

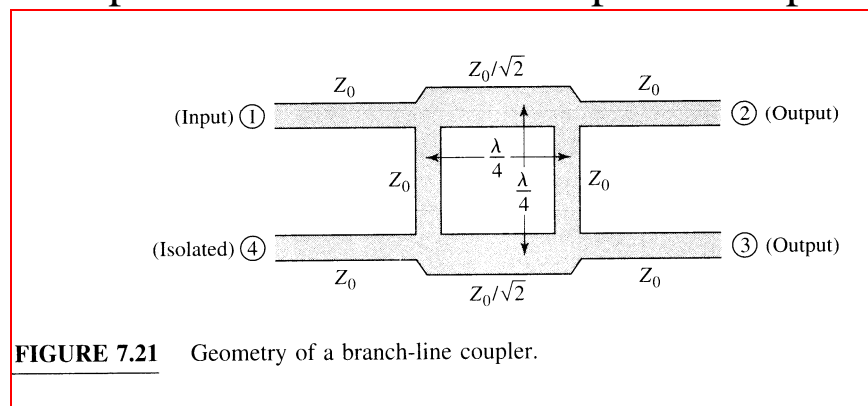
## Lecture 26: Quadrature (90°) Hybrid.

Back in Lecture 23, we began our discussion of dividers and couplers by considering important general properties of three- and four-port networks. This was followed by an analysis of three types of three-port networks in Lectures 24 and 25.

We will now move on to **(reciprocal) directional couplers**, which are four-port networks. As in the text, we will consider these specific types of directional couplers:

1. Quadrature (90°) Hybrid,
2. 180° Hybrid,
3. Coupled Line, and
4. Lange Coupler.

We will begin with the **quadrature (90°) hybrid**. Fig 7.21 shows this coupler implemented with microstrip as a 1:1 power divider:

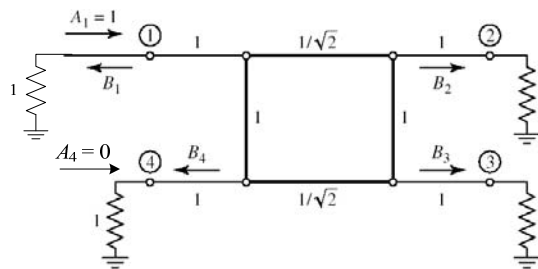


Because of the physical symmetry, we can simplify the analysis of this circuit considerably using **even-odd mode analysis**. This

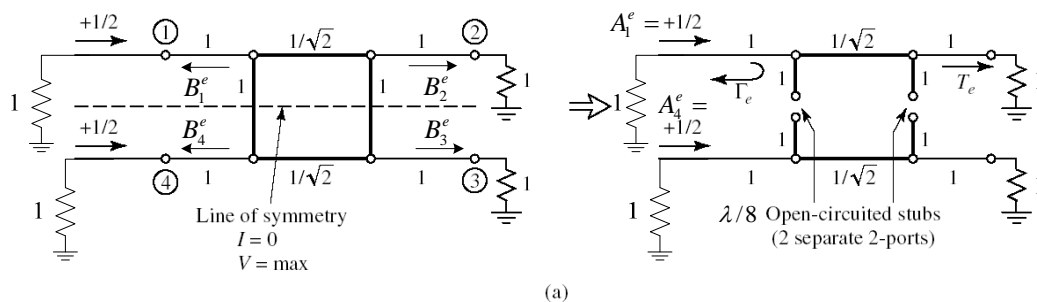
process is similar to what we did in the last lecture with the Wilkinson power divider.

## Even-Odd Mode Analysis of the Quadrature Hybrid

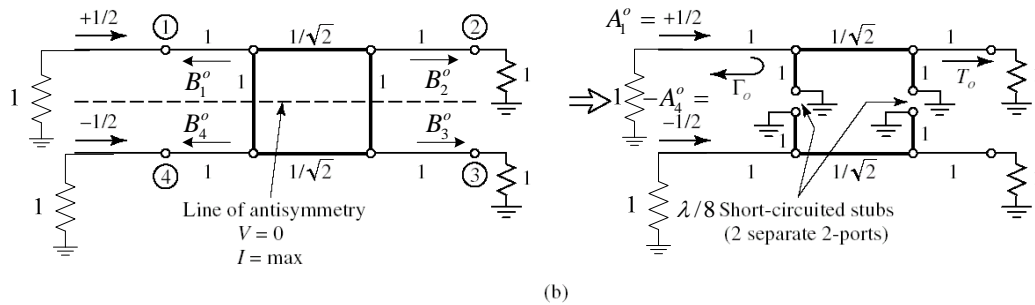
The normalized (wrt  $Z_0$ ) TL circuit is shown in Fig 7.22, minus the return lines:



A **symmetric** (even mode) excitation of this circuit is shown in Fig. 7.23a:



and an **anti-symmetric** (odd mode) excitation is shown in Fig. 7.23b:



Observe that the **circuit and its boundary conditions remain the same** in both the even and odd mode configurations. It is only the excitation that changes. Because of this and the circuit being linear, by superposition the total solution is **simply the sum** of the even and odd mode voltage wave amplitude solutions.

Each solution (even and odd) is simpler to determine than the complete circuit, which is why we employ this technique.

- **Even mode.** Because the voltages and currents must be the same above and below the line of symmetry (LOS) in Fig 7.23a, then  $I = 0$  at the LOS  $\Rightarrow$  open circuit loads at the ends of  $\lambda/8$  stubs, as shown.

Referring to the definition of  $B_i$  ( $i = 1, \dots, 4$ ) in Fig 7.22, we can write from Fig 7.23a that for the even mode excitation:

$$B_1^e = \Gamma_e A_1^e, \quad B_2^e = T_e A_1^e \quad (1a)$$

$$B_3^e = B_2^e = T_e A_1^e, \quad B_4^e = B_1^e = \Gamma_e A_1^e \quad (1b)$$

where  $A_1^e = 1/2$ , and  $\Gamma_e$  and  $T_e$  are the reflection and transmission coefficients for the even mode configuration. (We'll solve for these coefficients shortly.)

- **Odd mode.** Because the voltages and currents must have opposite values above and below the LOS in Fig 7.23b, then  $V = 0$  along the LOS  $\Rightarrow$  short circuit loads at the ends of  $\lambda/8$  stubs, as shown.

Then, 
$$B_1^o = \Gamma_o A_1^o, \quad B_2^o = T_o A_1^o \quad (2a)$$

$$B_3^o = -B_2^o = -T_o A_1^o, \quad B_4^o = -B_1^o = -\Gamma_o A_1^o \quad (2b)$$

where  $A_1^o = 1/2$  and  $\Gamma_o$  and  $T_o$  are reflection and transmission coefficients for the odd mode configuration.

- **Total solution.** The total solution is the sum of the voltages and voltage wave amplitudes in both circuits. From this fact, we can deduce that the total  $B_i$  coefficients will be the sum of (1) and (2):

$$B_1 = B_1^e + B_1^o = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \quad (7.62a),(3)$$

$$B_2 = B_2^e + B_2^o = \frac{1}{2}T_e + \frac{1}{2}T_o \quad (7.62b),(4)$$

$$B_3 = B_3^e + B_3^o = \frac{1}{2}T_e - \frac{1}{2}T_o \quad (7.62c),(5)$$

$$B_4 = B_4^e + B_4^o = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \quad (7.62d),(6)$$

Likewise, the incident wave coefficients are

$$A_1 = A_1^e + A_1^o = \frac{1}{2} + \frac{1}{2} = 1$$

$$A_4 = A_4^e + A_4^o = \frac{1}{2} - \frac{1}{2} = 0$$

These match the assumed excitation in the original circuit on p. 2.

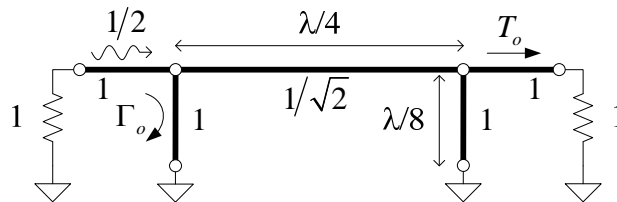
To finish the calculation of the  $S$  parameters for the quadrature hybrid, we need to determine the **reflection and transmission coefficients** for the even- and odd-mode configurations. These are two-port networks that are much easier to solve than the original four-port Quadrature Hybrid.

Your text shows that the solutions for  $\Gamma_e$  and  $T_e$  are

$$\Gamma_e = 0 \quad \text{and} \quad T_e = \frac{-1}{\sqrt{2}}(1 + j) \quad (7.64), (7), (8)$$

Here we'll derive **solutions for  $\Gamma_o$  and  $T_o$** .

From Fig 7.23b:



We have three cascaded elements, so we'll use  $ABCD$  parameters to solve for the overall  $S$  parameters of this circuit.

- **Elements 1 and 3.** These are short circuit stubs of length  $\lambda/8$ , which appear as the shunt impedance

$$Z_{\text{in}} = jZ_0 \tan \beta l \quad \text{where} \quad \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$$

Therefore,  $\frac{Z_{\text{in}}}{Z_0} = j$ , or  $Y_N = -j$

From the inside flap of your text:

$$ABCD = \begin{bmatrix} 1 & 0 \\ Y_N & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \quad (9)$$

- **Element 2.** This is a  $\lambda/4$ -length of TL where

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

From the inside flap of your text:

$$ABCD = \begin{bmatrix} \cos \beta l & j \frac{Z_0/\sqrt{2}}{Z_0} \sin \beta l \\ j \frac{Z_0}{Z_0/\sqrt{2}} \sin \beta l & \cos \beta l \end{bmatrix} = \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix}$$

Cascading these three  $ABCD$  matrices we find the overall  $ABCD$  matrix for odd mode excitation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \quad (10)$$

Using Table 4.2, we can convert these to  $S$  parameters (with  $Z_0 = 1$  for the normalized TL):

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = \frac{1/\sqrt{2} \cdot \frac{1+j-j-1}{1+j+j+1}}{1/\sqrt{2} \cdot \frac{1+j-j-1}{1+j+j+1}} = 0 \quad (11)$$

$$\begin{aligned}
 S_{21} &= \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1/\sqrt{2}} \cdot \frac{1}{1 + j + j + 1} \\
 &= \frac{2\sqrt{2}}{2 + 2j} = \frac{\sqrt{2}}{1 + j}
 \end{aligned} \tag{12}$$

Since the ports are matched, then:

$$\Gamma_o = S_{11} = 0 \tag{7.66a),(13)}$$

and

$$T_o = S_{21} = \frac{\sqrt{2}}{1 + j} \cdot \frac{1 - j}{1 - j} = \frac{1}{\sqrt{2}}(1 - j) \tag{7.66b),(14)}$$

Finally, using (7), (8), (13), and (14) in (3)-(6) we find:

- $B_1 = 0$  (7.67a),(15)

- $B_2 = \frac{-1}{2\sqrt{2}}(1 + j) + \frac{1}{2\sqrt{2}}(1 - j) = \frac{-j}{\sqrt{2}}$  (7.67b),(16)

- $B_3 = \frac{-1}{2} \frac{1}{\sqrt{2}}(1 + j) - \frac{1}{2\sqrt{2}}(1 - j) = \frac{-1}{\sqrt{2}}$  (7.67c),(17)

- $B_4 = \frac{1}{2} \cdot 0 - \frac{1}{2} \cdot 0 = 0$  (7.67d),(18)

These  $B_i$  form the first column of the  $S$  matrix for the Hybrid Quadrature.

When properly interpreted, these results tell us much about the circuit. In particular, when **port 1 is excited** and all other ports terminated in matched loads, then:

- $B_1 = 0 \Rightarrow$  port 1 is matched.

- $B_2 = -j/\sqrt{2} \Rightarrow -90^\circ$  phase shift from port 1 to port 2, and one half of the time average input power is delivered to port 2.
- $B_3 = -1/\sqrt{2} \Rightarrow -180^\circ$  phase shift from port 1 to port 3 ( $90^\circ$  phase shift between ports 3 and 2), and one half of the input power is delivered to port 3. (Hence, 1:1 power division.)
- $B_4 = 0 \Rightarrow$  no power output to port 4.

Because of the high degree of symmetry, we can treat any port as the input port. Then, the isolation is the other port on the same “side” as the input and the outputs are the two ports on the other “side” of the circuit.

Employing this concept and the results above, we can construct the **other three columns** in the full  $S$  matrix for the quadrature ( $90^\circ$ ) hybrid by simply transposing rows of the first column:

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \quad (7.61),(19)$$

That is, the first column in (19) is the results from (15)-(18) when the input was assumed at port 1. In the second column, we can directly deduce that the **outputs** are at ports 1 and 4, the **input** is at port 2 and the **isolation** is at port 4. Further transposition of the rows in column 1 produces columns 3 and 4.



**Example N26.1.** Design a branch line hybrid coupler using 100- $\Omega$  microstrip on 32-mil RO4003C for a center frequency of 2.5 GHz. Include the effects of copper and substrate losses.

Because there are two different characteristic impedances needed for the 90° hybrid device, two different widths of microstrip must be computed (because  $W/d$  depends on  $Z_0$ ) and two different  $\lambda/4$ -lengths must be determined (because  $\epsilon_{r,e}$  depends on  $W/d$ ).

- $Z_0 = 100 \Omega$  sections. Using LineCalc,  $W = 18.02$  mil and  $\epsilon_{r,e} = 2.424$ . The guide wavelength at this frequency is then

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8}{2.5 \times 10^9 \cdot \sqrt{2.424}} = 7.702 \text{ cm}$$

Hence, this branch line coupler should have 100- $\Omega$  lines with length =  $\lambda/4 = 1.93$  cm.

- $Z_0/\sqrt{2} = 70.71 \Omega$  sections. Using LineCalc,  $W = 39.62$  mil and  $\epsilon_{r,e} = 2.545$ . The guide wavelength is then

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8}{2.5 \times 10^9 \cdot \sqrt{2.545}} = 7.517 \text{ cm}$$

Hence, this branch line coupler should have 70.71- $\Omega$  lines with length =  $\lambda/4 = 1.88$  cm.

The following  $S$  parameter results were obtained for this design using ADS.

