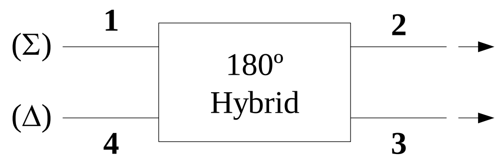
# Lecture 27: The 180º Hybrid.

The second reciprocal directional coupler we will discuss is the 180º hybrid. As the name implies, the outputs from such a device can be 180º out of phase.

There are two primary objectives for this lecture. The first is to show that the *S* matrix of the 180º hybrid is

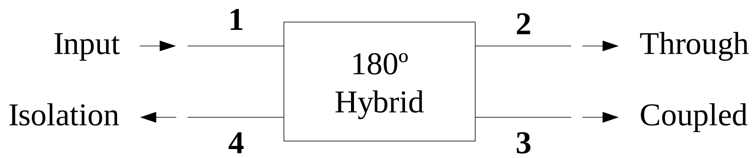
$$\boxed{[S]=\frac{-j}{\sqrt{2}}\begin{bmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 0 & -1\\ 1 & 0 & 0 & 1\\ 0 & -1 & 1 & 0\end{bmatrix}} \tag{7.101),(1}$$

with reference to the port definitions in Fig. 7.41:



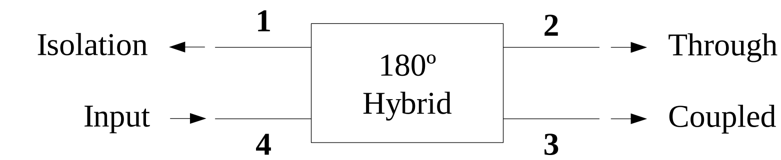

The second primary objective is to illustrate the three common ways to operate this device. These are:

1. In-phase power splitter:



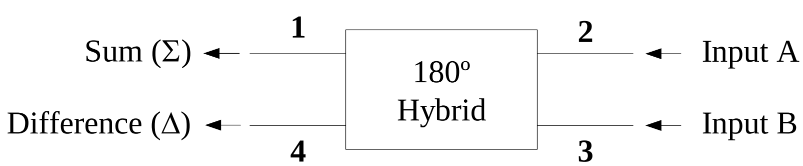

With input at port 1 and using column 1 of [*S*], we can deduce that port 1 is matched, the outputs are ports 2 and 3 (which are in phase with each other) and port 4 is the isolation port.

## 2. Out-of-phase power splitter:




With input at port 4 and using column 4 of [*S*], we can deduce that port 4 is matched, the outputs are ports 2 and 3 (which are completely out of phase with each other) and port 1 is the isolation port.

## 3. Power combiner:




With inputs at ports 2 and 3 and using columns 2 and 3 of [*S*], we can deduce that both ports 2 and 3 are matched, port 1 will provide the sum of the two input signals and port 4 will provide the difference.

Because of this, ports 1 and 4 are sometimes called the sum and difference ports, respectively.

There are different ways to physically implement a 180º hybrid, as shown in Fig. 7.42. We'll focus on the ring hybrid and specifically consider the first two applications described above. There is less symmetry in the *S* matrix (1) for the 180º hybrid than the quadrature hybrid so we expect less physical symmetry as well.

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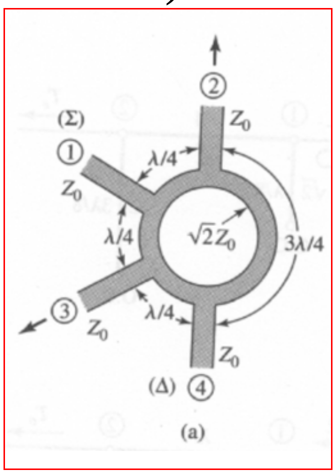
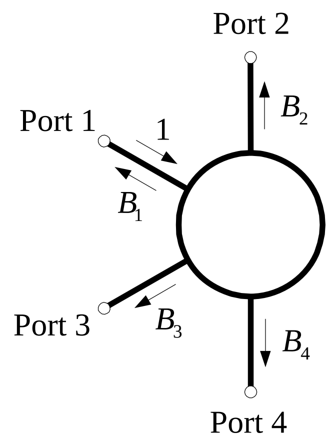
# Ring Hybrid

The ring hybrid (aka the rat race) is shown in Fig. 7.42a:

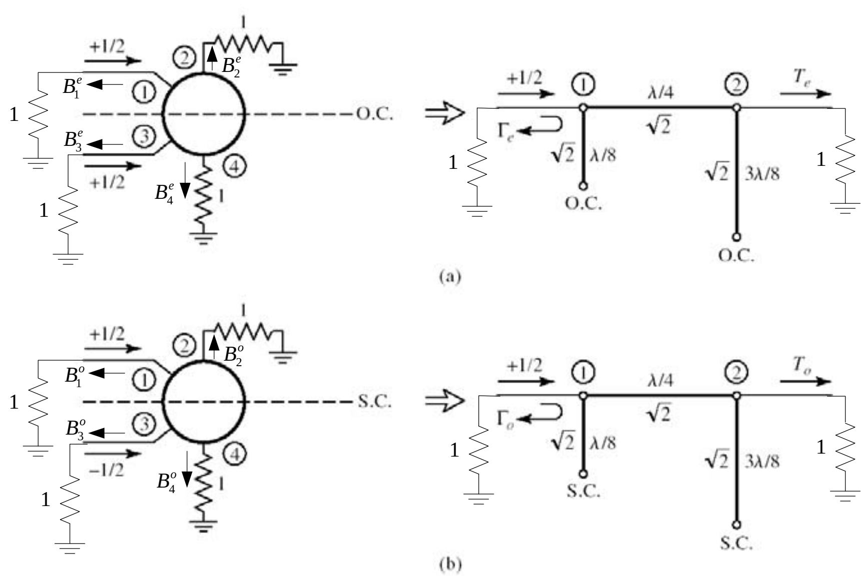



We'll analyze this structure using the same even-odd mode approach we applied to the Wilkinson power divider and the branch line coupler in the previous two lectures. In the present case, the physical symmetry plane bisects ports 1 and 2 from 3 and 4 in the figure above.

1. In-phase power splitter. Assume a unit amplitude voltage wave incident on port 1:




As in Lecture 26, <u>proper</u> symmetric and anti-symmetric <u>excitations</u> of this device are required to produce the even and odd mode problems, as shown in Fig. 7.44:




Notice that we're treating the curved portions of the rat race device as straight sections of TLs. Ignoring this curvature may be a reasonable assumption.

Similar to what we derived in Lecture 26,

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad (7.102a),(2)$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad (7.102b),(3)$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad (7.102c),(4)$$

$$B_4 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad (7.102d),(5)$$

Each of the even and odd solutions for $B_i$ $(i = 1,\ldots,4)$ can be found by cascading *ABCD* matrices, then converting to *S* parameters. Since the ports are terminated by matched loads, we can directly determine $\Gamma_{\substack{e\\o}}$ and $T_{\substack{e\\o}}$ from these *S* parameters.

As given in the text,

$$\Gamma_e = -\Gamma_o = \frac{-j}{\sqrt{2}} \qquad (7.104a,c),(6)$$

$$T_e = T_o = \frac{-j}{\sqrt{2}} \qquad (7.104b,d),(7)$$

Using these values in (2)-(5) produces

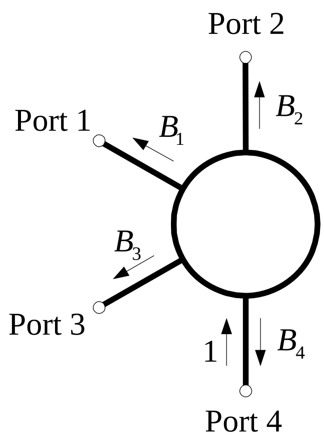
$$B_1 = B_4 = 0 \qquad (7.105a,d),(8)$$

$$B_2 = B_3 = \frac{-j}{\sqrt{2}} \qquad (7.105b,c),(9)$$

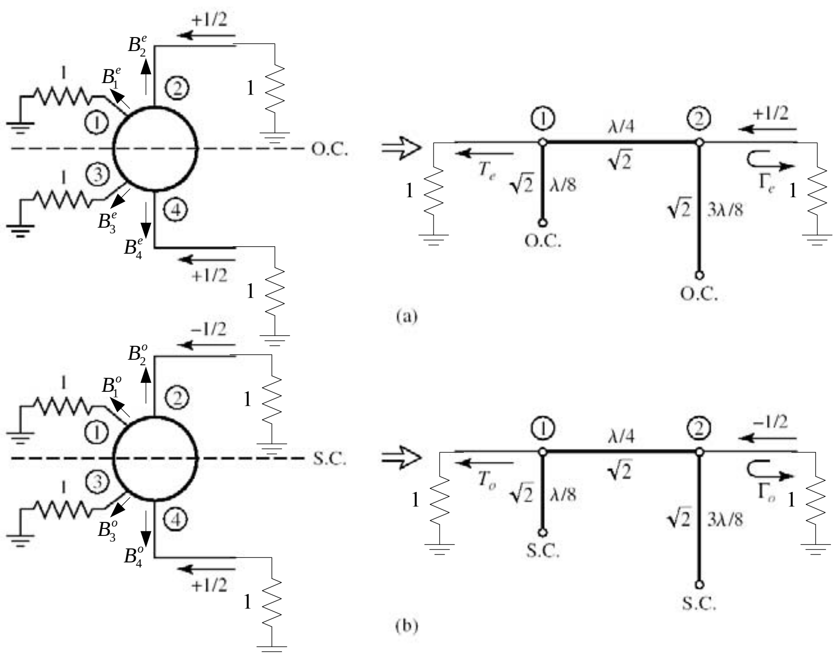
These results in (8) and (9) form the first column of [*S*] in (1). They indicate that with an input at port 1 and all output ports terminated by matched TLs and loads, the signal is equally divided in phase at ports 2 and 3, while none is delivered to port 4.

Using the physical symmetry of the circuit and exciting now at port 3, we can appropriately transpose the rows of column 1 to obtain the third column of [*S*] in (1).

2. Out-of-phase power splitter. Assume a unit amplitude voltage wave is incident on port 4.

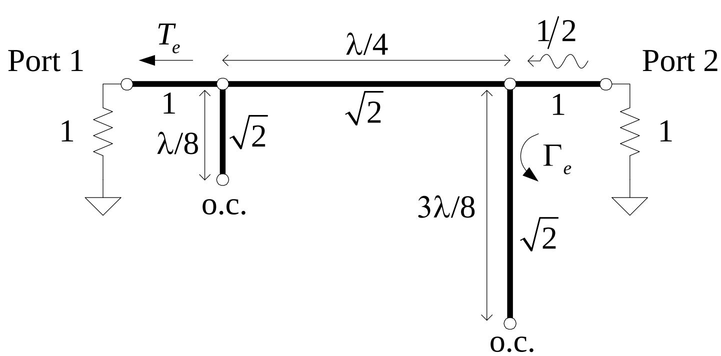



To generate symmetric and anti-symmetric problems, we'll excite the circuit at ports 2 and 4, as shown in Fig. 7.45:




These two excitations sum to +1 at port 4 and 0 at port 2, as required.

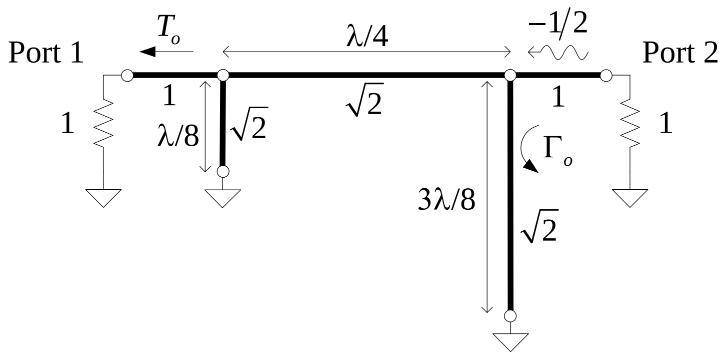
From Fig. 7.45a, the even mode problem is




From this figure (and the even symmetry), we can write

$$B_1^e = \frac{1}{2} T_e = B_3^e \quad \text{and} \quad B_2^e = \frac{1}{2} \Gamma_e = B_4^e \qquad (10),(11)$$

From Fig. 7.45b, the odd mode problem is




From this figure (and the odd symmetry), we can write

$$B_1^o = \frac{-1}{2} T_o = -B_3^o \quad \text{and} \quad B_2^o = \frac{-1}{2} \Gamma_o = -B_4^o \qquad (12),(13)$$

Summing (10)-(13), we find

$$B_1 = B_1^e + B_1^o = \frac{1}{2} T_e - \frac{1}{2} T_o \qquad (7.106a),(14)$$

$$B_2 = B_2^e + B_2^o = \frac{1}{2} \Gamma_e - \frac{1}{2} \Gamma_o \qquad (7.106b),(15)$$

$$B_3 = B_3^e + B_3^o = \frac{1}{2} T_e + \frac{1}{2} T_o \qquad (7.106c),(16)$$

$$B_4 = B_4^e + B_4^o = \frac{1}{2} \Gamma_e + \frac{1}{2} \Gamma_o \qquad (7.106d),(17)$$

Cascading *ABCD* matrices and converting to *S* parameters, the text shows that

$$B_1 = B_4 = 0 \qquad (7.109a,d),(18)$$

$$B_2 = -B_3 = \frac{j}{\sqrt{2}} \qquad (7.109b,c),(19)$$

These values form the fourth column of [*S*] in (1). They indicate that with excitation at port 4 and all output ports terminated by matched TLs and loads, port 1 is isolated and the signal is equally split between output ports 2 and 3 with a 180° phase shift between them.

Once again, using the physical symmetry of the circuit and exciting now at port 2, we can appropriately transpose the rows of column 4 to obtain the second column of [*S*] in (1).

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## Design of 180° Hybrid

The ring hybrid is extremely easy to design. One first computes the effective permittivities and strip widths for the $Z_0$ and $\sqrt{2}Z_0$ sections of the device on a chosen substrate. Then after choosing a center frequency, the physical lengths of the $\lambda/4$ and $3\lambda/4$ portions can be calculated, again using the effective permittivities. That's basically it.

Typical $|S_{1j}|$ results for this device are shown in Fig. 7.46:

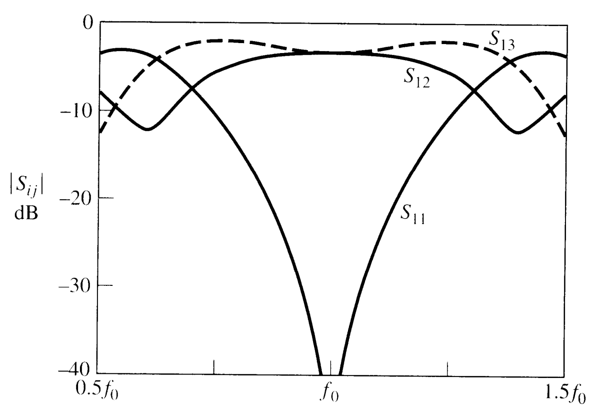


**FIGURE 7.46** $S$ parameter magnitudes versus frequency for the ring hybrid of Example 7.9.

Can you interpret the meaning of these results? How do you expect $|S_{14}|$ and $|S_{32}|$ to behave?