

Lecture 22: Measurement Errors. TRL Calibration of a VNA.

A VNA measures both the magnitude and phase of S parameters. However, there will invariably be significant errors in these microwave measurements that must be “**removed**” somehow if we are to obtain accurate results.

As we discussed in Lecture 18, there are **three general types of errors**:

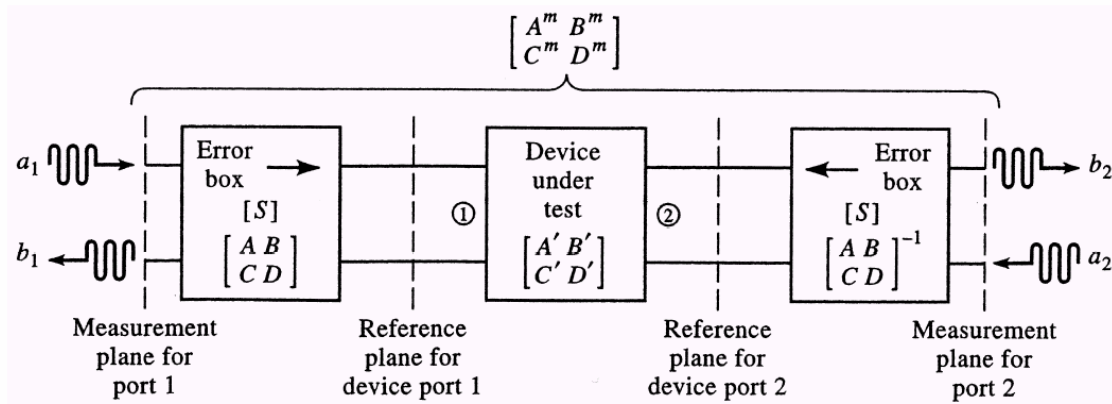
1. **Systematic**: repeatable errors due to imperfections in components, connectors, test fixtures, etc.
2. **Random**: vary unpredictability with time and cannot be removed. From noise, connector repeatability, etc.
3. **Drift**: caused by changes in systems characteristics after a calibration has been performed due to temperature, humidity, and other environmental variables.

Using well-designed and maintained equipment in an unchanging environment is about all we can do to minimize random errors. A similar environment helps minimize drift errors, or the network analyzer can be recalibrated.

The effects of **systematic errors** can be **largely removed** from the S parameters using “measurement calibration.” (In the context of microwave measurements, “measurement calibration” has a

much different meaning than “instrument calibration” for low-frequency equipment.)

To do this measurement calibration, we need to **assume a general mathematical model** for the effects of systematic errors, such as that shown in Fig 4.20:



(Fig. 4.20)

We'll define $[S^m]$ as the S parameters that are actually measured by the VNA. These include all of the errors we mentioned earlier.

The **error boxes** (with parameters $[S]$) and how these are specifically connected to the DUT form the *model* of the systematic errors. The parameters $[S']$ are those we desire to know. These are the S parameters of the DUT, which also, unfortunately, contain random errors.

The purpose of network analyzer measurement calibration is to determine the numerical values of all the S (or $ABCD$) parameters in the error model at each frequency of interest.

For coaxial measurements, we often use precision Short, Open, Load, and Thru (**SOLT**) standards as loads connected to the test ports. With these known standards as loads, we make several S -parameter measurements to construct enough equations from which we can numerically determine the error parameters.

Thru-Reflect-Line (TRL) Calibration

SOLT standards are difficult to implement for VNA measurements of microstrip and similar circuits. Instead, the **Thru-Reflect-Line (TRL) method** is more commonly used.

The TRL calibration method is very cleverly designed. It doesn't rely on precisely known standards and it uses only three simple connections to completely characterize the error model.

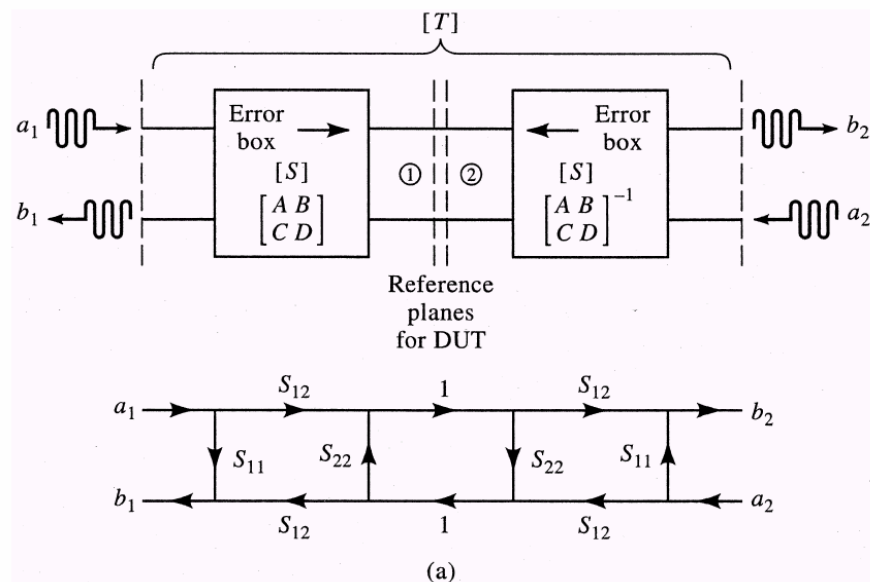
The **three connections for TRL calibration** of microstrip are:

1. **Thru.** Directly connect port 1 to 2, at the desired reference planes, using matched microstrip.

2. **Reflect.** Terminate a microstrip connected to each port with a load that produces a large reflection, say an open or short. These can be imperfect loads.
3. **Line.** Connect the two ports together through a microstrip approximately $\lambda/4$ longer than the Thru (at the center frequency).

We will step through each of these connections and outline the solutions for the S parameters using signal flow diagrams.

1. Thru Standard. The configuration for this measurement is shown in Fig 4.21a. The measured S matrix is defined as $[T]$:



(Fig. 4.21a)

Notice that:

- a. The $[S]$ matrices for the two error boxes are assumed to be **identical**. This simplifies things for us right now, though this is not assumed in actual VNA TRL “cal kits.”

- b. The reflection planes for the DUT are **coincident**. Consequently, this is called a “zero length Thru.” You’ll use this in the lab.
- c. $S_{21} = S_{12}$ for a reciprocal error box.

We can solve the signal flow graph for the elements of $[T]$, noting that $a_2 = 0$ because the measurement is made with a matched termination. (Note that this is a big assumption in the laboratory setting!)

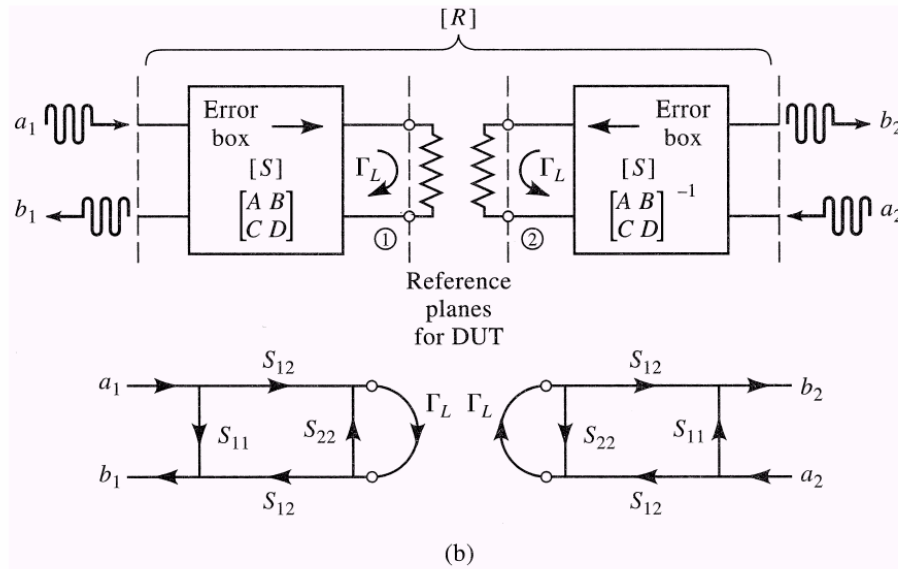
As you’ll determine in your homework:

$$T_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = S_{11} + \frac{S_{22}S_{12}^2}{1 - S_{22}^2} = T_{22} \quad (4.80a),(1)$$

and

$$T_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} = \frac{S_{12}^2}{1 - S_{22}^2} = T_{21} \quad (4.80b),(2)$$

2. Reflect Standard. The configuration for this measurement is shown in Fig. 4.21b. The measured S matrix is defined as $[R]$:



(Fig. 4.21b)

Notice that:

- The lines are terminated by an **unknown load**, with Γ_L to be determined, if that is of interest. This knowledge of Γ_L isn't necessary for calibration, however.
- The reference planes for the load are the **same locations** as the "zero length Thru." This will also be the situation in the lab.

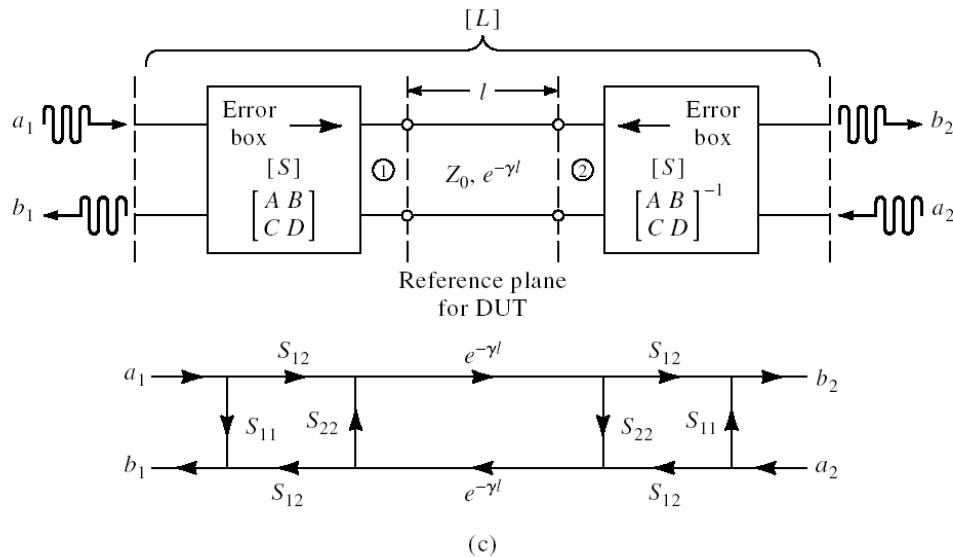
We solved this type of SFG before in the previous lecture. The solution is

$$R_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = S_{11} + \frac{S_{12}^2 \Gamma_L}{1 - S_{22} \Gamma_L} = R_{22} \quad (4.81), (3)$$

Since the two ports are not connected together, then

$$R_{21} = R_{12} = 0 \quad (4)$$

3. Line Standard. The configuration for this measurement is shown in Fig 4.21c. The measured S matrix is defined as $[L]$:



(Fig. 4.21c)

Notice that:

- The length l (beyond the Thru) need **not** be specified.
- The line may be **lossy**.

As you'll show in the homework:

$$L_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = S_{11} + \frac{S_{22}S_{12}^2 e^{-2\gamma l}}{1 - S_{22}^2 e^{-2\gamma l}} = L_{22} \quad (4.82a),(5)$$

and

$$L_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} = \frac{S_{12}^2 e^{-\gamma l}}{1 - S_{22}^2 e^{-2\gamma l}} = L_{21} \quad (4.82b),(6)$$

We now have five equations – (1), (2), (3), (5), and (6) – to solve for the five unknowns S_{11} , S_{22} , S_{12} , $e^{-\gamma l}$, and Γ_L .

These solutions are fairly easy to derive, as shown in the text, and are given in equations (4.84), and (4.86)-(4.89).

So, we can now determine $[S']$ (the S parameters of the DUT) from $[S^m]$ (the measured parameters), as shown in Fig 4.20.

Since we're cascading two-ports, we'll use $ABCD$ matrices to simplify the calculations. Table 4.2 is useful when making these conversions.

From Fig 4.20, we see the measured $ABCD$ parameters are given in terms of the error box and DUT parameters by

$$\begin{bmatrix} A^m & B^m \\ C^m & D^m \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\text{Error box}} \cdot \underbrace{\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}}_{\text{DUT}} \cdot \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1}}_{\text{Error box}} \quad (7)$$

The last matrix is an inverse because we've “switched” ports 1 and 2 by the connection shown in Fig. 4.20.

Next, we pre-multiply (7) by $\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1}$:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} A^m & B^m \\ C^m & D^m \end{bmatrix} = [I] \cdot \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \quad (8)$$

then post-multiply (8) by $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} A^m & B^m \\ C^m & D^m \end{bmatrix} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot [I] \quad (9)$$

Therefore,

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} A^m & B^m \\ C^m & D^m \end{bmatrix} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (4.90),(10)$$

At this point, the DUT parameters are now “**de-embedded**” from the measured parameters by this TRL calibration.

The Agilent 8753ES VNA will **automatically** perform these numerical calculations for you and after calibration will present the “corrected” S parameters.

The error box coefficients $ABCD$ are functions of frequency so these TRL measurements and calculations are performed at every frequency point of interest.

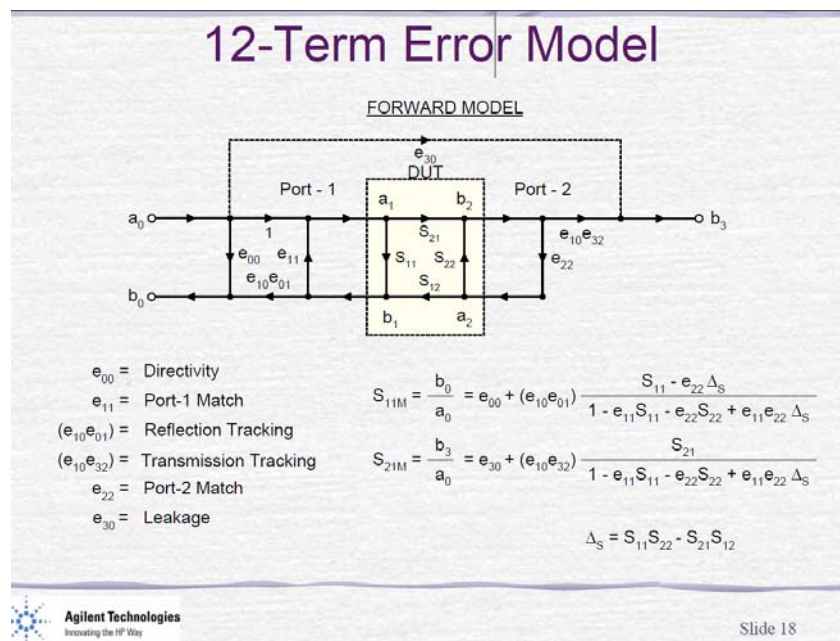
Actual VNA Calibration

The example shown in this lecture with the simple four term error correction model in Fig. 4.20 is really of **little practical use in the laboratory**. It does serve as an excellent example of how measurement calibration is performed, however.

The main shortcoming of in this simple four term error correction model is that not enough sources of error have been

allowed. One glaring example, is that because the same four terms are used for both error correction boxes in Fig. 4.20, we're implicitly assuming the measurement equipment and the setup is **symmetrical**. That is certainly not something to likely be encountered in the laboratory, and we have found this assumption to be inaccurate.

The VNA you will be using in the laboratory has a much more sophisticated error correction model, likely involving 12 terms that are computed at each frequency of interest using properly applied standards. **This lecture gives you a good working idea of how that is accomplished.**



12-Term Error Model

REVERSE MODEL

e'_{33} = Directivity
 e'_{11} = Port-1 Match
 $(e'_{23}e'_{32})$ = Reflection Tracking
 $(e'_{23}e'_{01})$ = Transmission Tracking
 e'_{22} = Port-2 Match
 e'_{03} = Leakage

$$S_{22M} = \frac{b'_3}{a'_3} = e'_{33} + (e'_{23}e'_{32}) \frac{S_{22} - e'_{11}\Delta_S}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta_S}$$

$$S_{12M} = \frac{b'_0}{a'_3} = e'_{03} + (e'_{23}e'_{01}) \frac{S_{12}}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta_S}$$

$$\Delta_S = S_{11}S_{22} - S_{21}S_{12}$$

12-Term Error Model

$$S_{11} = \frac{\left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right) \left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right) e'_{22}\right] - e'_{22} \left(\frac{S_{21M} - e'_{30}}{e'_{10}e'_{32}}\right) \left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

$$S_{21} = \frac{\left(\frac{S_{21M} - e'_{30}}{e'_{10}e'_{32}}\right) \left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right) (e'_{22} - e'_{22})\right]}{D}$$

$$S_{22} = \frac{\left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right) \left[1 + \left(\frac{S_{11M} - e'_{00}}{e'_{10}e'_{01}}\right) e'_{11}\right] - e'_{11} \left(\frac{S_{21M} - e'_{30}}{e'_{10}e'_{32}}\right) \left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

$$S_{12} = \frac{\left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right) \left[1 + \left(\frac{S_{11M} - e'_{00}}{e'_{10}e'_{01}}\right) (e'_{11} - e'_{11})\right]}{D}$$

$$D = \left[1 + \left(\frac{S_{11M} - e'_{00}}{e'_{10}e'_{01}}\right) e'_{11}\right] \left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23}e'_{32}}\right) e'_{22}\right] - \left(\frac{S_{21M} - e'_{30}}{e'_{10}e'_{32}}\right) \left(\frac{S_{12M} - e'_{03}}{e'_{23}e'_{01}}\right) e'_{22}e'_{11}$$

This is the result of solving the four simultaneous measured s-parameter equations. Note that each actual s-parameter calculated requires measuring all four S-parameters as well as knowing the 12 error terms.

Source: Doug Rytting, "Network Analyzer Error Models and Calibration Methods," Agilent Technologies.