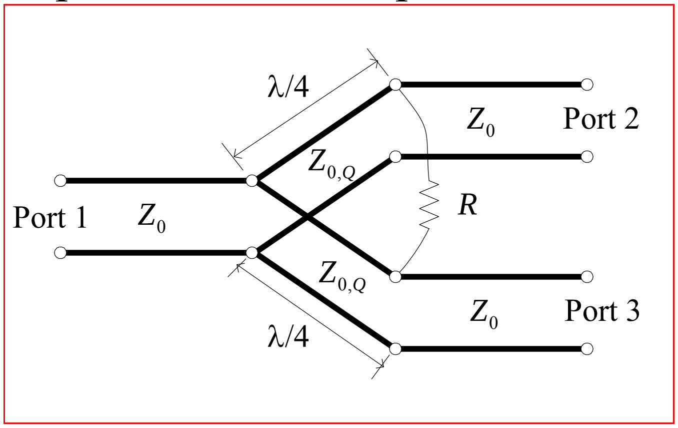
Lecture 25: Wilkinson Power Divider.

The next three port network we will consider is the Wilkinson power divider, in particular the 1:1 power divider (Fig 7.8b):



This is a popular power divider because it is easy to construct and has some extremely useful properties:

1. Matched at all ports,
2. Large isolation between output ports,
3. Reciprocal,
4. Lossless when output ports are matched.

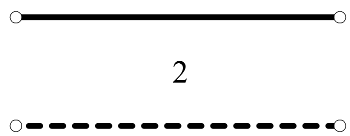
There is much symmetry in this circuit that we can exploit to make the S parameter calculations easier. Specifically, we will excite this circuit in two very special configurations (symmetrically and anti-symmetrically), then add these two solutions for the total solution.

This mathematical process is called an "even-odd mode analysis." It is a technique used in many branches of science such as quantum mechanics, antenna analysis, etc.

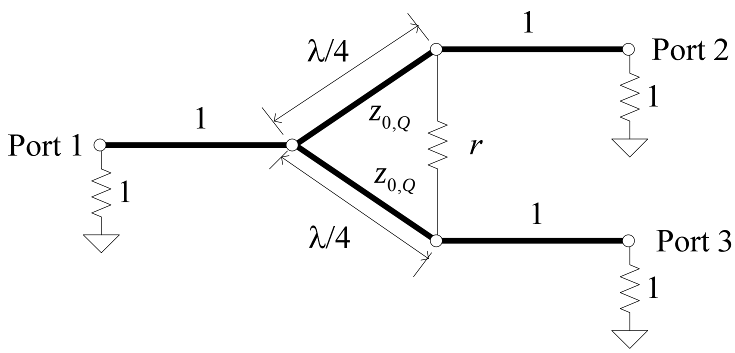
We will now show that for a 1:1 Wilkinson power divider, $Z_{0,Q} = \sqrt{2} Z_0$ and $R = 2Z_0$. To simplify matters, as in the text, we will:

1. Normalize all impedances to Z_0,
2. Not draw the return line for the TL.

For example, a TL with characteristic impedance $2Z_0$ will be delineated as



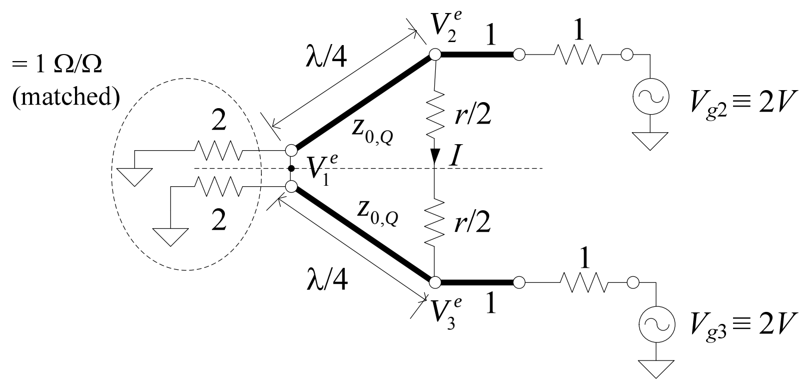
Hence, the Wilkinson power divider shown in the first figure above, and with matched terminations, can be drawn as



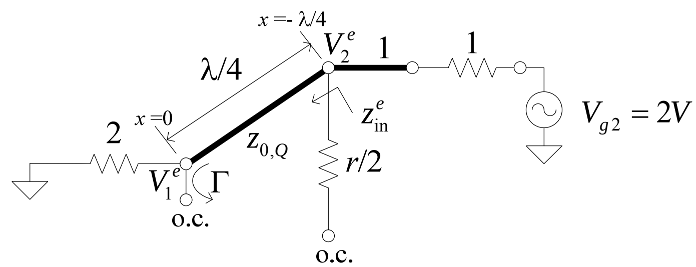
Even-Odd Mode Analysis of the Wilkinson Power Divider

In the even-odd mode analysis for the S parameters, we will first excite this network symmetrically at the two output ports, followed by an anti-symmetrical excitation.

- **Symmetric excitation (even mode):**



Notice that $I = 0$ because we have symmetrical excitation. Hence, $V_2^e = V_3^e$ and we can bisect this circuit as shown to simplify the analysis (Fig. 7.10a):



We can recognize this circuit as a QWT. Consequently,

$$z_{in}^e = \frac{z_{0,Q}^2}{2} \tag{7.33),(1}$$

or

$$z_{0,Q} = \sqrt{2 z_{in}^e} \tag{2}$$

We want the output ports to be matched. Therefore,

$$z_{in}^e = 1 \quad \Rightarrow \quad z_{0,Q} = \sqrt{2} \tag{3}$$

Since $z_{in}^e = 1$, then by voltage division at the output port

$$V_2^e = \frac{z_{in}^e}{z_{in}^e + 1} V_{g2} = \frac{1}{2} V_{g2} = V \tag{4}$$

Next, to find V_1^e we'll use the TL equation

$$V(x) = V^+\left(e^{-j\beta x} + \Gamma e^{j\beta x}\right)$$

so that

$$V(0) = V^+(1+\Gamma) = V_1^e \tag{5}$$

Therefore,

$$V(-\lambda/4) = V^+\left(e^{j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}} + \Gamma e^{-j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}}\right)$$

$$= jV^+(1-\Gamma) = V_2^e \underbrace{=}_{(4)} V \qquad \text{(7.34a),(6)}$$

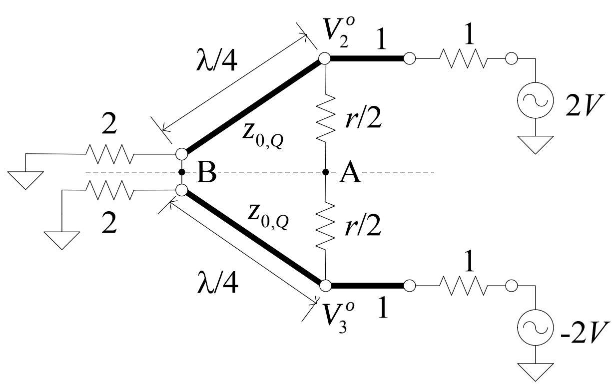
Γ is the reflection coefficient at port 1 seen looking towards the normalized load of 2 Ω/Ω. Therefore,

$$\Gamma = \frac{2 - z_{0,Q}}{2 + z_{0,Q}} \underbrace{=}_{(3)} \frac{2-\sqrt{2}}{2+\sqrt{2}} \tag{7}$$

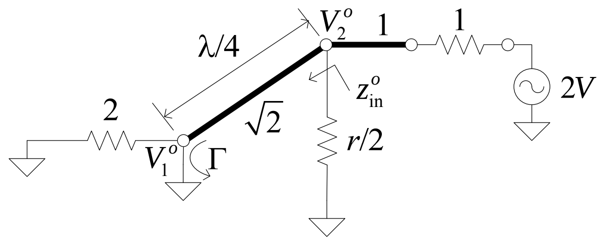
Substituting V^+ from (6) into (5) and using (7) we find that

$$V_1^e = \frac{V}{j(1-\Gamma)}\cdot(1+\Gamma) \underbrace{=}_{(7)} -jV\sqrt{2} \tag{8}$$

- **Anti-symmetric excitation (odd mode):**



Since the circuit is fed anti-symmetrically, $V_3^o = -V_2^o$ and the voltage = 0 at points A and B. Hence, to simplify the analysis, we can bisect the circuit with grounds as shown (Fig 7.10b):



For z_{in}^o, notice that the load is a short circuit and the TL is $\lambda/4$ long (1/2 rotation around the Smith chart). This means $z_{\text{in}}^o = \infty$. Therefore, to match port 2 (and 3) for odd mode excitation, select

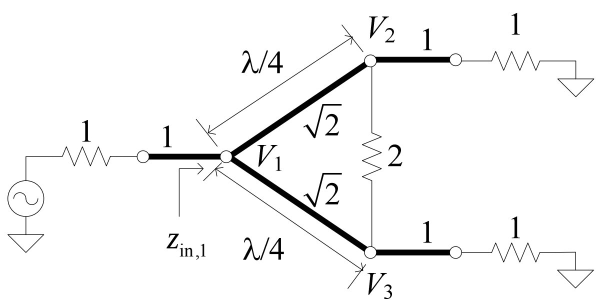
$$\frac{r}{2} = 1 \quad \Rightarrow \quad r = 2 \ [\Omega/\Omega] \tag{9}$$

Further, because $z_{\text{in}}^o = \infty$, then with $r = 2$ and port 2 matched:

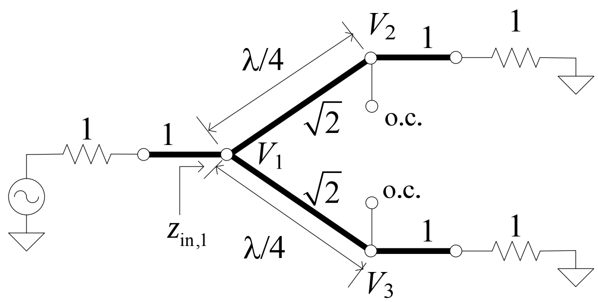
$$V_2^o = \frac{r/2}{r/2+1} \cdot 2V \underbrace{=}_{(9)} V \tag{10}$$

Even and odd solutions are eigenvectors. Any solution can be determined by summing appropriately weighted eigenvectors.

With this information, we'll be able to deduce most of the S parameters. But first, let's determine $z_{\text{in},1}$ so we can compute S_{11}. Terminating ports 2 and 3 gives the circuit in Fig. 7.11a:



By symmetry, $V_2 = V_3$ so we can bisect the circuit like we did in the even mode analysis (Fig 7.11b):



This input impedance is that from a parallel combination of two matched QWTs:

$$z_{\text{in},1} = z_{0,Q} \| z_{0,Q} = \left(\sqrt{2}\right)^2 \| \left(\sqrt{2}\right)^2 = 1 \qquad (7.36),(11)$$

In other words, we have a matched input at port 1.

Also, notice that the effects of *r* no longer appear so this circuit is ideally lossless when matched at both output ports.

S Parameters of the Wilkinson Power Divider

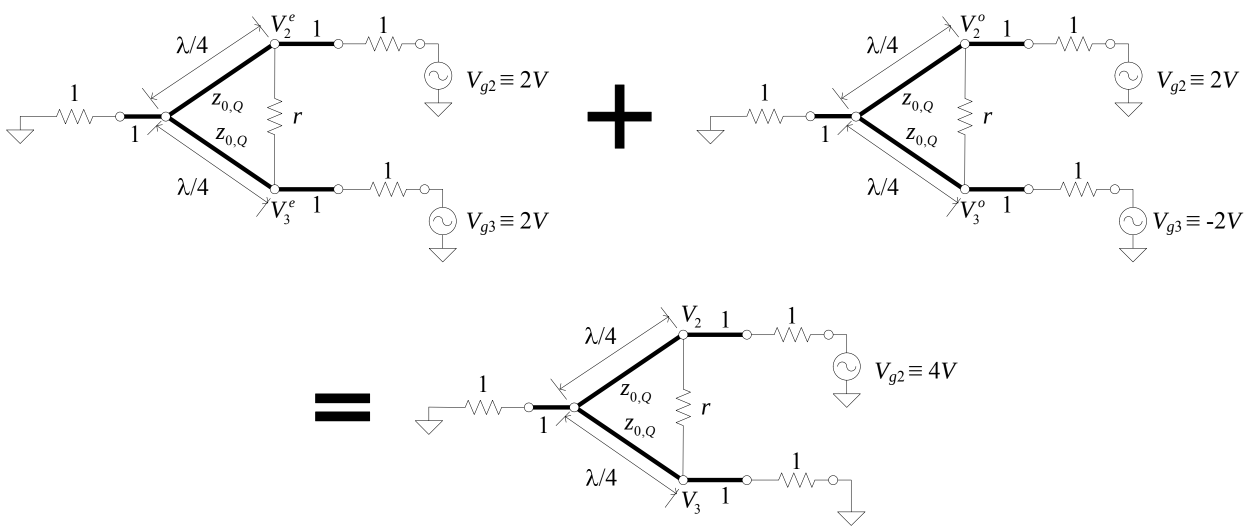
Finally, we will determine the *S* parameters of the Wilkinson power divider.

- S_{11}. From (11),

$$S_{11} = 0 \tag{12}$$

- S_{22} and S_{33}. We'll compute S_{22} here, while by symmetry, $S_{33} = S_{22}$.

  A circuit with a voltage source applied to only port 2 can be obtained by simply adding the even and odd mode excitation problems together:



  To be specific, what we're adding together are the voltages (and currents) everywhere in the identical two circuits that have different excitations.

By definition

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+ = V_3^+ = 0}$$

The voltage wave amplitudes V_2^- and V_2^+ are the sum of the respective voltages from the even and odd mode excitation circuits. That is,

$$S_{22} = \left.\frac{V_2^{e,-} + V_2^{o,-}}{V_2^{e,+} + V_2^{o,+}}\right|_{V_1^+ = V_3^+ = 0}$$

Earlier in (3), we chose $z_{0,Q} = \sqrt{2}$ in the even mode solution so that port 2 was matched, meaning $V_2^{e,-} = 0$. Likewise, in the odd mode solution we chose $r = 2$ in (9) to match port 2, meaning $V_2^{o,-} = 0$. Using these two results means that

$$S_{22} = 0 = S_{33} \tag{13}$$

The last equality is valid due to the symmetry in the Wilkinson power divider circuit with respect to ports 2 and 3.

- S_{12} and S_{21}. By definition

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+ = V_3^+ = 0}$$

We can use the previous figure with excitation at port 2 for this solution to S_{12}. We'll add voltages together from the even and odd mode solutions, similar to what we did in the solution for S_{22}.

Because port 1 is matched in both the even and odd mode circuits ($V_1^{e,+} = V_1^{o,+} = 0$), the total voltage at port 1 is just V_1^-:

$$V_1^- = V_1 = V_1^e + V_1^o \tag{14}$$

Similarly, since port 2 is matched in both the even and odd mode circuits ($V_2^{e,-} = V_2^{o,-} = 0$) then

$$V_2^+ = V_2 = V_2^e + V_2^o \tag{15}$$

Consequently, using (14) and (15) along with (4), (8), (10), and $V_1^o = 0$ (p. 5) we find

$$S_{12} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{-jV\sqrt{2} + 0}{V + V} = -j\frac{\sqrt{2}}{2}$$

or

$$S_{12} = -\frac{j}{\sqrt{2}} = S_{21} \tag{16}$$

The last equality arises because the Wilkinson power divider is a reciprocal network.

- S_{13} and S_{31}. Using a similar approach as that for S_{12} and S_{21}, it can be shown that

$$S_{13} = S_{31} = -\frac{j}{\sqrt{2}} \tag{17}$$

- S_{32} and S_{23}. By definition

$$S_{32} = \left.\frac{V_3^-}{V_2^+}\right|_{V_1^+ = V_3^+ = 0} = \left.\frac{V_3^{e,-} + V_3^{o,-}}{V_2^{e,+} + V_2^{o,+}}\right|_{V_1^+ = V_3^+ = 0}$$

But, from the symmetry of the circuit and the odd and even nature of the solutions, we know

$$V_3^{e,-} = V_2^{e,-} \quad \text{and} \quad V_3^{o,-} = -V_2^{o,-}$$

such that,

$$S_{32} = \left. \frac{V_2^{e,-} - V_2^{o,-}}{V_2^{e,+} + V_2^{o,+}} \right|_{V_1^+ = V_3^+ = 0} \tag{18}$$

Using (4) and (10) in (18), we find:

$$S_{32} = \frac{V - V}{V + V} = 0 = S_{23} \tag{19}$$

This last result shows that at the design frequency there is complete isolation between the output ports! Nice.

Remember that these S parameters for the Wilkinson divider are only applicable at the design frequency since we used QWTs.

Recap

Let's reflect on this Wilkinson power divider design for a moment. We listed on page 1 four properties we wanted to build into this power divider. We began with only two degrees of freedom in the circuit on page 1: $Z_{0,Q}$ and R. Both of these were used to obtain three matched ports.

So how are the other three conditions satisfied since we've used up all the degrees of freedom available to us? First, the circuit is obviously reciprocal since it is constructed from metal and dielectric materials only.

The remaining two conditions (large isolation and lossless when the output ports are matched) are met because of the symmetric nature of the circuit! Consequently, it is very important to ensure that Wilkinson power divider circuits maintain this **physical symmetry** when they are constructed.