_2} \quad \checkmark \text{Yes}$$

Hence, we've proved the second Kuroda identity in Table 8.7.

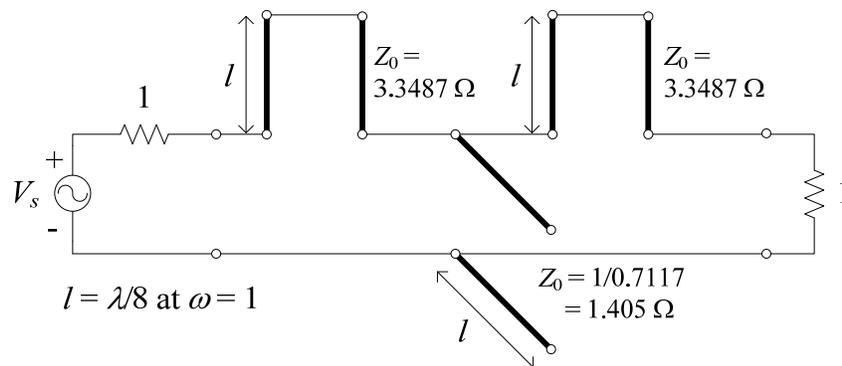
It is perhaps best to illustrate the **use** of Kuroda's identities by an example.

Example N31.2 (Text example 8.5). Design a stub low pass filter on microstrip for $f_c = 4$ GHz, a 50- Ω system impedance, and 3-dB equi-ripple in the passband using a third order filter.

As in the text, we'll choose the series-first topology of Fig. 8.25c. From Table 8.4 we find $g_1 = g_3 = 3.3487$, $g_2 = 0.7117$ and $g_4 = 1$. The low pass prototype is then:



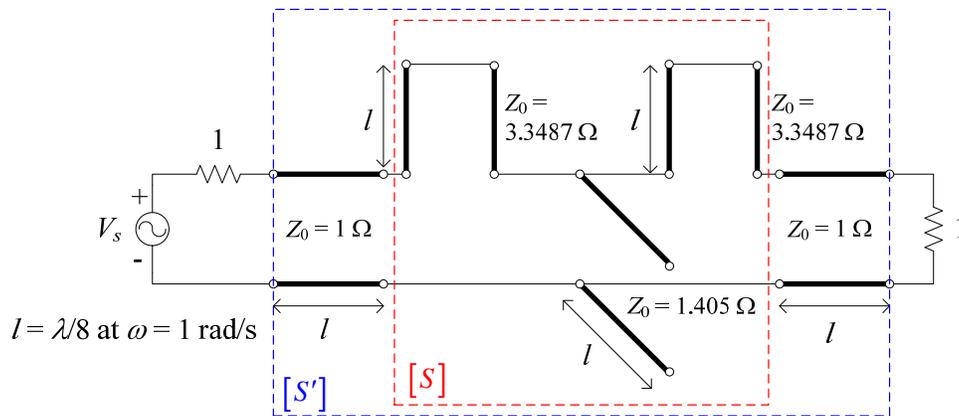
Next, we **synthesize** stub TLs to provide the equivalent reactances at the center frequency equal to these lumped circuit elements. Using (7) and (8):



We probably couldn't actually get this circuit to operate correctly in the lab because there's no physical separation between the stubs. If we were to build it with a "small" separation between them, there would be extensive coupling between the stubs. (Additionally, how would we implement the series connected stubs in this circuit with microstrip or stripline?)

So what do we do? One approach is to use **Kuroda's identities** to transform this impractical circuit into an equivalent, and more practical, one.

But there are no “unit elements” in this circuit, so how can we use Kuroda’s identities? We can **add these to either end of the circuit** without affecting the power loss factor P_{LR} , provided their characteristic impedances are $Z_0 = 1$. Adding unit elements (U.E.’s) on each end gives (Fig. 8.36c):



These U.E.’s don’t affect the filter performance because their characteristic impedances are matched to the system impedances at the source and load ends.

To see how this is an accurate statement, the original filter can be described by the S parameters $[S]$ as outlined in the figure above. As the phase planes are moved along the U.E.’s towards the source and load, as we learned in Lecture 16, only the phases of the S parameters will change, not the magnitudes.

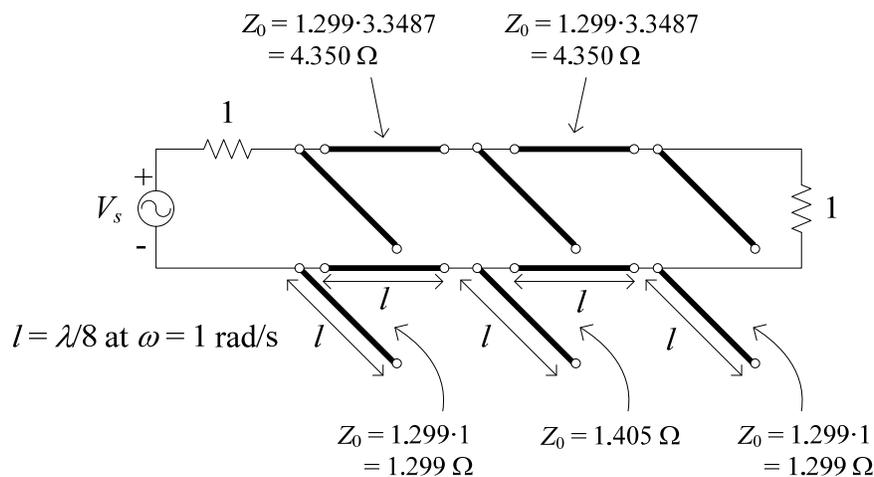
The new filter with $[S']$ parameters will have the same magnitudes of the S parameters of the original filter. It is only the magnitudes that are of interest in many filters, including low pass, high pass, band pass, and bandstop filters. Consequently,

the two U.E.'s that we added on each end of this low pass filter prototype will not alter the overall filter performance as measured by the power loss ratio P_{LR} .

We can now apply Kuroda's identity (b) on the left and on the right. In both cases,

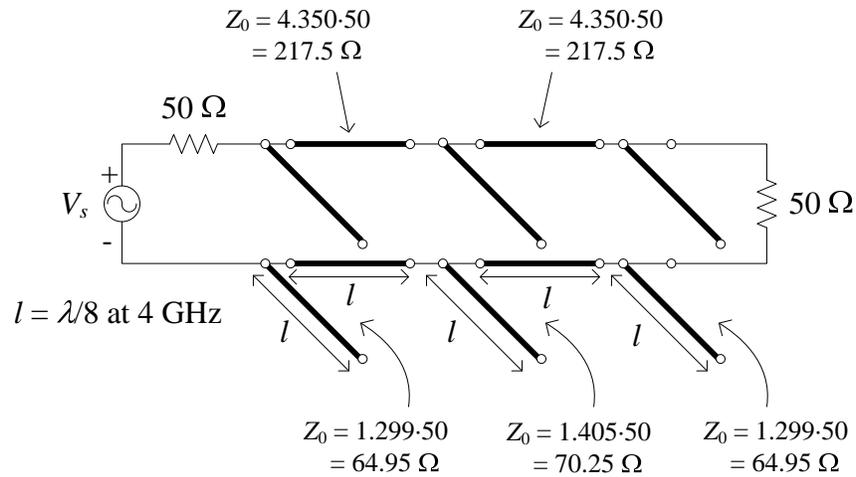
$$n^2 = 1 + \frac{Z_2}{Z_1} = 1 + \frac{1}{3.3487} = 1.299$$

giving (Fig. 8.36d):

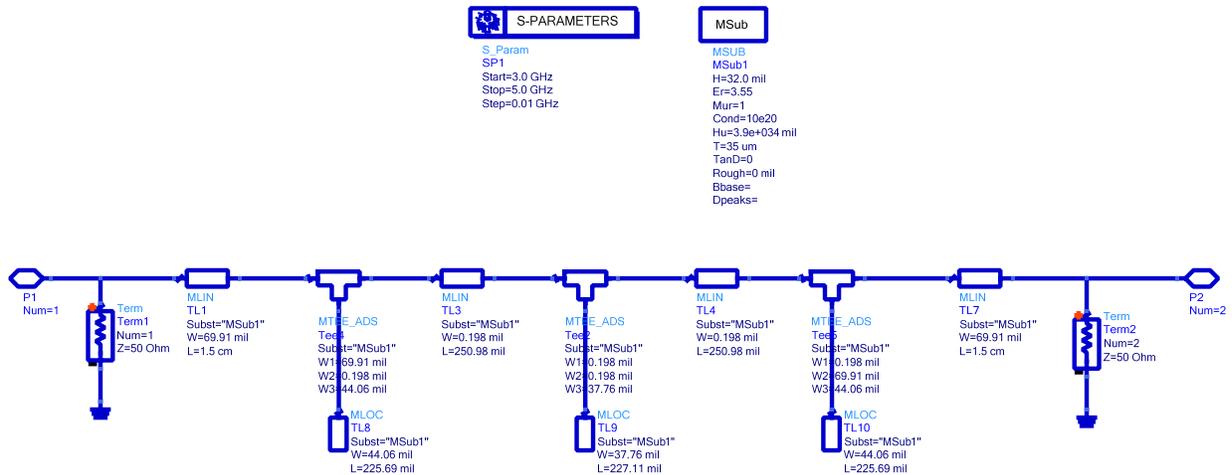


The use of Kuroda's identities has **served its purposes** since we now have separated the stubs by electrically significant dimensions that would allow their fabrication. We have also converted the series stubs to parallel stubs.

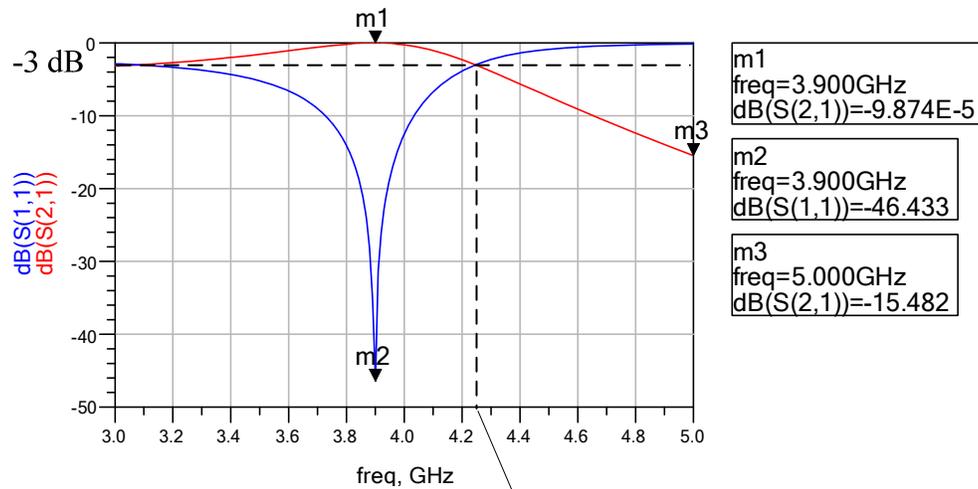
The final step is to impedance- and frequency-scale the circuit. To do this, we multiply all impedances by 50 and scale the TLs to $\lambda/8$ at 4 GHz using $\epsilon_{r,e}$ of the microstrip (Fig. 8.36e):



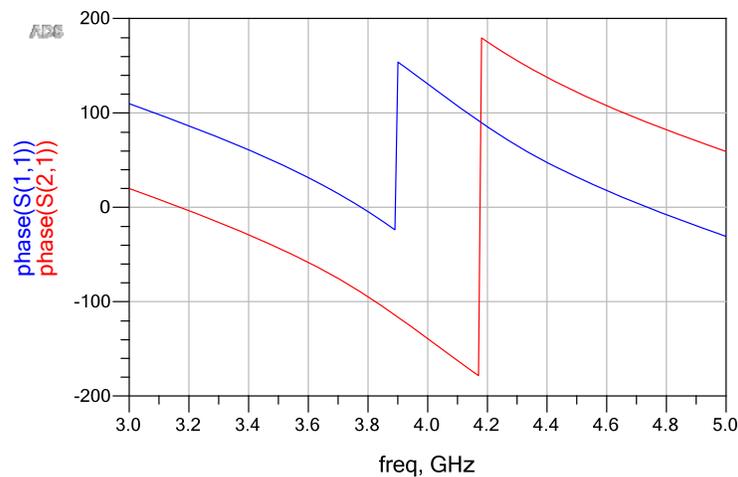
This microstrip circuit was simulated in ADS for **lossless** 32-mil Rogers 4003C laminate ($\epsilon_r = 3.55$):



and the S parameters are:



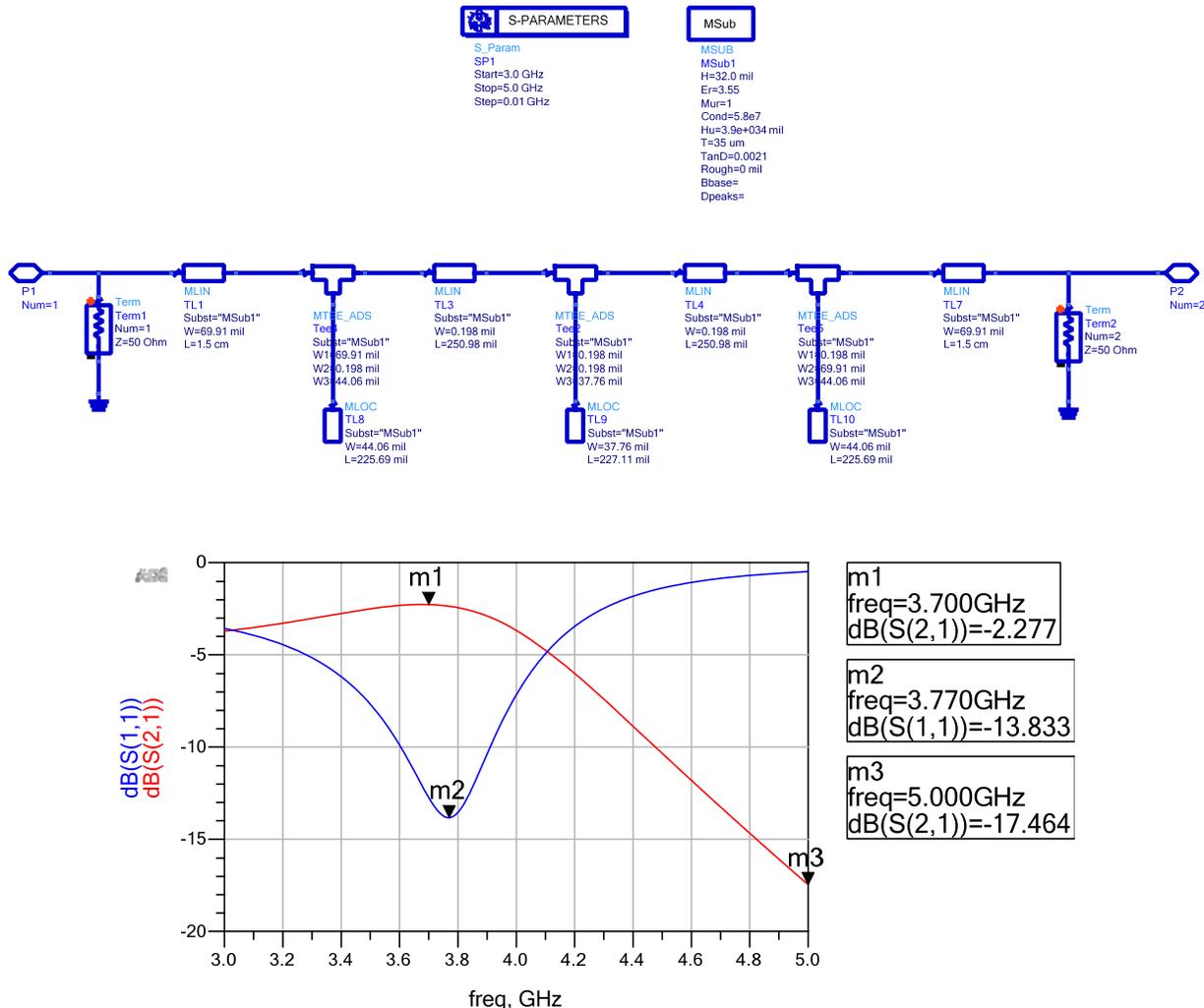
~4.25 \neq 4.5 GHz. The circuit needs some “tweaking”.



At 5 GHz, $\omega/\omega_c = f/f_c = 5/4 = 1.25$. From Fig. 8.27(b), the attenuation is ~ 31 dB for an ideal low pass filter. Above, we see for this stub filter design that the attenuation is only ~ 15.5 dB. The **out-of-band response is not as good** as an ideal low pass filter because the effective “L’s” and “C’s” of the stub filter **change with frequency**.

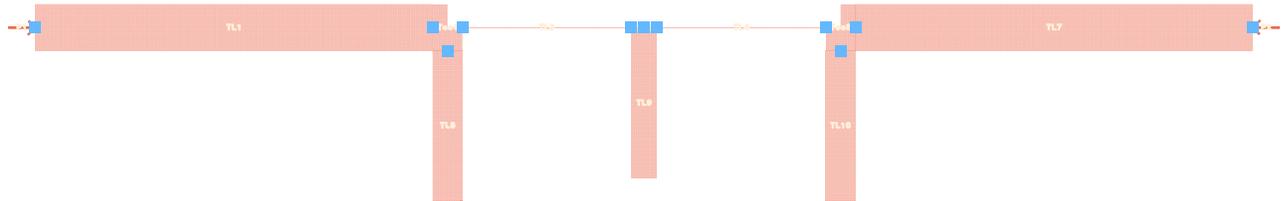
A further example of this is shown in Fig. 8.37.

Furthermore, losses in the copper and the substrate greatly affect this filter response. Below is the magnitude of the S parameters with losses in the Rogers 4003C board included:



In fact, the true response will likely not even be this “good.” We weren’t able to use accurate models for the tees in ADS since the **width ratios** were **too large**.

In the layout below, the two TLs that interconnect the stubs are only 0.198-mil wide while the other TLs are on the order of tens of mils.



This example illustrates one of the chief **disadvantages** of the stub filters: they often require physically unrealistic strip widths. Even for a large 50% bandwidth, the bandpass filter in this example would be extremely difficult to manufacture.