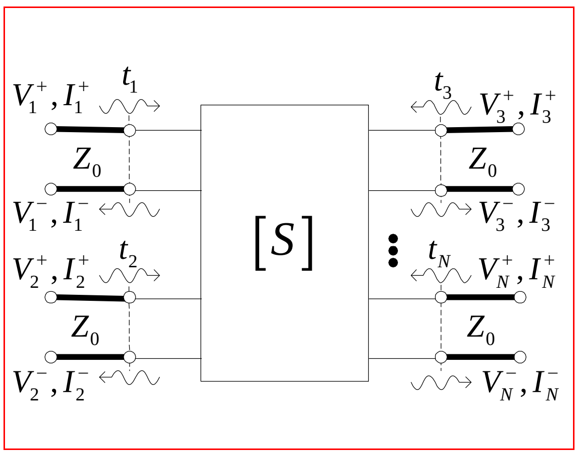
Lecture 15: *S* Parameters and the Scattering Matrix.

While *Z* and *Y* parameters can be useful descriptions for networks, *S* and *ABCD* parameters are even more widely used in microwave circuit work. We'll begin with the scattering (or *S*) parameters.

As we'll see shortly in this course, *S* parameters play a hugely important role in microwave engineering. It may be helpful to know that as voltage and current are to electrical circuit analysis, *S* parameters are to microwave network analysis.

Consider again the multi-port network from the last lecture, which is connected to *N* transmission lines as:



Rather than focusing on the *total* voltages and currents (i.e., the sum of "+" and "-" waves) at the terminal planes $t_1, \ldots, t_n$, the ***S* parameters** are formed from ratios of reflected and incident voltage wave amplitudes.

When the characteristic impedances of all TLs connected to the network are the same (as is the case for the network shown above), then the S parameters are defined as

$$\begin{bmatrix} V_1^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

or

$$\left[V^-\right] = [S] \cdot \left[V^+\right] \qquad (4.40),(1)$$

where $[S]$ is called the scattering matrix.

As we defined in the last lecture, the terminal planes are the "phase = 0" planes at each port. That is, with

$$V_n(z_n) = V_n^+ e^{-j\beta_n(z_n - t_n)} + V_n^- e^{j\beta_n(z_n - t_n)} \qquad n = 1, \ldots, N$$

then at the terminal plane t_n

$$V_n(z_n = t_n) \equiv V_n = V_n^+ + V_n^-$$

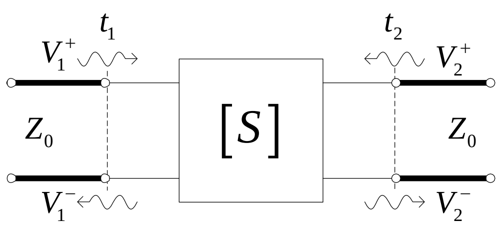
Each S parameter in (1) can be computed as

$$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+ = 0,\ \forall k \neq j} \qquad (4.41),(2)$$

Notice in this expression that the wave amplitude ratio is defined "from" port j "to" port i:

$$S_{i\,j}$$

Let's take a close look at this definition (2). Imagine we have a two-port network:



Then, using (2),
$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0}$$

Simple enough, but how do we make $V_2^+ = 0$? This requires that:

1. There is no source on the port-2 side of the network, and
2. Port 2 is matched so there are no reflections from this port.

Consequently, with $V_2^+ = 0 \;\Rightarrow\; S_{11} = \Gamma_{11}$, which is the voltage reflection coefficient at port 1.

Next, using (2) once again, consider
$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

Again, with a matched load at port 2 so that $V_2^+ = 0$, then
$$S_{21} = T_{21}$$
which is the voltage transmission coefficient from port 1 to port 2.

It is very important to realize it is a mistake to say S_{11} is the voltage reflection coefficient at port 1. Actually, S_{11} is such a reflection coefficient only when $V_2^+ = 0$.

As we'll see in the following example, if port 1 is not matched, then the reflection coefficient at port 1 will generally depend not only on S_{11}, but also on all other S parameters, as well as the load impedance.

An advantage of using S parameters compared to other parameter types is that matched loads are used for terminating the ports rather than opens and shorts. In some circuits this difference is critical. For example, with transistor amplifiers a nearly matched load may be necessary for the amplifier to operate correctly, whereas an open or short load may render the amplifier nonfunctional.

Example N15.1: (Similar to text example 4.5.) A two-port network has the following S matrix referred to some system impedance Z_0:

$$[S] = \begin{bmatrix} 0.1 & 0.8\measuredangle 90^\circ \\ 0.8\measuredangle 90^\circ & 0.2 \end{bmatrix} \tag{3}$$

If a short circuit is connected to port 2, what is the resulting return loss at port 1?

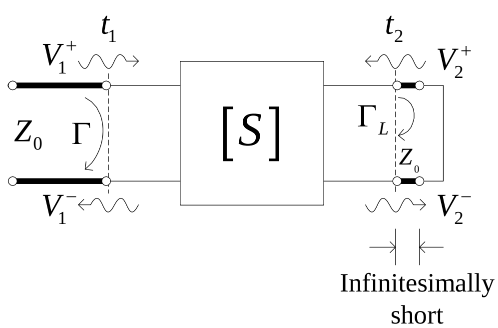
From the definition (1)

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

we find that

$$\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ \end{aligned} \tag{4}$$

How can we incorporate the short circuit load into these equations? Start with TLs connected to both ports as:



From this circuit, we see that

$$\Gamma_L = \frac{V_2^+}{V_2^-}$$

It is important to realize this definition for Γ_L is the inverse of what you may have first thought, because of the assumed direction of the "incident" waves for S parameters: which are always *into* the port.

So, for a short circuit load, $\Gamma_L = -1 \Rightarrow V_2^+ = -V_2^-$. Using this result in (4) we find

$$V_1^- = S_{11}V_1^+ - S_{12}V_2^- \tag{5}$$

$$V_2^- = S_{21}V_1^+ - S_{22}V_2^- \quad (6)$$

Our desired result is the input reflection coefficient $\Gamma = V_1^- / V_1^+$. Rearranging (6), we obtain

$$V_2^- + S_{22}V_2^- = S_{21}V_1^+$$

or

$$V_2^- = \frac{S_{21}}{1+S_{22}}V_1^+$$

Substituting this into (5) yields

$$V_1^- = S_{11}V_1^+ - \frac{S_{12}S_{21}}{1+S_{22}}V_1^+$$

so that

$$\boxed{\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}}} \quad (7)$$

This is the input reflection coefficient expression for a two-port terminated in a short circuit.

Substituting the numerical values for the S parameters in (3) we find $\Gamma = 0.633$, so that $\text{RL} = -20\log_{10}|\Gamma| = 3.97$ dB.

Again, it is crucial to realize that the input reflection coefficient of a two-port network is generally not S_{11}! In the above example, $S_{11} = 0.1$ while $\Gamma = 0.633$. Remember that $\Gamma = S_{11}$ **only** when all other ports are terminated in matched loads.

By similar reasoning, $S_{ij} = T_{ij}$, $i \neq j$, is valid only when all ports are matched.

From this discussion, we can surmise that as we vary load or source impedances connected to a two-port network, the *S* parameters of the network **do not change**. However, the reflection and transmission coefficients of waves on TLs connected to this two-port network **will** generally change.