Lecture 30: Scaling of Low Pass Prototype Filters. Stepped Impedance Low Pass Filters.

In the last lecture, we discussed the design of prototype low pass filters where $R_s = R_L = 1\ \Omega$ and $\omega_c = 1$ rad/s. Of course, one generally is not going to implement the prototype filter. So what good is it?

It is possible to scale and transform the low pass prototype filter to obtain a low pass, high pass, band pass, and band stop filters for any impedance "level" ($R_s = R_L$) and cutoff frequency. Nice!

The process of filter design has three basic steps as discussed in the last lecture: (1) collect the filter specifications, (2) design the low pass prototype filter, (3) scale and convert the prototype.

The first two steps were performed in the previous lecture. We'll now consider the last step, beginning with scaling.

Scaling Low Pass Prototype Filters

There are two types of scaling for low pass prototype circuits, impedance scaling and frequency scaling:

1. **Impedance Scaling.** Since the filter is a linear circuit, we can multiply all the impedances (including the terminating resistances) by some factor without changing the transfer function of the filter. Of course, the input and output impedances *will* change.

   If the desired source and load impedances equal R_0, then

   - $X_L{}' = R_0 X_L = \omega\left(R_0 L\right)$. Therefore, $L' = R_0 L$. (8.64a),(1)
   - $X_C{}' = R_0 X_C = -\dfrac{1}{\omega}\left(\dfrac{R_0}{C}\right)$. Therefore, $C' = \dfrac{C}{R_0}$. (8.64b),(2)
   - $R_s{}' = R_0 \cdot 1 = R_0$. (8.64c),(3)
   - $R_L{}' = R_0 \cdot R_L = R_0 R_L$. (8.64d),(4)

   The primed quantities are the scaled quantities while the unprimed are those from the low pass prototype circuit (i.e., the unscaled quantities).

2. **Frequency Scaling.** As defined for the prototype $\omega_c = 1$ rad/s. To scale for a different low pass cutoff frequency, we substitute

$$\omega \to \frac{\omega}{\omega_c} \qquad (8.65),(5)$$

   where ω_c is the desired cutoff frequency of the low pass filter.

   Applying this to the inductive and capacitive reactances in the prototype filter we find

- $X_L{}' = \omega L\big|_{\omega \to \frac{\omega}{\omega_c}} = \omega\left(\dfrac{L}{\omega_c}\right)$. Therefore, $L' = \dfrac{L}{\omega_c}$. (8.66a),(6)
- $X_C{}' = \dfrac{1}{\omega C}\bigg|_{\omega \to \frac{\omega}{\omega_c}} = \dfrac{1}{\omega}\left(\dfrac{\omega_c}{C}\right)$. Therefore, $C' = \dfrac{C}{\omega_c}$. (8.66b),(7)

For a one-step impedance and frequency scaling process, we can combine (1)-(4), (6), and (7) to obtain

- $L_k{}' = \dfrac{R_0 L_k}{\omega_c}$ (8.67a),(8)
- $C_k{}' = \dfrac{C_k}{\omega_c R_0}$ (8.67b),(9)
- $R_s{}' = R_0$ (10)
- $R_L{}' = R_0 R_L$ (11)

where $k = 1,\ldots,N$ as in Fig. 8.25. For example, in the circuit of Fig. 8.25a, $C_1 = g_1$, $L_2 = g_2$, $C_3 = g_3$, etc.

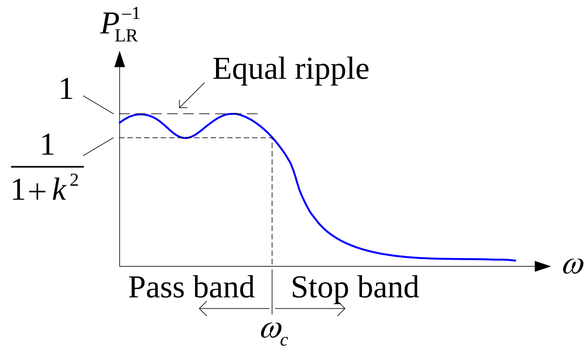
Example N30.1. Design a 3-dB, equi-ripple low pass filter with a cutoff frequency of 2 GHz, 50-Ω impedance level, and at least 15-dB insertion loss at 3 GHz.

The first step is to determine the order of the filter needed to achieve the required IL at the specified frequency. From equation (7) in the previous lecture for $\omega \gg \omega_c$

$$P_{\text{LR}} \approx \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N} \quad (12)$$

(This is just an approximation here since $\omega/\omega_c = 1.5$.)

What value do we use for k? From Fig. 8.21



we see that the passband ripple equals $1+k^2$. So, with A = ripple in dB, then

$$10\log\left(1+k^2\right) = A$$

so that

$$k = \sqrt{10^{A/10} - 1} \quad (13)$$

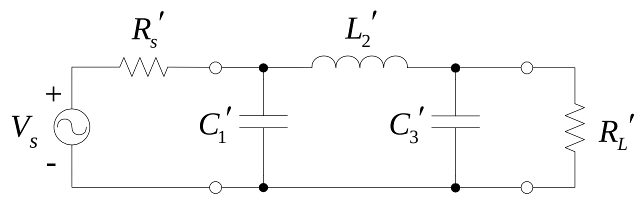
Consequently, for $A = 3$ dB then $k = 0.998 \approx 1$. Therefore, at $\omega/\omega_c = 1.5$, equation (12) becomes $P_{\text{LR}} \approx 3^{2N}/4$ so that

N	**10log*P*$_{\text{LR}}$**	**Fig. 8.27b w/ $\lvert\omega/\omega_c\rvert$-1=0.5**
1	3.5 dB	6 dB
3	22.6 dB	19 dB
5	41.7 dB	35 dB

The third column is the more accurate number since it originates from the plot in Fig. 8.27b. The second column is less accurate because we used (12) with $\omega/\omega_c = 1.5$, which is not $\gg 1$.

For this filter, we'll choose N = 3 to meet the IL specification. From Table 8.4 (3.0-dB ripple), we find the immitance values to be $g_1 = 3.3487$, $g_2 = 0.7117$, $g_3 = g_1$ and $g_4 = 1$.

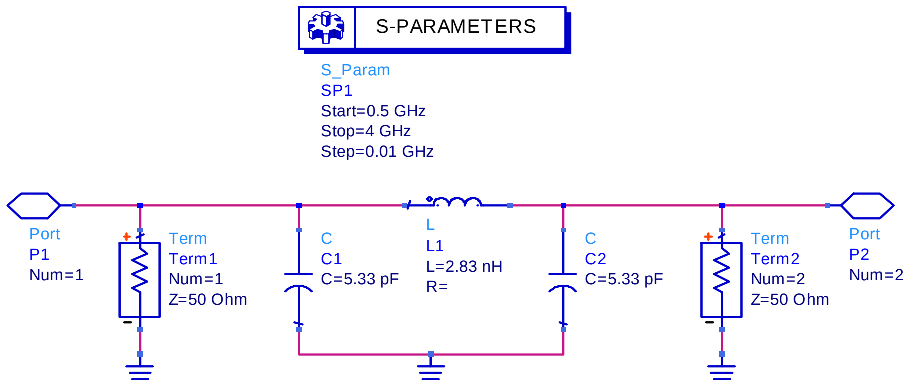
Using (8)-(11) with $R_0 = 50\ \Omega$, $f_c = 2$ GHz and arbitrarily choosing the prototype circuit having the fewest inductors

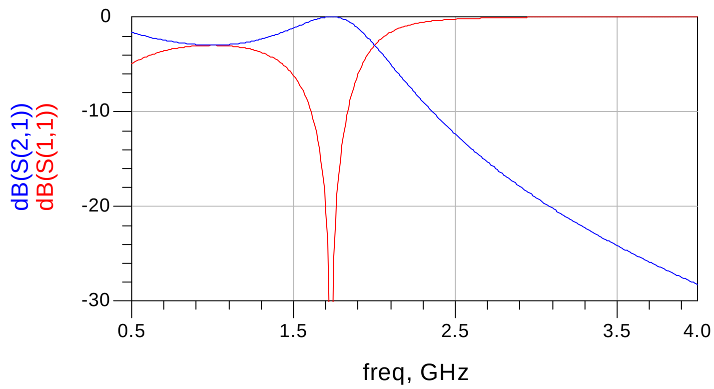


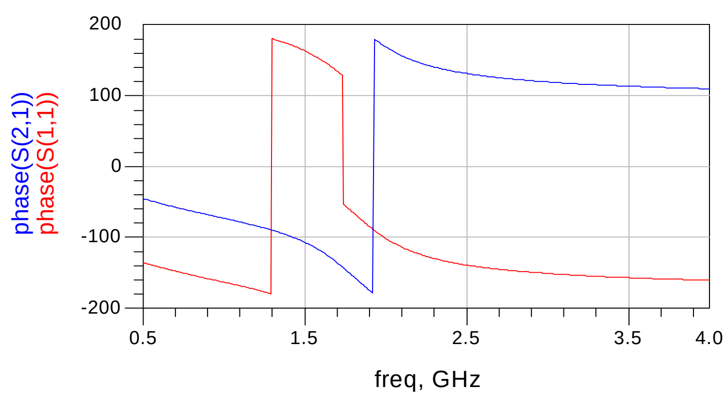
then,

- $R_s' = R_L' = R_0 = 50\ \Omega$
- $C_1' = C_3' = \dfrac{C_1}{\omega_c R_0} = \dfrac{g_1}{\omega_c R_0} = \dfrac{3.3487}{2\pi \cdot 2 \cdot 10^9 \cdot 50} = 5.33$ pF
- $L_2' = \dfrac{R_0 L_2}{\omega_c} = \dfrac{R_0 g_2}{\omega_c} = \dfrac{50 \cdot 0.7117}{2\pi \cdot 2 \cdot 10^9} = 2.83$ nH

The response of this filter was computed in ADS and is shown below. Note that $|S_{21}| = -3$ dB at 2 GHz and that $|S_{21}| = -19$ dB at 3 GHz, both which meet the original specifications for the filter.

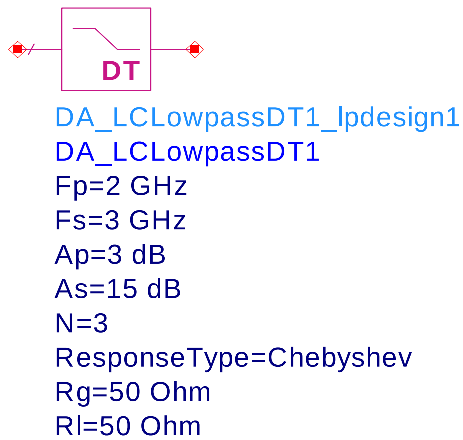


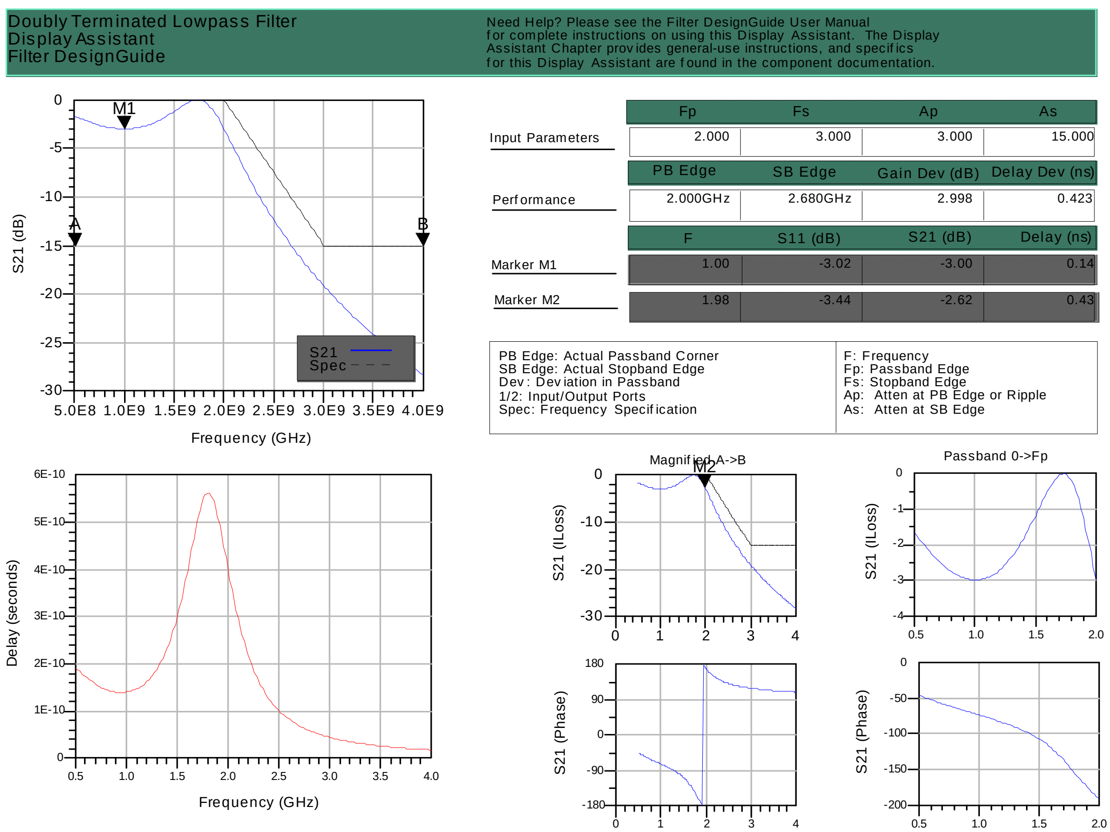




This filter was also designed in ADS using the Filter DesignGuide feature for automatic filter design. Using SmartComponents in ADS can greatly speed up the filter design process. Here using the low pass filter from the “Filter DG – All

Networks" palette. The filter is designed using the Filter DesignGuide, which is activated by pointing to DesignGuide -> Filter.





Stepped Impedance Low Pass Filters

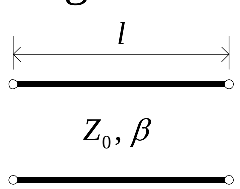
The next question is: How do we implement these filters in microwave circuits? Lumped components (such as SMT) can be used up to approximately 5-6 GHz, but their electrical size and the electrical distance between them may not be negligible!

Also, the losses of such components can be appreciable, which will limit the performance of filters.

We'll look at two methods for realizing low pass filters without lumped elements, (1) Stepped impedance and (2) Stubs.

To understand stepped impedance filters, we must first look at electrically short sections of TLs with either a very large or a very small characteristic impedance.

To begin, we'll first determine the equivalent T-network model for a length of TL. From the front flap of the text



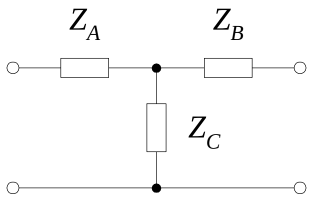
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & \cos\beta l \end{bmatrix} \quad (14)$$

From Table 4.2

$$Z_{11} = Z_{22} = \frac{A}{C} = -j\frac{Z_0}{\tan\beta l} = -jZ_0\cot\beta l \quad (8.81a),(15)$$

$$Z_{21} = Z_{12} = \frac{1}{C} = -j\frac{Z_0}{\sin\beta l} = -jZ_0\csc\beta l \quad (8.81b),(16)$$

Using results from Example 4.3 in the text, we can construct an equivalent T network for a length of TL as



where $$Z_C = Z_{21} = -jZ_0 \csc \beta l \quad (17)$$

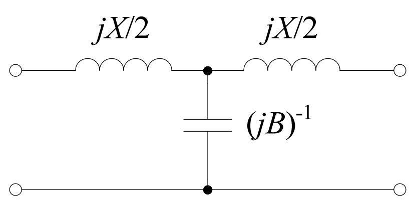
and $$Z_A = Z_{11} - Z_C = Z_B$$

$$= -jZ_0 \cot \beta l + jZ_0 \csc \beta l = jZ_0 \tan \frac{\beta l}{2} \quad (18)$$

using the trig identity

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha}$$

With $\beta l < \pi/2$, then from (18) the series elements have positive reactance ($\Rightarrow$ inductance) while from (17) the shunt element has negative reactance ($\Rightarrow$ capacitance). This leads to the equivalent circuit (Fig 8.39a):



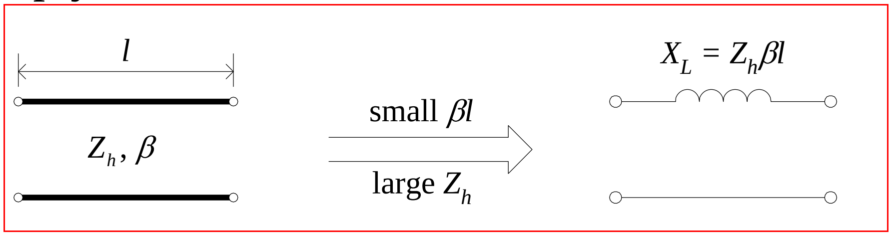
where $\frac{X}{2} = Z_0 \tan\left(\frac{\beta l}{2}\right)$ and $B = \frac{1}{Z_0} \sin \beta l$. (8.83),(19),(20) Now here's the interesting part. Suppose the TLs are very short (so that $\beta l < \pi/4$) and that Z_0 is **very large** and equal to Z_h. Then (19) and (20) become

$$\frac{X}{2} \approx Z_h \frac{\beta l}{2} \quad \Rightarrow \quad X \approx Z_h \beta l \qquad \text{(8.84a),(21)}$$

and

$$B \approx 0 \qquad \text{(8.84b),(22)}$$

This is simply a series inductance!

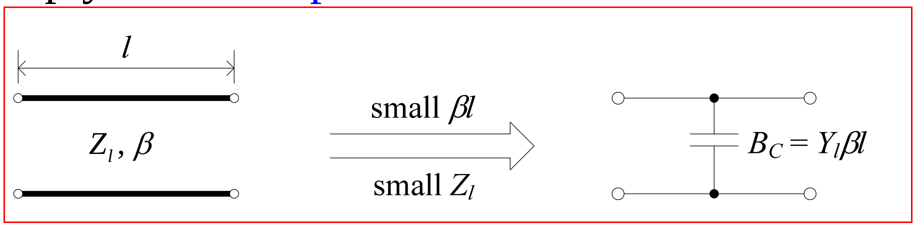


Conversely, suppose the line is short and that Z_0 is **very small** and equal to Z_l. Then (19) and (20) become

$$\frac{X}{2} \approx 0 \qquad \text{(8.85a),(23)}$$

$$B \approx \frac{\beta l}{Z_l} = Y_l \beta l \qquad \text{(8.85b),(24)}$$

This is simply a shunt capacitance!



The use of these electrically short high impedance and low impedance sections of TLs is the origin of the alternate name for these stepped impedance filters: Hi-Z, Low-Z filters.

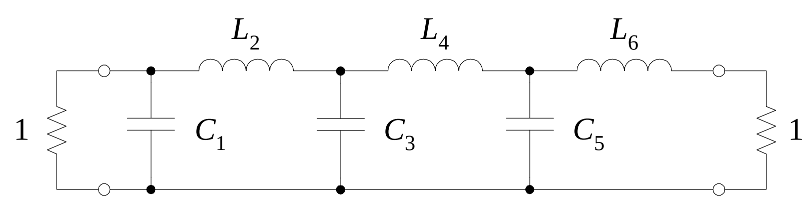
Example N30.2 (Similar to text example 8.6). Design a stepped impedance low pass filter with maximally flat response in the pass band, $f_c = 2.5$ GHz, and more than 20-dB IL at 4 GHz. Assume the impedance seen by both ports of the filter is 50 Ω and the highest practical Z_0 is 150 Ω while the lowest is 10 Ω.

For maximally flat low pass filter with $f \gg f_c$,

$$P_{LR} \approx \left(\frac{f}{f_c}\right)^{2N} = \left(\frac{4}{2.5}\right)^{2N} > 100$$

This implies that $N \geq 5$.

But, let's take a look at Fig 8.26. At $|f/f_c| - 1 = 0.6$, the attenuation is approximately 20 dB for N = 5 and 24 dB for $N = 6$. Hence, we'll choose N = 6 to be safe. Also, we'll arbitrarily choose a shunt element to be the first one on the left:



From Table 8.3 with N = 6:

$$g_1 = 0.5176 = C_1 = L_6$$
$$g_2 = 1.4142 = L_2 = C_5$$
$$g_3 = 1.9318 = C_3 = L_4$$

The next step is to determine the lengths of the high-Z and low-Z TLs to realize these immitance values. To do this, we first need to normalize the results from (21)-(24). From (21)

$$X_L' = \omega L' = Z_h \beta l \tag{25}$$

Then, using (8)

$$L' = \frac{R_0 L}{\omega_c} \quad \Rightarrow \quad L = \frac{\omega_c}{R_0} L'$$

and substituting (25) at $\omega = \omega_c$:

$$L = \frac{\omega_c}{R_0} \frac{Z_h}{\omega_c} \beta l = \frac{Z_h}{R_0} \beta l \qquad \text{(8.86a),(26)}$$

Similarly, one can show that

$$C = \frac{R_0}{Z_l} \beta l \qquad \text{(8.86b),(27)}$$

Remember, (26) and (27) are *normalized* parameters.

Equations (26) and (27) can be used to determine the TL electrical lengths needed to realize the required filter coefficients:

$$\boxed{\beta l_k = \frac{L_k R_0}{Z_h}} \quad \text{or} \quad \boxed{\beta l_k = \frac{C_k Z_l}{R_0}}.$$

Substituting the g_k values listed above into one of these two equations gives

$$\beta l_1 = 0.1036 \text{ rad} = 5.94^\circ$$
$$\beta l_2 = 0.4713 \text{ rad} = 27.01^\circ$$
$$\beta l_3 = 0.3864 \text{ rad} = 22.14^\circ$$
$$\beta l_4 = 0.644 \text{ rad} = 36.90^\circ$$

$$\beta l_5 = 0.2888 \text{ rad} = 16.20°$$
$$\beta l_6 = 0.1727 \text{ rad} = 9.89°$$

We can use LineCalc, for example, to compute the physical lengths from these electrical lengths once the specific details of the transmission line have been given.

A microstrip circuit implementation of this design is shown in Fig 8.40:

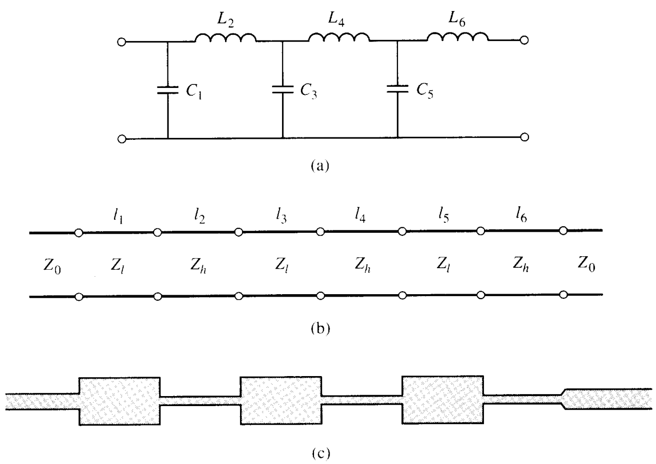
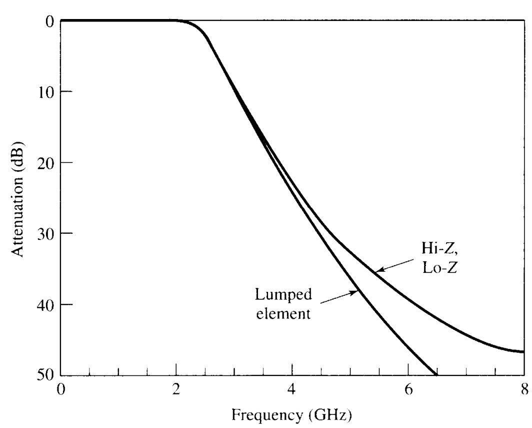


FIGURE 8.40 Filter design for Example 8.7. (a) Low-pass filter prototype circuit. (b) Stepped-impedance implementation. (c) Microstrip layout of final filter.

and representative results are shown in below.



The response of the microstrip Hi-Z, Lo-Z stepped impedance filter is compared with the ideal response of a lumped element design.