

## Lecture 34: Antenna Radiation Patterns. Directivity and Gain.

No physical antenna radiates uniformly in all directions. Rather, **antennas radiate EM waves better in certain directions than others**. In fact, many antennas will not radiate in some directions at all.

A **radiation pattern** is a plot that shows the **relative far field** strength of  $\bar{E}$  (or  $\bar{H}$ ) **versus angular direction** ( $\theta, \phi$ ) at a fixed distance from the antenna (again, in the far field). It is often a polar plot.

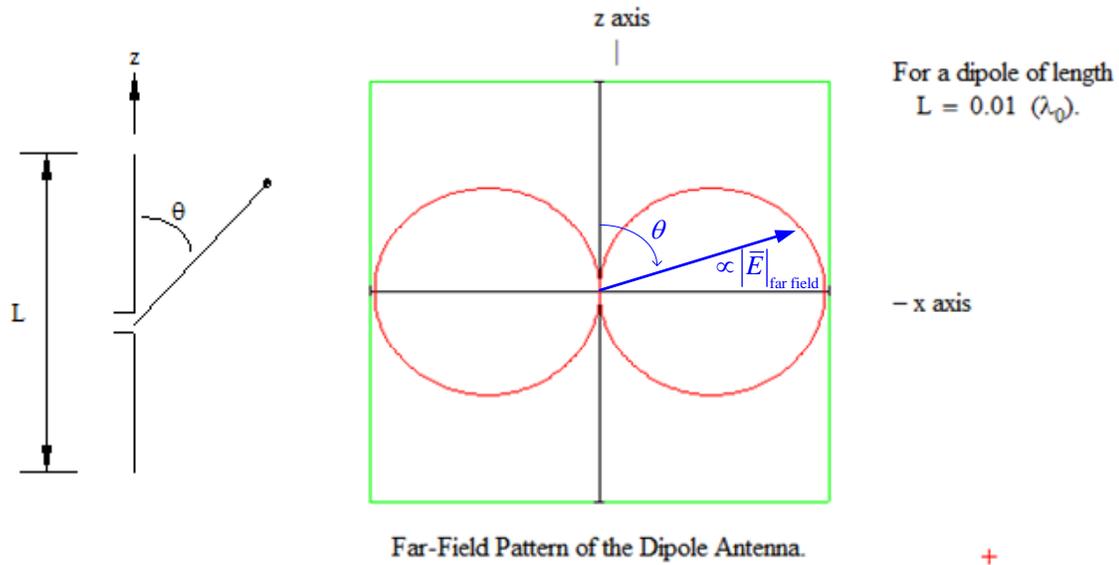
We'll take the Hertzian dipole antenna as an example. From equation (12) in the previous lecture we found in the far field that

$$\bar{E} \approx \hat{a}_\theta j \frac{I \Delta L}{4\pi} \beta \eta \sin \theta \frac{e^{-j\beta r}}{r} \quad [\text{V/m}] \quad (\beta r \gg 1) \quad (1)$$

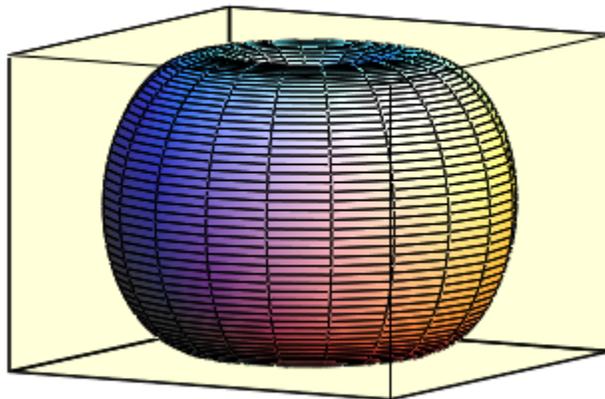
The magnitude of this phasor electric field is

$$|E_\theta| \approx \frac{|I| \Delta L}{4\pi} \beta \eta \frac{|\sin \theta|}{r} \quad (\beta r \gg 1) \quad (2)$$

What is of interest in an antenna radiation pattern is the variation of  $|\bar{E}|$  in  $\theta$  and  $\phi$  at a fixed  $r$ , which is in the far field of the antenna. In the case of the Hertzian dipole antenna in (2), this variation is simply  $|\sin \theta|$ :



Radiation Pattern of the Dipole Antenna.



For a dipole of length  
 $L = 0.01 (\lambda_0)$ .

+

This radiation pattern indicates a number of characteristics of the Hertzian dipole antenna:

1. Maximum radiation is at  $\theta = 90^\circ$ . This is called **broadside** radiation.
2. There is no variation in  $\phi$ , as expected.
3. There is **no radiation in directions along the ends of the antenna** ( $\theta = 0, 180^\circ$ ).

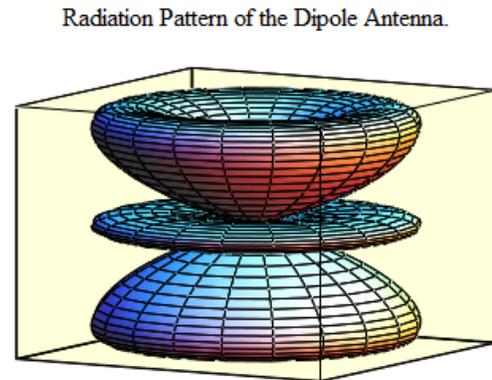
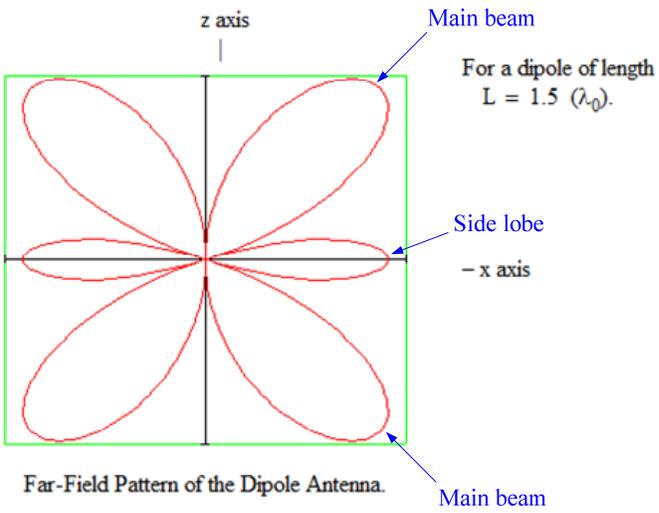
## Radiation Patterns of Longer Dipole Antennas

The radiation patterns of dipole antennas **get far more interesting as their electrical length increases**. We're only considering the electrically short Hertzian dipole antenna in this course, but we'll show a couple of radiation patterns for two other dipole antenna lengths without proof.

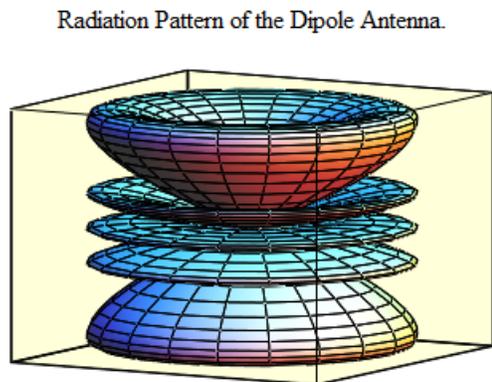
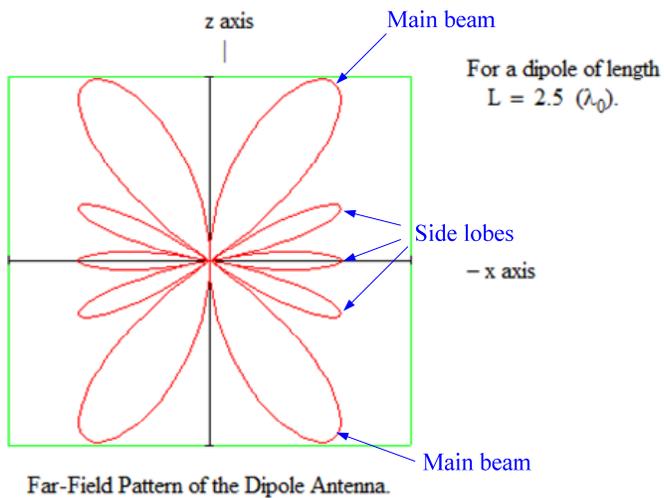
Dipole antenna radiation patterns stay roughly the same shape as the Hertzian dipole up to lengths  $\sim 0.5 \lambda$ . The main beam then sharpens for lengths from  $\sim 0.5 \lambda$  to  $1 \lambda$ . There is only one beam of radiation for these lengths. There are no so-called side lobes. (You can experiment with this yourself by adjusting the dipole antenna length  $L$  in the *VisualEM* worksheet "Radiation Pattern of a Dipole Antenna.")

For **lengths longer than  $1 \lambda$** , the radiation patterns change considerably with the addition of multiple main beams and the appearance of **side lobes**:

- $L = 1.5 \lambda$ :



- $L = 2.5 \lambda$ :



(Beamwidth? E plane, H plane?)

## Directivity and Gain

An important characteristic of an antenna is its ability to focus radiated power in a given direction in the far field.

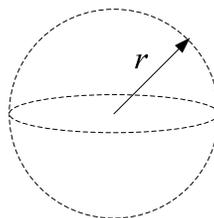
The **directivity**  $D(\theta, \phi)$  of an antenna is the ratio of the time average power density radiated in the  $(\theta, \phi)$  direction at some distance in the far field of the antenna to the time average power density at this same point if the total power were radiated “isotropically” (i.e., equally in all directions).

Mathematically, directivity is defined as

$$D(\theta, \phi) = \frac{\bar{S}_{av}(\theta, \phi) \cdot \hat{a}_r}{S_{av, \text{isotropic}}} \quad (3)$$

While there is **no such thing** as a physical antenna that radiates equally well in all directions (i.e., isotropically), we use this artifice as a normalization quantity in the denominator of (3) to assess the focusing ability of a given antenna.

If a **total power**  $P_{rad}$  is radiated isotropically by an antenna, then the time average power density anywhere on an imaginary sphere of radius  $r$



is

$$S_{av, \text{isotropic}} = \frac{P_{rad}}{4\pi r^2} \quad [\text{W/m}^2] \quad (4)$$

Using (4) in (3) along with the definition of  $\bar{S}_{av}$ , then (3) becomes

$$D(\theta, \phi) = \frac{1/2 \text{Re}[\bar{E}(\theta, \phi) \times \bar{H}^*(\theta, \phi)] \cdot \hat{a}_r}{P_{rad}/(4\pi r^2)} \quad (5)$$

The **gain** of an antenna is defined similarly as directivity. In particular, the gain  $G(\theta, \phi)$  of antenna is the ratio of the time average power density radiated to some point in the far field of an antenna to the **total time average power accepted by the antenna,  $P_{in}$** , when radiated isotropically.

Mathematically, antenna gain is defined as

$$G(\theta, \phi) = \frac{\bar{S}_{av} \cdot \hat{a}_r}{P_{in}/(4\pi r^2)} \quad (6)$$

The radiation efficiency  $e_r$  of an antenna is defined as

$$e_r \equiv \frac{P_{rad}}{P_{in}} \quad (7)$$

Using this in (6) and comparing to (5) we find that

$$G(\theta, \phi) = e_r D(\theta, \phi) \quad (8)$$

In short, the **gain  $G(\theta, \phi)$**  of antenna includes the effects of **losses in the antenna** (and other surrounding structures), if

present. The directivity of an antenna is determined solely by the pattern shape of the antenna.

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**Example N34.1.** Calculate the directivity of a Hertzian dipole antenna of length  $\Delta L$  with current  $\hat{a}_z I$  A. If the antenna is made of steel, is 62'' long, and 1/8'' in diameter, and is operating at 1 MHz, calculate the antenna gain.

In the far field of this antenna, from (15) in the previous lecture,

$$\bar{S}_{av} = \hat{a}_r \frac{1}{2} \frac{|E_\theta|^2}{\eta} \quad [\text{W/m}^2] \quad (9)$$

Substituting (2) into (9)

$$\bar{S}_{av} = \hat{a}_r \frac{\beta^2 \eta}{32\pi^2} |I|^2 (\Delta L)^2 \frac{\sin^2 \theta}{r^2} \quad (\beta r \gg 1) \quad (10)$$

The total power radiated by this antenna is found by integrating this power density over a closed surface, the simplest of which is an imaginary sphere centered on the dipole antenna [this was done earlier in (17) of Lecture 33]:

$$\begin{aligned}
 P_{rad} &= \oint_s \bar{S}_{av} \cdot d\bar{s} \\
 &\stackrel{(10)}{=} \int_0^{2\pi} \int_0^\pi \left( \hat{a}_r \frac{\beta^2 \eta}{32\pi^2} |I|^2 (\Delta L)^2 \frac{\sin^2 \theta}{r^2} \right) \cdot (\hat{a}_r r^2 \sin \theta d\theta d\phi) \\
 &= \frac{\beta^2 \eta}{32\pi^2} |I|^2 (\Delta L)^2 \underbrace{\int_0^{2\pi} \int_0^\pi \sin^3 \theta d\theta d\phi}_{2\pi \cdot \frac{4}{3}}
 \end{aligned}$$

Therefore,

$$P_{rad} = \frac{\beta^2 \eta}{12\pi} |I|^2 (\Delta L)^2 \quad [\text{W}] \quad (11)$$

Substituting (10) and (11) into the definition of directivity in (3), and using (4) gives

$$\begin{aligned}
 D(\theta, \phi) &= 4\pi r^2 \frac{\bar{S}_{av} \cdot \hat{a}_r}{P_{rad}} = 4\pi r^2 \frac{\frac{\beta^2 \eta}{32\pi^2} |I|^2 (\Delta L)^2 \frac{\sin^2 \theta}{r^2}}{\frac{\beta^2 \eta}{12\pi} |I|^2 (\Delta L)^2} \\
 &= 4\pi \frac{12\pi}{32\pi^2} \sin^2 \theta = \frac{3}{2} \sin^2 \theta
 \end{aligned}$$

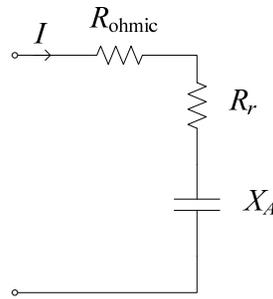
Consequently,

$$D(\theta, \phi) = \frac{3}{2} \sin^2 \theta$$

The **maximum directivity**,  $D$ , occurs for this antenna when  $\theta = 90^\circ$  at which

$$D = \frac{3}{2} \quad \text{or} \quad D_{dB} = 10 \log_{10} D = 1.76 \text{ dB}$$

To calculate the gain of the antenna, we first must compute its radiation efficiency. The efficiency of this antenna was computed previously in Example N33.1. As we saw in that example, the equivalent input circuit at the terminals of the Hertzian dipole antenna is



The total time average power delivered to the input terminals of this antenna is

$$P_{\text{in}} = \frac{1}{2}|I|^2 R_{\text{ohmic}} + \frac{1}{2}|I|^2 R_r \quad (12)$$

while the total radiated time average power is

$$P_{\text{rad}} = \frac{1}{2}|I|^2 R_r \quad (13)$$

Substituting (12) and (13) into the definition of radiation efficiency (7) gives

$$e_r = \frac{P_{\text{rad}}}{P_{\text{in}}} = \frac{\frac{1}{2}|I|^2 R_r}{\frac{1}{2}|I|^2 R_r + \frac{1}{2}|I|^2 R_{\text{ohmic}}} = \frac{R_r}{R_r + R_{\text{ohmic}}} \quad (14)$$

It was this last equation for  $e_r$  that was used previously in Example N33.1. There it was found that  $e_r = 8.95\%$  for this steel antenna.

Using (8) we find that

$$G(\theta, \phi) = 0.0895 \cdot 1.5 \cdot \sin^2 \theta = 0.134 \sin^2 \theta$$

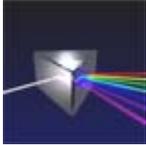
and the **maximum gain**,  $G$ , occurs for this antenna at  $\theta = 90^\circ$  where

$$G = 0.134 \quad \text{or} \quad G_{dB} = 10 \log_{10} G = -8.72 \text{ dB}$$

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## Conclusion

To conclude, it is important to realize that there is **no real “gain” associated with such antennas**. These are made of metal and are completely passive devices. There is no signal amplification. The “gain” that has been defined in this lecture refers to the **focusing properties of the antenna above that of a lossless, isotropic radiator**.



## Section 9.3 and Problem 9.3.6



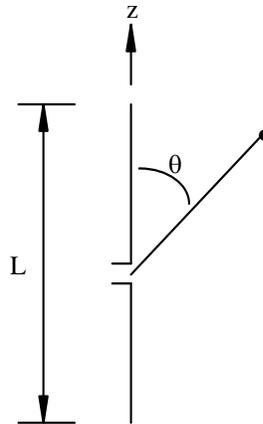
## Radiation Pattern of a Dipole Antenna

### Purpose

To compute and visualize the radiation pattern of linear dipole antennas. These radiation patterns are viewed in both two- and three-dimensional plots. A line plot of the radiation pattern is provided in order to accurately measure those  $\theta$  angles at which the far-zone electric field is zero.

### Enter parameters

The geometry of the linear dipole antenna is shown in Fig. 9.12 of the text and in the figure below:



Choose the length of the dipole antenna in units of free-space wavelengths:

$$L := 1.5 \quad \text{Length of the dipole } (\lambda_0).$$

In this worksheet, we will plot the radiation pattern produced by this linear dipole antenna. The variation of the *current* on this dipole antenna can be viewed in other worksheets. For example, the approximate current distribution is shown in the [radiation resistance](#) worksheet while a more accurate current distribution is shown in the [numerical analysis](#) worksheet.

### Background

The electric field in the far-zone region of a dipole antenna was found in Equation (66b) of Chap. 9 to be:

$$E_{\theta} = j\eta_0 I_m \frac{e^{-j\beta_0 r}}{2\pi r} F(\theta)$$

This far-zone electric field is independent of the spherical coordinate  $\phi$  due to the symmetry of the dipole current and the z-directed orientation of the dipole. Plotting the *radiation pattern* (or, equivalently, the far-zone electric field pattern) of this dipole antenna means to plot the angular variation of  $E_{\theta}$  and/or  $E_{\phi}$  at some fixed radius  $r$  away from the dipole. In the case of a dipole antenna,  $E_{\phi}$  is zero if the antenna is orientated as shown in the figure above.

Consequently, to construct the radiation pattern we need only plot the *pattern factor*  $F(\theta)$  which is given in Equation (65) of Chap. 9 of the text to be:

$$\beta_0 := 2 \cdot \pi$$

$$F(\theta) := \text{if} \left( \sin(\theta) \neq 0, \frac{\cos\left(\frac{\beta_0 \cdot L}{2} \cdot \cos(\theta)\right) - \cos\left(\frac{\beta_0 \cdot L}{2}\right)}{\sin(\theta)}, 0 \right)$$

It can be shown that  $F(\theta)$  goes to zero when  $\sin(\theta) = 0$  as is used in this definition of  $F(\theta)$ .

### Plot radiation pattern in the $y = 0$ plane

For the remainder of the worksheet, we will construct a number of plots to show the far-field pattern produced by this dipole antenna. We will first consider the far-field pattern in the  $y = 0$  plane ( $\phi = 0$  and  $180^\circ$ ).

Choose the number of points to plot the radiation pattern as a function of  $\theta$ :

$$\text{npts} := 359 \quad \text{Number of points to plot in } \theta.$$

$$\theta_{\text{start}} := 0 \quad \theta_{\text{end}} := 360 \quad \theta \text{ starting and ending angles } (^\circ).$$

Construct a list of  $\theta_{p_i}$  values at which to plot the pattern:

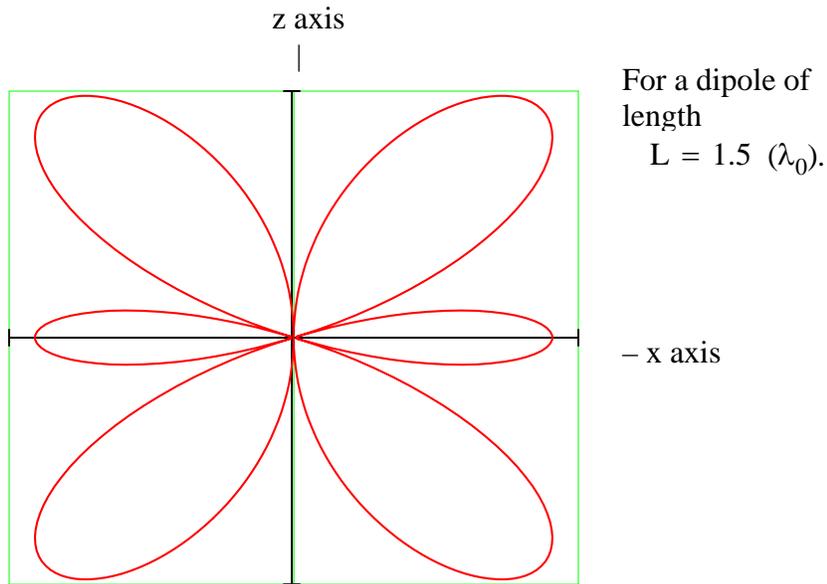
$$i := 0 \dots \text{npts} - 1 \quad \theta_{p_i} := \left( \theta_{\text{start}} + i \cdot \frac{\theta_{\text{end}} - \theta_{\text{start}}}{\text{npts} - 1} \right) \cdot \text{deg}$$

Note that while the spherical coordinate  $\theta$  varies only from  $0$  to  $180^\circ$ , here we let  $\theta$  sweep from  $0$  to  $360^\circ$  in order to simultaneously plot the radiation pattern for both  $\phi = 0$  and  $180^\circ$ . This is simply a trick that yields the same results as letting  $\theta$  vary from  $0$  to  $180^\circ$  with  $\phi$  separately equal to  $0$  and then  $180^\circ$ .

Now compute the radiation pattern:

$$F_{x_i} := \left| F(\theta_{p_i}) \right| \cdot \sin(\theta_{p_i}) \quad F_{z_i} := \left| F(\theta_{p_i}) \right| \cdot \cos(\theta_{p_i})$$

Plot the radiation pattern in the  $y = 0$  plane:

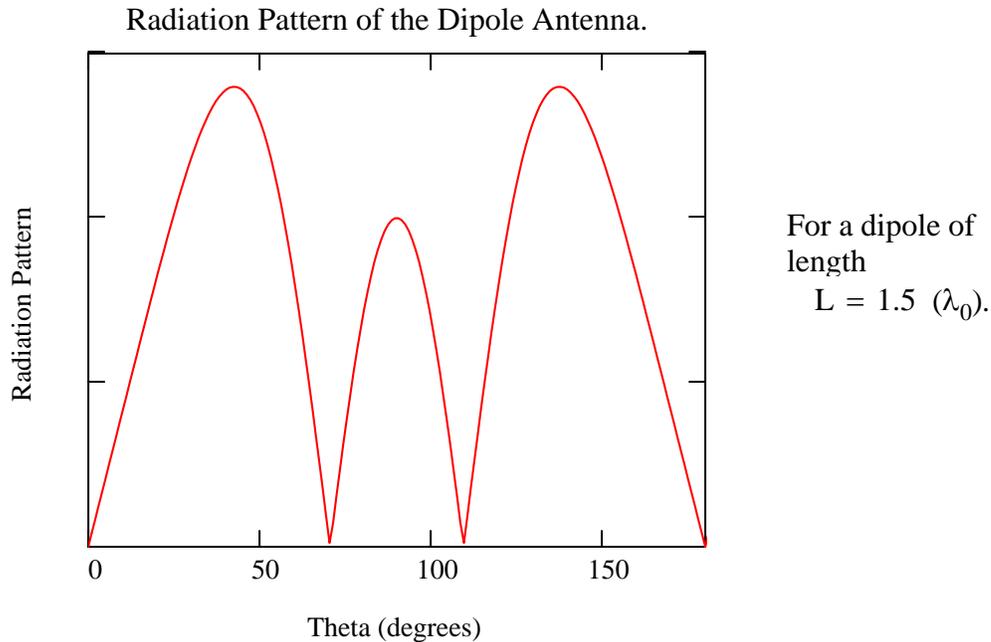


Far-Field Pattern of the Dipole Antenna.

The dipole antenna can be pictured as situated at the origin of this plot and orientated vertically. Regardless of the length of the dipole antenna,  $L$ , you will observe that this far-field pattern of the electric field has *nulls* at  $\theta = 0$  and  $\theta = 180^\circ$ . As requested in the Prob. 9.3.6, this fact can be proven directly from the far-electric field expression given in (66b) of the text. For clarification, a **null** in the electric field pattern means that an observer far from the dipole would measure a negligibly small electric field from the antenna in this particular direction regardless of the orientation of any receiving antenna and for all time.

You can verify – both with this worksheet and analytically using (66b) – that when the length of the dipole is less than  $1 \lambda_0$ , an observer would measure no far-zone electric field from this antenna only when viewing the dipole from either of its ends. However, as you can observe for yourself by changing the length of the dipole,  $L$ , there will be nulls in the electric field pattern at other observation angles if the length of the dipole is greater than  $1 \lambda_0$ .

It is requested in Prob. 9.3.6 that a mathematical expression be obtained for all  $\theta$  angles where a null occurs in the radiation pattern of long dipole antennas. To assist in confirming your predictions of these angles where a null occurs, a line plot of the radiation pattern is shown below:



You can accurately measure directly from this plot those  $\theta$  angles where the radiation pattern vanishes, i.e., the nulls of the radiation pattern. 

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### Three-dimensional radiation pattern

To help appreciate the three-dimensional variation of this far-zone electric field pattern, we will present it in the form of a surface plot. This surface is constructed by simply rotating the polar plot of the radiation pattern (shown in the first plot above) around the  $z$  axis (which is the the vertical axis in that plot):

$n_\phi := 20$       Number of points to plot in the  $\phi$  direction.

$n_\theta := 100$       Number of points to plot in the  $\theta$  direction.

Compute the pattern function  $F(\theta)$  at the specified  $\phi$  and  $\theta$  angles:

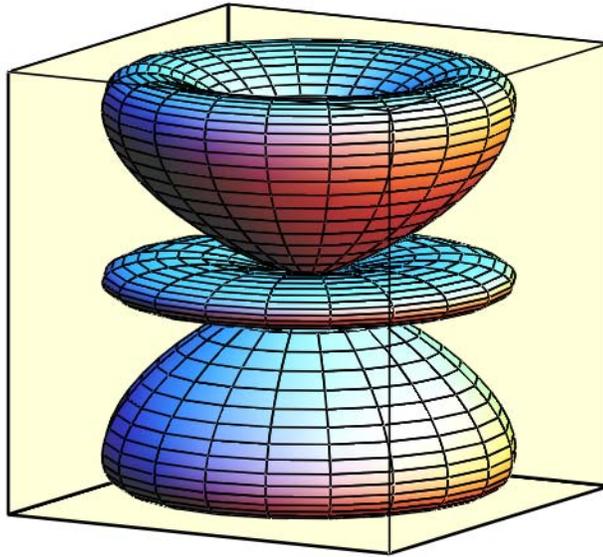
$$j := 0..n_\phi \quad \phi_j := j \cdot \frac{2 \cdot \pi}{n_\phi} \quad k := 0..n_\theta \quad \theta_k := k \cdot \frac{\pi}{n_\theta}$$

$$X_{j,k} := |F(\theta_k)| \cdot \sin(\theta_k) \cdot \cos(\phi_j) \quad Y_{j,k} := |F(\theta_k)| \cdot \sin(\theta_k) \cdot \sin(\phi_j)$$

$$Z_{j,k} := |F(\theta_k)| \cdot \cos(\theta_k)$$

Plot the radiation pattern:

## Radiation Pattern of the Dipole Antenna.



For a dipole of length  
 $L = 1.5 (\lambda_0)$ .

(X, Y, Z)

The dipole can be pictured as located at the center of this plot and orientated vertically as before. Of course, this radiation pattern is shown for distances very far away from the dipole and, consequently, the dipole is essentially a point located at the origin of this (and any other) radiation pattern plot.

We can see in this plot that the radiation pattern of such linear dipoles is invariant with the spherical  $\phi$  coordinate due to the symmetry of the dipole and its current. However, there is generally a great deal of variation of this pattern with the spherical  $\theta$  coordinate. You may wish to experiment further with this worksheet by varying the dipole antenna length,  $L$ , from small values ( $< 0.5 \lambda_0$ ) to larger values ( $> 1 \lambda_0$ ).

Alternatively, we will construct a "cutaway" view of this radiation pattern which may help resolve the side lobes of this pattern for long dipole antennas:

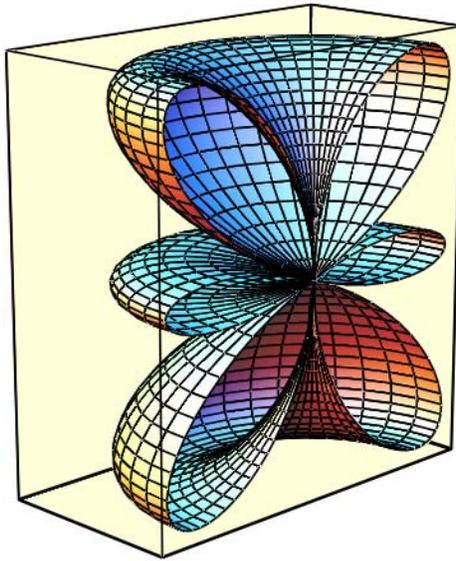
$$j := 0..n_\phi \quad \phi_j := j \cdot \frac{\pi}{n_\phi} \quad k := 0..n_\theta \quad \theta_k := k \cdot \frac{\pi}{n_\theta}$$

$$X_{j,k} := |F(\theta_k)| \cdot \sin(\theta_k) \cdot \cos(\phi_j) \quad Y_{j,k} := |F(\theta_k)| \cdot \sin(\phi_j) \cdot \sin(\theta_k)$$

$$Z_{j,k} := |F(\theta_k)| \cdot \cos(\theta_k)$$

Plot the "cutaway" view of the radiation pattern from  $\phi = 0$  to  $180^\circ$ :

Radiation Pattern of the Dipole Antenna.



For a dipole of  
length  
 $L = 1.5 (\lambda_0)$ .

(X, Y, Z)

Again, the dipole can be pictured as located at the center of this plot and is orientated vertically.

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End of worksheet.

