Lecture 35: Antenna Effective Aperture. Friis Equation.

Up to this point in our discussion about antennas we have only discussed their transmitting characteristics. In a communication system, we need both transmitting and receiving antennas. There are many, many similarities between the transmitting and receiving properties of antennas, but there is one difference, which we'll discuss in this lecture.

Antenna Effective Aperture

In a receiving mode, antennas function as transducers to convert some of the energy in a passing electromagnetic wave into the motion of charges (i.e., a current) in an electrical circuit connected to the antenna.

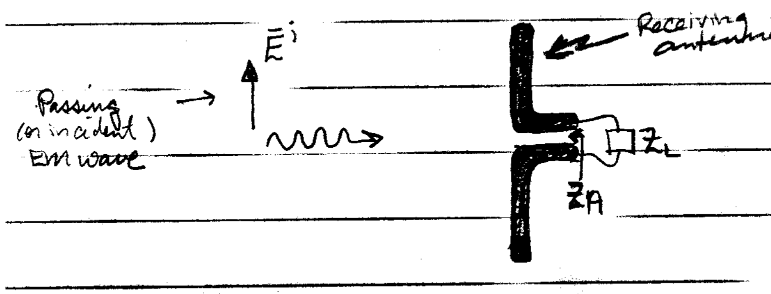


Fig. 1

We define the **maximum effective aperture** of an antenna, A_{em}, as the ratio of the maximum time average power (P_R) delivered to a conjugate matched load ($Z_L = Z_A^*$) connected to a lossless receiving antenna to the time average power density of an incident EM wave (linearly polarized UPW) illuminating the antenna, S_{av}^{i}:

$$A_{em} = \frac{P_R}{S_{av}^{i}} \quad [\text{m}^2] \tag{1}$$

(The adjective "maximum" in maximum effective aperture indicates the assumption there are no losses in the antenna.)

Alternatively, from (1)

$$P_R = A_{em} S_{av}^{i}$$

which states that the time average power delivered to a conjugate matched load connected to a lossless receiving antenna equals the power flow through an area equal to A_{em} of an incident UPW.

It can be shown (through a rather involved derivation found in many introductory antenna texts) that for any antenna, the maximum directivity D is related to the maximum effective aperture as

$$D = \frac{4\pi}{\lambda^2} A_{em} \tag{2}$$

In the case that there are losses in the antenna, the power delivered to the matched load is further reduced to the fraction e_r (i.e., the radiation efficiency) of what would have been received in the lossless situation according to

$$A_e = e_r A_{em} \tag{3}$$

where A_e is the effective aperture of the receiving antenna.

Using (3) in (2) and the definition of antenna gain ($G = e_r D$), we arrive at

$$G = \frac{4\pi}{\lambda^2} A_e \tag{4}$$

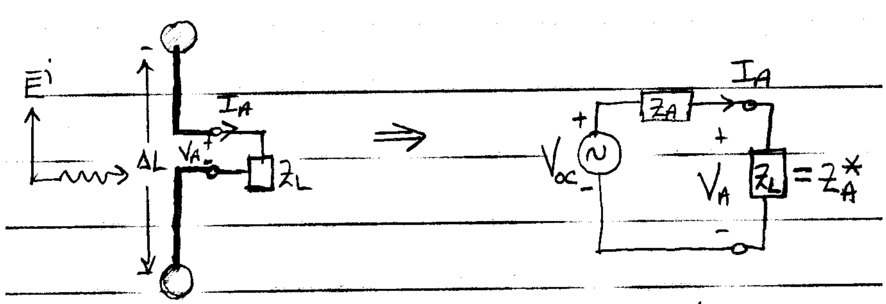
In words, this extremely important equation (4) states that the ratio of an antenna gain as a transmitter and its effective aperture area as a receiver is a constant (dependent on wavelength) for **any** antenna. Amazing!

Example N35.1. Verify equation (2) for a Hertzian dipole antenna.

Imagine a uniform plane wave (UPW) is incident on a Hertzian dipole antenna as shown in Fig. 1. Following the definition of A_{em} in (1), we first calculate S_{av}^i for the UPW

$$S_{av}^i = \frac{1}{2} \frac{\left|E^i\right|^2}{\eta} \tag{5}$$

To calculate P_R, we assume the antenna terminals are connected to a matched load:



Assuming the Hertzian dipole antenna is aligned with $\bar{E}^i$, then it can be shown that

$$V_{OC} = -E^i \cdot \Delta L \quad [\text{V}] \tag{6}$$

For a conjugate matched load ($Z_L = Z_A^*$), maximum time average power will be delivered to the load circuit attached to the antenna. Using voltage division in this circumstance

$$V_A = \frac{Z_L}{Z_L + Z_A} V_{OC} \underbrace{=}_{(6)} -\frac{Z_A^*}{Z_A^* + Z_A} E^i \Delta L$$

If $Z_A = R_A + jX_A$, then

$$V_A = -\frac{Z_A^*}{2R_A} E^i \Delta L \tag{7}$$

The time average power delivered to this load is then

$$\begin{aligned} P_R &= \frac{1}{2}\text{Re}\left(\frac{\left|V_A\right|^2}{Z_L^*}\right) \underbrace{=}_{(7)} \frac{1}{2}\text{Re}\left[\frac{Z_A Z_A^*}{Z_A \cdot 4R_A^2}\left|E^i\right|^2 (\Delta L)^2\right] \\ &= \frac{\left|E^i\right|^2 (\Delta L)^2}{8R_A} = \frac{\left|E^i\right|^2 (\Delta L)^2}{8R_r} \end{aligned} \tag{8}$$

where $R_A = R_r$ for a lossless antenna.

Substituting (5) and (8) into the definition of A_{em} in (1):

$$A_{em} = \frac{\dfrac{\left|E^i\right|^2 (\Delta L)^2}{8R_r}}{\dfrac{1}{2}\dfrac{\left|E^i\right|}{\eta}} = \frac{1}{4}\frac{\eta}{R_r}(\Delta L)^2 \qquad (9)$$

We solved for R_r in (20) of Lecture 33 to be

$$R_r = \frac{2\pi}{3}\eta\left(\frac{\Delta L}{\lambda}\right)^2 \qquad (10)$$

Substituting (10) into (9) we find

$$A_{em} = \frac{\eta(\Delta L)^2}{4}\frac{3}{2\pi\eta}\left(\frac{\lambda}{\Delta L}\right)^2 = \frac{3}{2}\frac{\lambda^2}{4\pi} \qquad (11)$$

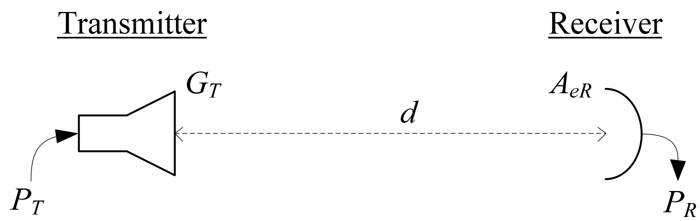
As we saw in Example N34.1, $D = 3/2$ for the Hertzian dipole antenna. Using this in (11), we find that

$$A_{em} = \frac{\lambda^2}{4\pi}D$$

which verifies (2) for the Hertzian dipole antenna.

The Friis Equation

We will now combine the use of both the transmitting and receiving properties of antennas. Antennas are often used in a wireless communication system, or often called a communication "link," as illustrated in the figure below. There is an important equation that is used to design such communication links, called the Friis equation. We imagine that two antennas are oriented towards each other for maximum transmitted and received power:



The Friis equation is simple to derive. First, if $P_T = P_{\text{in}}$ is the time average power accepted by (or delivered to) a transmitting antenna, we saw from equation (6) in Lecture 34 that the gain of the transmitting antenna G_T is defined as

$$G_T = \frac{S_{av}}{P_{\text{in}} \big/ \left(4\pi r^2\right)} \tag{12}$$

We can rearrange this equation (12) for an expression of the radiated time average power density, S_{av}, at the position of a receiving antenna as

$$S_{av} = \frac{G_T P_T}{4\pi d^2} \tag{13}$$

where d is the separation distance between the transmitting and receiving antennas.

The receiving antenna will capture some of this incident power density in (13) converting it to time average power delivered to a conjugate matched load, according to the definition of antenna aperture in (1) as

$$S_{av}^{i} = \frac{P_R}{A_{eR}} \tag{14}$$

where A_{eR} is the antenna aperture of the receiving antenna.

Assuming the receiving antenna is in the far field of the transmitting antenna, then the two S_{av}'s in (13) and (14) are the same.

Consequently, using (14) in (13) gives

$$P_R = P_T \frac{G_T A_{eR}}{4\pi d^2} \quad \text{[W]} \tag{15}$$

As we saw in (4), however, effective aperture and antenna gain are related to each other by the same factor for all antennas. Using (4) here in (15) gives

$$P_R = P_T \frac{G_T G_R \lambda^2}{(4\pi d)^2} \quad \text{[W]} \tag{16}$$

Equation (16) – and to a lesser extent (15) – is called the **Friis equation**. It can be used to design communication links, which we'll illustrate in the next example.

Example N35.2. Consider a direct broadcast satellite (DBS) television system typical of Dish Network or DirecTV. These typically use transmitting satellites with 120 W of radiated power, frequencies between 12.2 GHz and 12.7 GHz, and an EIRP of ~55 dBW in each 24-MHz transponder that handles several compressed digital video channels. The receiving system uses an 18” (0.46 m) diameter, offset-fed reflector antenna. What is the approximate receiver power delivered to a matched load?

We’ll use (16) for this calculate at a mid-band frequency of f = 12.45 GHz.

The Effective Isotropic Radiated Power (EIRP) is defined as

$$\boxed{\text{EIRP} = P_T G_T \quad [\text{W}]} \tag{17}$$

It is the same time average power in a certain direction that would be radiated by an isotropic radiator ($G_i = 1$) it if had an input power of $P_T G_T$.

For this example, $\text{EIRP} = 55$ dBW.

We can write the Friis equation in terms of dBW. Taking $10\log_{10}$ of (16) we obtain

$$P_R(\text{dB}) = P_T(\text{dB}) + G_T(\text{dB}) + G_R(\text{dB}) + 20\log_{10}(c_0) - 20\log_{10}(f) - 20\log_{10}(4\pi d)$$

or

$$P_R(\text{dB}) = P_T(\text{dB}) + G_T(\text{dB}) + G_R(\text{dB}) + 169.5$$
$$-20\log_{10}(f) - \underbrace{20\log_{10}(4\pi)}_{=21.98} - 20\log_{10}(d)$$

Therefore

$$\boxed{\begin{aligned} P_R(\text{dB}) = P_T(\text{dB}) + G_T(\text{dB}) + G_R(\text{dB}) - 20\log_{10}(f) \\ -20\log_{10}(d) + 147.6 \end{aligned}} \qquad (18)$$

In this example

$$P_T(\text{dBW}) = 10\log_{10}(120) = 20.8 \text{ dBW}$$

and using the definition of EIRP in (17)

$$G_T(\text{dB}) = \text{EIRP}(\text{dBW}) - P_T(\text{dBW}) = 55 - 20.8 = 34.2 \text{ dB}$$

For geosynchronous orbit, a typical "slant path" distance is $d = 38{,}000$ km.

From (4), $$G_R = \frac{4\pi}{\lambda^2} A_{eR}$$

For this parabolic dish receiver antenna and its very large electrical size (~20 wavelengths in diameter), A_{eR} is approximately 70% of the physical aperture area of the dish. Therefore, assuming a circular reflector antenna

$$G_R = \frac{4\pi}{0.024^2} 0.7\left[\pi\left(\frac{0.46}{2}\right)^2\right] = 2{,}538 = 34.0 \text{ dB}$$

Using these values in (18):

$$P_R(\text{dB}) = 20.8 + 34.2 + 34.0 - \underbrace{20\log_{10}\left(12.5\times 10^{9}\right)}_{=201.9}$$

$$-\underbrace{20\log_{10}\left(38{,}000\times 10^{3}\right)}_{=151.6} + 147.6$$

or $P_R(\text{dB}) = -116.9 \text{ dBW} \Rightarrow P_R = 2.04\times 10^{-12} \text{ W}$.

This is a very small power! Without the combined gains of the two antennas (~68.2 dB = 34.2 dB + 34.0 dB) this received signal would be hopelessly lost in noise.

This example illustrates one use of the Friis equation: What antenna gains are required for a communication link? There are other applications, of course.