Lecture 23: Basic Properties of Dividers and Couplers.

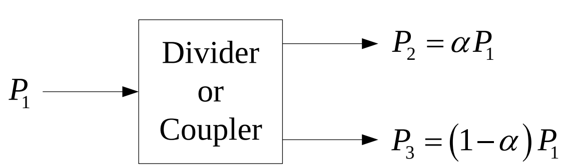
For the remainder of this course we're going to investigate a plethora of microwave devices and circuits – both passive and active.

To begin, during the next six lectures we will focus on different types of power combiners, power dividers, and directional couplers.

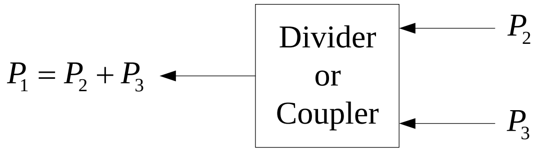
Such circuits are ubiquitous and highly useful. Applications include:

- Dividing (combining) a transmitter (receiver) signal to (from) many antennas.
- Separating forward and reverse propagating waves (can also use for a sort of matching).
- Signal combining for a mixer.

As a simple example, a two-way power splitter would have the form (Fig 7.1a):



where $\alpha \in \mathbb{R}$ and $0 \le \alpha \le 1$. The same device can often be used as a power combiner (Fig. 7.1b):



We see that even the simplest divider and combiner circuits are three-port networks. It is common to see dividers and couplers with even more ports than that.

But before we consider specific examples, it will be beneficial for us to consider some general properties of three- and four-port networks.

Basic Properties of Three-Port Networks

As we'll show here, it's not possible to construct a three-port network that is:

1. lossless,
2. reciprocal, and
3. matched at all ports.

This basic property of three-ports limits our expectations for power splitters and combiners. We must design around it.

To begin, a three-port network has an S matrix of the form:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad (7.1),(1)$$

If the network is matched at every port, then $S_{11} = S_{22} = S_{33} = 0$. (It is important to understand that "matched at every port" means Γ_1, Γ_2, and $\Gamma_3 = 0$ when all other ports are terminated in Z_0.)

If the network is reciprocal, then $S_{21} = S_{12}$, $S_{31} = S_{13}$, and $S_{32} = S_{23}$. Consequently, for a matched and reciprocal three-port, its S matrix has the form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \qquad (7.2),(2)$$

Note there are only three different S parameters in this matrix.

Lastly, if the network is lossless, then $[S]$ is unitary. Applying (4.53a) to (2), we find that

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad (7.3a),(3)$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad (7.3b),(4)$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad (7.3c),(5)$$

and applying (4.53b) we find that:

$$S_{13}^* S_{23} = 0 \qquad (7.3d),(6)$$

$$S_{23}^* S_{12} = 0 \qquad (7.3e),(7)$$

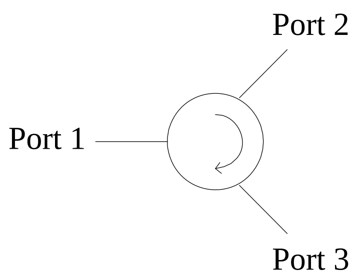
$$S_{12}^* S_{13} = 0 \qquad (7.3f),(8)$$

From (6)-(8), it can be surmised that at least two of the three S parameters must equal zero. If this is the case, then not all of the equations (3)-(5) can be satisfied. [For example, say $S_{13} = 0$. Then (6) and (8) are satisfied. For (7) to be satisfied and $S_{23} \neq 0$, we must have $S_{12} = 0$. But with S_{12} and S_{13} both zero, then (3) cannot be satisfied.]

Our conclusion then is that a three-port network cannot be lossless, reciprocal, and matched at all ports. Bummer. This finding has wide-ranging ramifications.

However, one can realize such a network if any of these three constraints is loosened. Here are three possibilities:

1. Nonreciprocal three-port. In this case, a lossless three-port that is matched at all ports can be realized. It is called a circulator (Fig 7.2a):



$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

   Notice that $S_{ij} \neq S_{ji}$.

2. Match only two of the three ports. Assume ports 1 and 2 are matched. Then,

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \qquad (7.7),(9)$$

3. Lossy network. All ports can be simultaneously matched and the network reciprocal.

Basic Properties of Four-Port Networks

Unlike three-ports, it **is possible** to make a lossless, matched, and reciprocal four-port network. These are called directional couplers.

The *S* matrix of a reciprocal and matched four-port has the form

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \qquad (7.9),(10)$$

Incorporating the fact that the network is lossless puts further constraints on these *S* parameters, as discussed in the text.

As described in Section 7.1 of the text, there are two commonly used realizations of directional couplers:

1. The Symmetrical Coupler. The *S* matrix for this device is

$$[S]=\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \qquad (7.17),(11)$$

where $\alpha, \beta \in \mathbb{R}$ and $\alpha^2 + \beta^2 = 1$. It is obvious from the S matrix that the network is reciprocal and matched. It can also be shown that $[S]$ is unitary, which means this four-port is also lossless.

We will study this coupler later as the Quadrature (90º) Hybrid.

2. The Asymmetrical Coupler. The S matrix for this device is

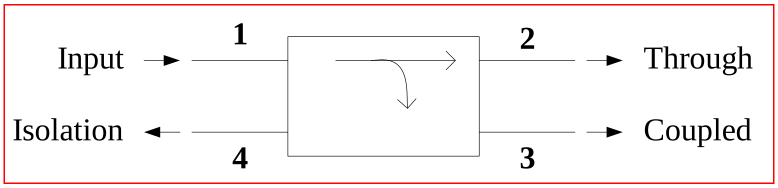
$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix} \qquad (7.18),(12)$$

We can see from this S matrix that the network is matched and reciprocal. It can also be shown that the network is lossless.

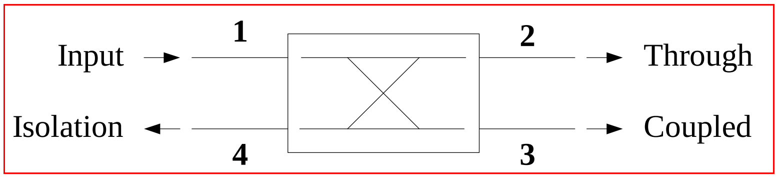
We will study this coupler later as the 180º Hybrid.

Directional Couplers

We will now take a quick look at the operation of a directional coupler. Common circuit symbols are (Fig 7.4):



or



The arrows indicate the assumed directions of time average power flow.

As we saw in (11) and (12), the S matrix of the symmetrical and antisymmetrical directional couplers has the form

$$[S] = \begin{bmatrix} 0 & \_ & \_ & 0 \\ \_ & 0 & 0 & \_ \\ \_ & 0 & 0 & \_ \\ 0 & \_ & \_ & 0 \end{bmatrix}$$

where '_' indicates a non-zero value.

We can deduce the operation of this network directly from the S matrix, assuming all the ports are matched.

For example, if power enters port 1, then from column 1 of $[S]$ this power splits between ports 2 ($S_{21} \neq 0$) and 3 ($S_{31} \neq 0$), while no power is delivered to port 4 ($S_{41} = 0$). Since $S_{11} = 0$, there will be no reflected power from port 1.

Alternatively, if power enters from port 2, then from the second column of [S] we deduce that the signal splits between ports 1 ($S_{12} \neq 0$) and 4 ($S_{42} \neq 0$), but none to port 3 ($S_{32} = 0$).

Of course, no directional coupler is ideal and the S matrix above is only approximately realized in practice.

The performance of directional couplers is characterized by the following three values. For these definitions, port 1 is assumed the input, ports 2 and 3 the outputs, and port 4 is the isolated port.

1. Coupling, C:

$$C \equiv 10\log_{10}\frac{P_1}{P_3} = 10\log_{10}\frac{1}{|S_{31}|^2} \text{ dB}$$

or, using (11) and (12):

$$\boxed{C = -20\log_{10}|\beta| \text{ dB}} \qquad (7.20a),(13)$$

2. Directivity, D:

$$D \equiv 10\log_{10}\frac{P_3}{P_4} = 10\log_{10}\frac{P_3/P_1}{P_4/P_1} = 10\log_{10}\frac{|S_{31}|^2}{|S_{41}|^2} \text{ dB}$$

or, using (11) and (12):

$$\boxed{D = 20\log_{10}\frac{|\beta|}{|S_{14}|} \text{ dB}} \qquad (7.20b),(14)$$

If the directional coupler is ideal, then $D \to \infty$.

3. Isolation, *I*:

$$I \equiv 10\log_{10}\frac{P_1}{P_4} = 10\log_{10}\frac{1}{\left|S_{41}\right|^2} \text{ dB}$$

$$\boxed{I = -20\log_{10}\left|S_{14}\right| \text{ dB}} \qquad (7.20c),(15)$$

Similarly, if the directional coupler is ideal, then $I \to \infty$.

These three quantities are related by

$$\boxed{I = D + C \text{ dB}} \qquad (7.21),(16)$$

4. Insertion Loss, *L*:

$$L \equiv 10\log_{10}\frac{P_1}{P_2} = 10\log_{10}\frac{1}{\left|S_{21}\right|^2} \text{ dB}$$

or, using (11) and (12):

$$\boxed{L = -20\log_{10}\left|\alpha\right| \text{ dB}} \qquad (7.20d),(17)$$