Lecture 29: Microwave Filter Design by the Insertion Loss Method.

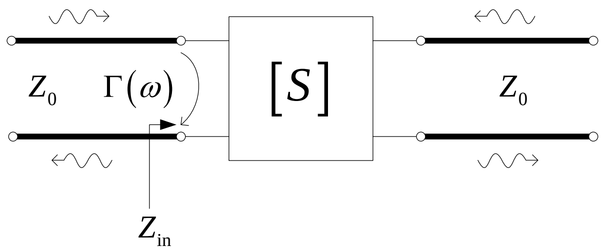
The next major topic we're going to cover in this course is microwave filter design. Its theoretical basis is **exactly** the same as low frequency analog filters, which you saw in your electronics courses.

For example, you'll see Butterworth and Chebyshev filters, but designed to operate at microwave frequencies. Implementation of these filters is different, however. For example, we won't use discrete inductors and capacitors.

Insertion Loss Method

We will begin this process with the design of analog filters, but perhaps with more detail than you've seen before in other courses.

There are different methods for systematically designing filters, but the insertion loss method is probably the most prominent. In this technique, the relative power loss due to a lossless filter with reflection coefficient $\Gamma(\omega)$:



is specified in the power loss ratio P_{LR} defined as:

$$P_{\text{LR}} = \frac{P_{\text{inc}}}{P_{\text{load}}} = \frac{P_o}{P_o\left[1-\left|\Gamma(\omega)\right|^2\right]}$$

or

$$P_{\text{LR}} = \left[1-\left|\Gamma(\omega)\right|^2\right]^{-1} \qquad (8.49),(1)$$

If both the load and source ports are matched for this network, then $P_{\text{LR}} = 1/\left|S_{21}\right|^2$.

In Section 4.1, the text shows that $\left|\Gamma(\omega)\right|^2$ is an even function of ω. This implies that $\left|\Gamma(\omega)\right|^2$ can only be expanded in a polynomial series in ω^2.

In particular, for a linear and time invariant system, $\left|\Gamma(\omega)\right|^2$ is the rational function

$$\left|\Gamma(\omega)\right|^2 = \left|\frac{Z_{\text{in}}(\omega)-Z_0}{Z_{\text{in}}(\omega)+Z_0}\right|^2$$

meaning it can be expressed as a quotient of (real and nonnegative) polynomials $M(\omega^2)$ and $N(\omega^2)$. For physically realizable filters, we choose:

$$\left|\Gamma(\omega)\right|^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)} \qquad (8.51),(2)$$

Using (2) in (1) $$P_{\text{LR}} = 1 + \frac{M\left(\omega^2\right)}{N\left(\omega^2\right)} \quad (8.52),(3)$$

This is valid for any linear, time invariant system that is an even function of ω.

Types of Low Pass Filters

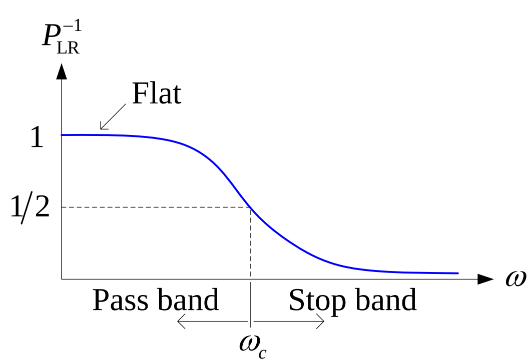
There are four types of low pass filters discussed in the text that are all based on (3):

1. Maximally Flat, Butterworth, Binomial Filter. For this type of low pass filter:

$$P_{\text{LR}} = 1 + k^2\left(\frac{\omega}{\omega_c}\right)^{2N} \quad (8.53),(4)$$

where $N =$ filter order and $\omega_c =$ cutoff frequency.

If $k = 1$, then $P_{\text{LR}} = 2$ at $\omega = \omega_c$, which is the 3-dB frequency:



For large ω and with $k = 1$, then

$$P_{\text{LR}} \approx k^2 \cdot \left(\frac{\omega}{\omega_c}\right)^{2N} = 1^2 \cdot \left(\frac{\omega}{\omega_c}\right)^{2N} \tag{5}$$

From this result we learn that the insertion loss IL, defined as

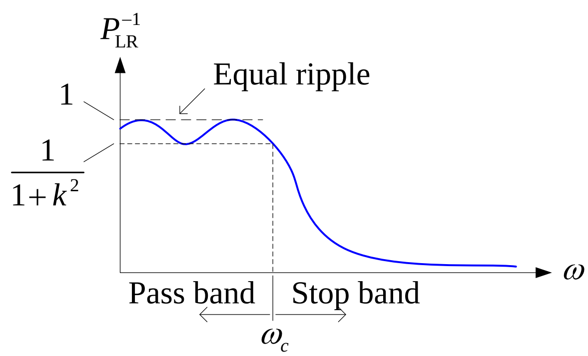
$$\text{IL} = 10\log\left(P_{\text{LR}}\right), \tag{8.50}$$

increases by $20N$ dB/decade in the stop band for the maximally flat low pass filter.

2. Equal Ripple or Chebyshev Filter. For this type of low pass filter:

$$P_{\text{LR}} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right) \qquad (8.54),(6)$$

where $T_N(x)$ is the Chebyshev polynomial. A typical plot of P_{LR}^{-1} in (6) is



Generally, N is chosen to be an odd integer when the source and load impedances are equal (two-sided filters).

For large ω/ω_c and using the large argument form of T_N, (6) becomes

$$P_{\text{LR}} \approx \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N} \tag{7}$$

As with the Butterworth filter, (7) also increases at 20N dB/decade, but with the extra factor

$$\frac{2^{2N}}{4} \tag{8}$$

compared to (5). Consequently, there is more so-called "roll off" with the Chebyshev low pass filter. For example,

$$N = 3 \quad \Rightarrow \quad \log_{10}\left(\frac{2^{2\cdot 3}}{4}\right) = 12.0 \text{ dB}$$

$$N = 5 \quad \Rightarrow \quad \log_{10}\left(\frac{2^{2\cdot 5}}{4}\right) = 24.1 \text{ dB}$$

This is a very sizeable increase in the stop band attenuation of the Chebyshev filter over the Butterworth.

3. Elliptic Filter. This type of low pass filter has an equi-ripple response in both the pass band and the stop band. It has a "faster" roll off than the previous two filters.

4. Linear Phase Filter. If it's important that there be no signal distortion, then the phase of the filter must be linear in the passband.

   Why? It's the group delay that contributes to signal distortion of a transient waveform.

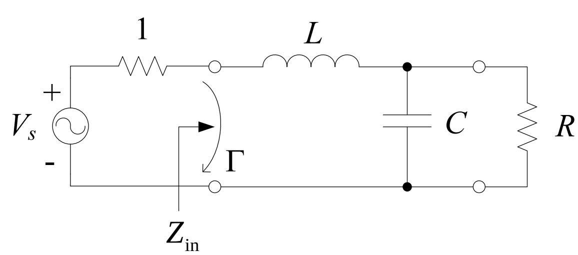
General Procedure for Filter Design

The general procedure for designing a filter using the insertion loss method can be summarized in three steps (Fig 8.23):

1. Filter specifications. These include the cutoff frequency, the stop-band attenuation, the pass-band insertion loss, the pass-band behavior, etc.
2. Design a "low pass **prototype**" circuit. In such a prototype, $R = 1\ \Omega$ and $\omega_c = 1$ rad/s. Filter tables are used for this step, or perhaps a computer package.
3. Scale and conversion. Finally, the filter is scaled to the proper impedance level and, if desired, to a high pass, band pass, or band stop topology.

Prototype Circuit for the Low Pass Filter

We'll consider this whole process in an example in the next lecture. Before that, we'll derive the L and C values for a second order, low pass "prototype" filter, as shown in Fig 8.24:



This will help demystify the origin of filter table coefficients that we'll see in the next lecture.

From this above figure

$$Z_{\text{in}} = j\omega L + \frac{1}{j\omega C} \| R = j\omega L + \frac{R}{1 + j\omega RC} \quad (9)$$

The reflection coefficient at the input port is

$$\Gamma = \frac{Z_{\text{in}} - 1}{Z_{\text{in}} + 1} \quad (10)$$

From the definition of P_{LR} in (1) and using (10)

$$P_{\text{LR}} = \left[1 - \frac{Z_{\text{in}} - 1}{Z_{\text{in}} + 1} \cdot \frac{Z_{\text{in}}^* - 1}{Z_{\text{in}}^* + 1}\right]^{-1} = \frac{\left|Z_{\text{in}} + 1\right|^2}{2\left(Z_{\text{in}} + Z_{\text{in}}^*\right)} \quad (8.59),(11)$$

Your text, in equation (8.60), gives expressions for the numerator and denominator of (11) in terms of *R*, *L*, and *C*.

For a maximally flat low pass filter, then from (4) and with $k = 1$

$$P_{\text{LR}} = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N} \underbrace{=}_{\substack{N=2 \\ \omega_c = 1}} 1 + \omega^4 \quad (12)$$

Equating (11) and (12) and after some algebra, the text shows

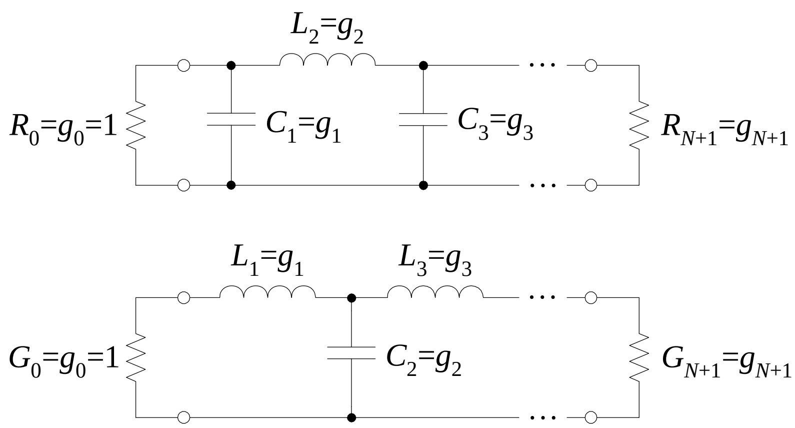
$$L = C = \sqrt{2} \quad \text{and} \quad R = 1 \quad (13)$$

This is second row of Table 8.3.

This circuit is a "prototype" in that

1. The source and load resistances = 1 Ω, and
2. $\omega_c = 1$ rad/s

The two topologies for low pass prototype circuits are shown in Fig 8.25:



Both give identical responses. The "immitance" values, g, for a low pass filter are defined as

$$g_k = \begin{cases} \text{inductance for series elements} \\ \text{capacitance for shunt elements} \end{cases} \qquad (k = 1, \ldots, N)$$

Filter tables can be used to determine these parameters in prototype circuits for maximally flat, equi-ripple, and other types of filters. A great reference for this and related filter topics is:

G. L. Matthaei, L. Young and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*. Norwood, MA: Artech House, 1980.

The following filter tables are listed in the text:

- Table 8.3 – Maximally flat low-pass,
- Table 8.4 – Equi-ripple w/ 0.5 dB and 3 dB ripple,
- Table 8.5 – Linear phase (maximally flat time delay).