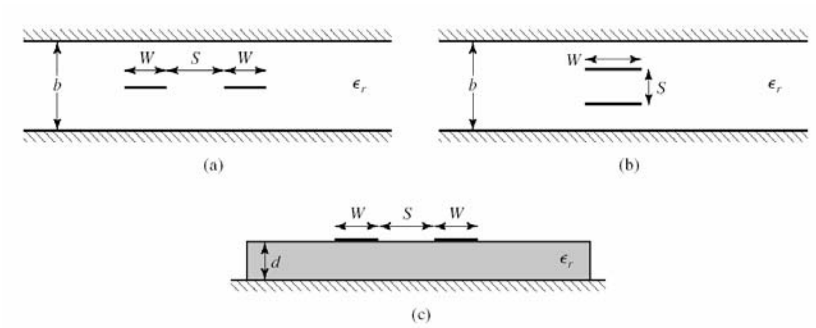
Lecture 28: Coupled Line and Lange Directional Couplers.

These are the final two directional couplers we will consider. They are closely related and based on two TLs that interact with each other, but are not physically connected.

Coupled Line Directional Coupler

When two TLs are brought near each other, as shown in the figure below (Fig. 7.26), it is possible for power to be coupled from one TL to the other.



This can be a serious problem on PCBs where lands are close together and carry signals changing rapidly with time. EMC engineers face this situation in high speed digital circuits and in multiconductor TLs.

For coupled line directional couplers, this coupling between TLs is a useful phenomenon and is the physical principle upon which the couplers are based.

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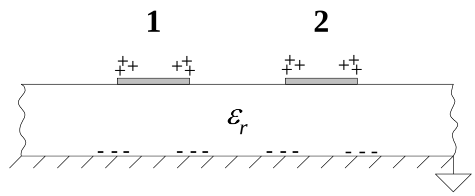
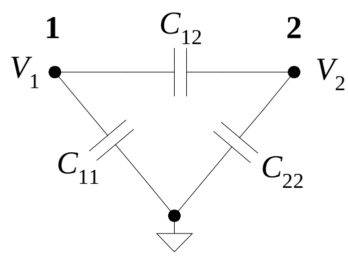
Consider the geometry shown in Fig 7.27:



When voltages are applied, charge distributions will be induced on all of the conductors. The voltages and total charges are related to each other through capacitance coefficients C_{ij}:



By definition ($Q = CV$):

$$Q_1 = C_{11}V_1 + C_{12}V_2$$
$$Q_2 = C_{21}V_1 + C_{22}V_2$$

where

- C_{11} =capacitance of conductor 1 with conductor 2 present but grounded.
- C_{22} =capacitance of conductor 2 with conductor 1 present but grounded.
- C_{12} =mutual capacitance between conductors 1 and 2. (If the construction materials are reciprocal, then $C_{21} = C_{12}$.)

By computing only these capacitances and the quasi-TEM mode wave speed, we'll be able to analyze these coupled line problems. Why? Assuming TEM modes, then

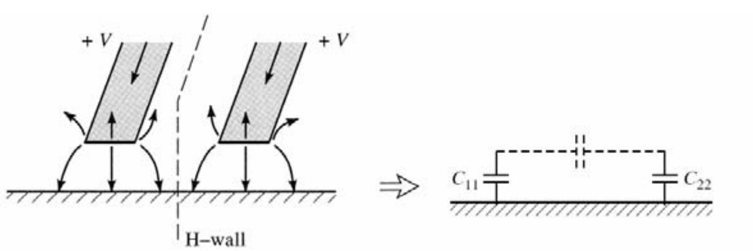
$$Z = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C} \tag{1}$$

Notice that L doesn't appear here. Hence we only need v_p and C, as conjectured. This is a widely used approach in all TL problems, not just microstrip or coupled lines.

Even-Odd Mode Characteristic Impedances

To simplify the problem analysis, we'll assume the two strips are identical and located on top of a dielectric. Because of the symmetry, we can use an even-odd mode solution approach.

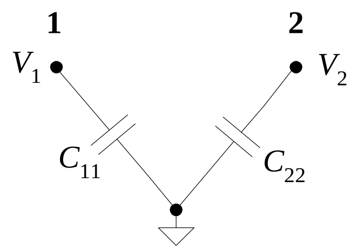
- **Even mode**. The voltages and currents are the same on both strips, as shown in Fig 7.28a:



(a)

Hence, the electric field has even symmetry about the plane of symmetry (POS). This plane is called an "H-wall."

Notice that no $\bar{E}$ field lines from conductor 1 (2) terminate on conductor 2 (1). Consequently, the two halves are decoupled, as we expect. This leads us to the equivalent circuit:



We define the effective capacitance to ground for either conductor in this configuration as

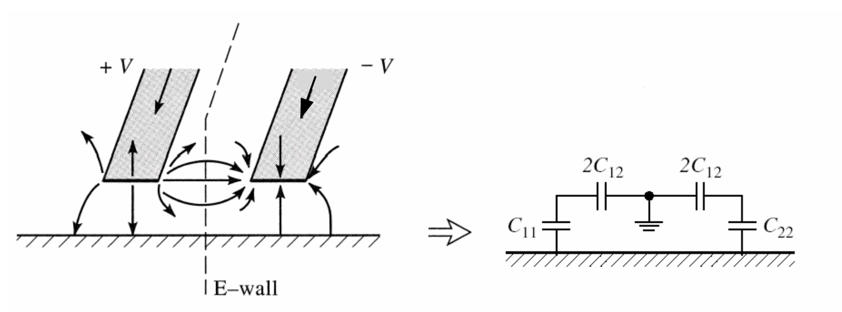
$$C_e = C_{11} = C_{22} \qquad (7.68),(2)$$

Then using (1)

$$Z_{0e} = \frac{1}{v_{pe} C_e} \qquad (7.69),(3)$$

which is the characteristic impedance of either TL when both are operated in the even mode. This is called the even mode characteristic impedance.

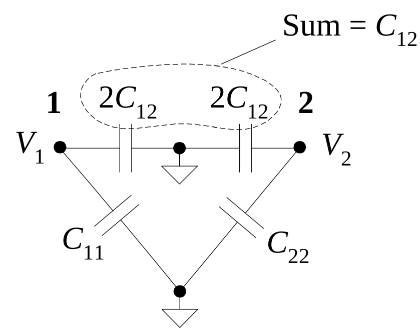
- **Odd mode**. The voltages and currents are opposite on each line as shown in Fig 7.28(b):



(b)

Notice here that the electric field lines are perpendicular to the POS. Therefore, similar to image theory, we can consider the POS as an equipotential surface (or an "E wall"). That is, as a ground plane where $V = 0$.

This approach leads to the equivalent capacitance circuit:



The effective capacitance to ground of either conductor in this configuration is then

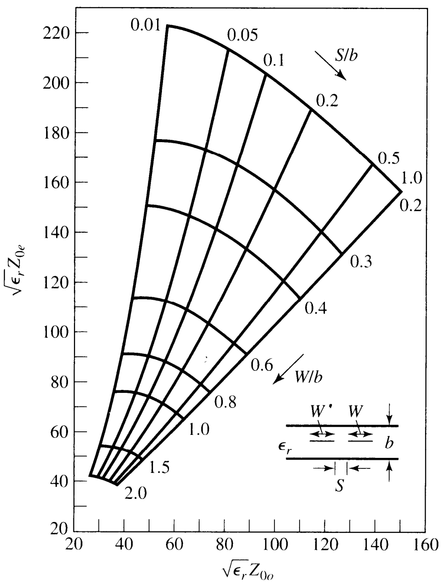
$$C_o = 2C_{12} \| C_{11} = C_{11} + 2C_{12} \qquad (7.70),(4)$$

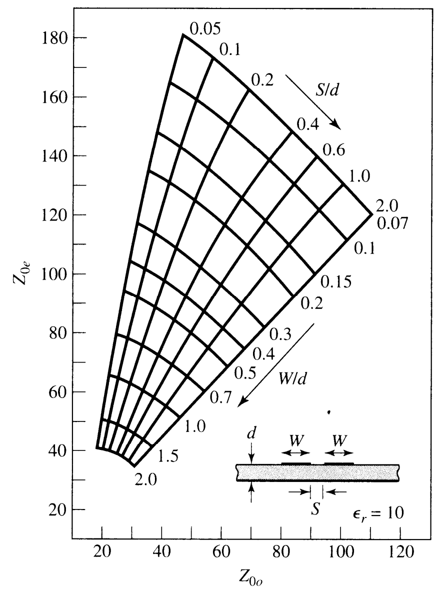
so that, from (1)

$$\boxed{Z_{0o} = \frac{1}{v_{po} C_o}} \qquad (7.71),(5)$$

This is the odd mode characteristic impedance. It is the characteristic impedance seen when a voltage wave is launched on the structure with odd symmetry, as shown in Fig. 7.28b.

The computation of C_e and C_o required in (3) and (5) is a bit involved. Alternatively, the text presents two examples of graphical design data for specific geometries. Fig 7.29 contains design data for striplines and Fig 7.30 for microstrips with $\varepsilon_r = 10$.

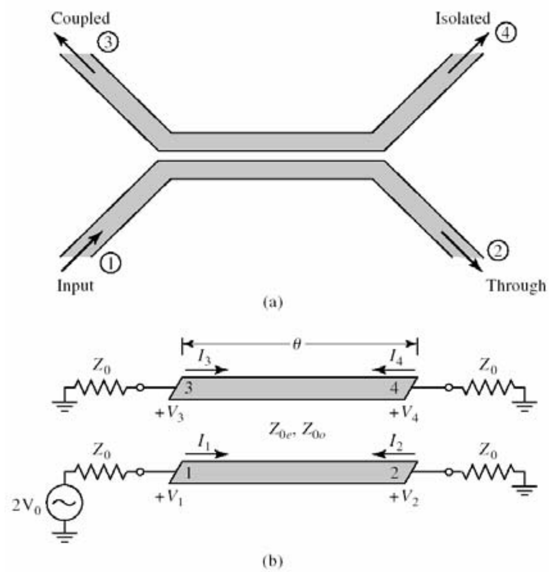




Recall that striplines support pure TEM modes, while microstrips do not. To use the stripline design curves, scale the even and odd mode impedances upward by $\sqrt{\varepsilon_r}$, while the microstrip design curves apply only for $\varepsilon_r = 10$.

Even-Odd Mode Solution for Coupled Line Directional Coupler

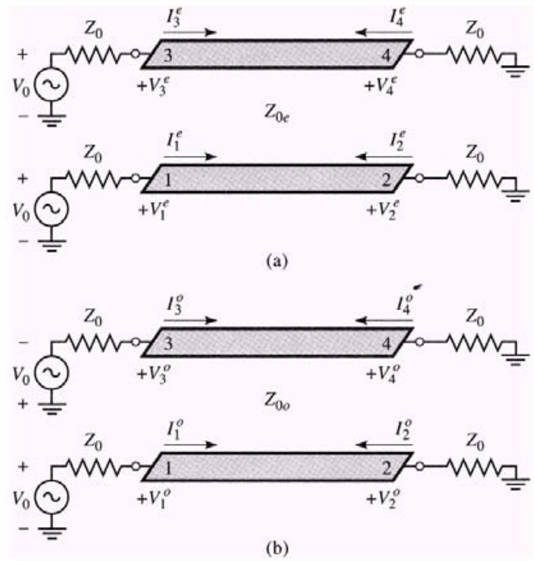
A coupled line directional coupler (in stripline or microstrip) is show in Fig 7.31. It is excited at port 1 and terminated with Z_0 at all ports.



As mentioned earlier, we will apply an even-odd mode analysis for this coupler. Recall that in order to add these two solutions, both problems must have the same boundary conditions. That's why we terminate the TLs with Z_0 rather than $Z_{0,e}$ or $Z_{0,o}$.

The effects of mutual coupling between the conductors has been efficiently accounted for by the characteristic impedances $Z_{0,e}$ and $Z_{0,o}$ in the even and odd modes, respectively.

The even and odd mode problems are shown in Fig 7.32:



By symmetry, we can deduce that for the even mode problem

$$I_3^e = I_1^e,\ I_4^e = I_2^e \tag{6}$$
$$V_3^e = V_1^e,\ V_4^e = V_2^e$$

while for the odd

$$I_3^o = -I_1^o,\ I_4^o = -I_2^o \tag{7}$$
$$V_3^o = -V_1^o,\ V_4^o = -V_2^o$$

By definition, $$Z_{\text{in}} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} \qquad (7.72),(8)$$

These TL problems in Fig 7.32 are simple and easy to solve. As shown in the text, by choosing

$$\boxed{Z_0 = \sqrt{Z_{0e} Z_{0o}}} \qquad (7.77),(9)$$

leads to $$Z_{\text{in}} = Z_0 \qquad (7.78),(10)$$

Furthermore, with (9) then port 1 is matched and given the symmetry of the structure, **all** ports will then be matched!

As shown in the text,

$$V_1 = V \tag{11}$$

$$V_2 = V \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \tag{7.84),(12}$$

$$V_3 = V \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta} \tag{7.82),(13}$$

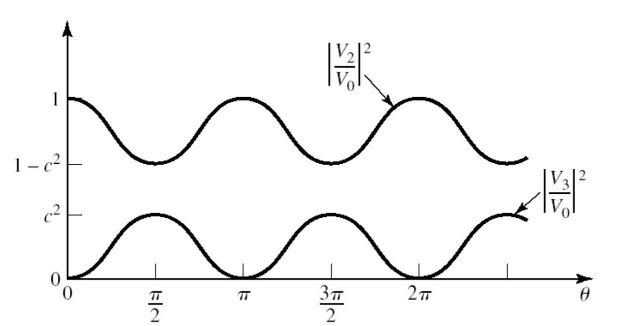
$$V_4 = 0 \text{ (total isolation)} \tag{7.83),(14}$$

where $\theta = \beta l$ is the electrical length and

$$\boxed{C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}} \tag{7.81),(15}$$

is the voltage coupling coefficient. (Here, "C" stands for "coupling," not "capacitance.")

Typical plots of $|V_2/V|^2$ and $|V_3/V|^2$ are shown in Fig 7.33:



Notice from these plots that we can maximize $|V_3/V|^2$ and simultaneously minimize $|V_2/V|^2$ when $\theta = \pi/2, 3\pi/2, \ldots$ so that $l = \lambda/4,\ 3\lambda/4, \ldots$. In these cases, (11)-(14) become

$$V_1 = V$$

$$V_2 = -jV\sqrt{1-C^2} \qquad (7.86),(16)$$

$$V_3 = CV \qquad (7.85),(17)$$

$$V_4 = 0$$

Because of the 90° phase shift between V_3 and V_2, this device can also be used as a quadrature hybrid, but without a 1:1 power division at the output.

Lastly, it can be easily shown that power is conserved, which is a valuable check of the analysis.

Example N28.1 (Similar to text example 7.7). Design a 20-dB, single section, coupled line directional coupler using stripline. Assume a 0.158-cm ground plane separation, $\varepsilon_r = 2.56$, $Z_0 = 50$ Ω, and a center frequency of 3 GHz.

From (17), $C = V_3/V$ is the coupling coefficient, which must be less than one for a passive device. In this case,

$$-20 = 20\log_{10} C \quad \Rightarrow \quad C = 0.1 \qquad (18)$$

Next, we need to determine Z_{0e} and Z_{0o}. From (9) and (15),

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}} = 50\sqrt{\frac{1.1}{0.9}} = 55.28\ \Omega \qquad (7.87a),(19)$$

and $$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}} = 50\sqrt{\frac{0.9}{1.1}} = 45.23\ \Omega \qquad (7.87b),(20)$$

These values can be used in Fig 7.29 to determine W and S for the stripline dimensions. With $\varepsilon_r = 2.56$, then

$$\sqrt{\varepsilon_r}Z_{0e} = 88.4\ \Omega \quad \text{and} \quad \sqrt{\varepsilon_r}Z_{0o} = 72.4\ \Omega$$

Using these values in Fig 7.29 we find

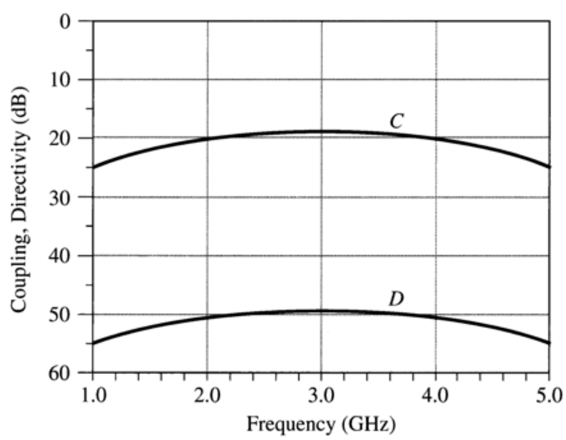
$$\frac{W}{b} \approx 0.72 \quad \text{and} \quad \frac{S}{b} \approx 0.34.$$

Given that $b = 0.158$ cm, then

$$W = 0.114\ \text{cm} \quad \text{and} \quad S = 0.054\ \text{cm}.$$

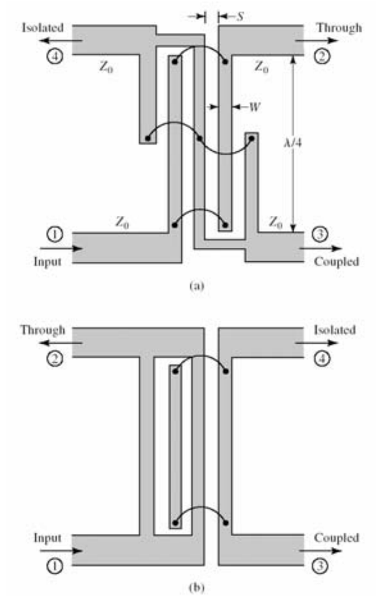
This value for S is rather small and may lead to fabrication difficulties with basic milling machines.

The coupling and directivity for this coupled line directional coupler computed by a CAD package are shown in Fig. 7.34:



Lange Coupler

This device is shown in Fig 7.38:



This is one method for obtaining higher coupling coefficients (up to approximately 2-3 dB or so) than what is possible with regular coupled line directional couplers.